# FORECASTING FOR THE ORDERING AND STOCK-HOLDING OF CONSUMABLE SPARE PARTS

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# ABSTRACT

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A modern military organisation like the Royal Air Force (RAF) is dependent on readily available spare parts for in-service aircraft and ground systems in order to maximise operational capability. Parts consumed in use, or otherwise not economically repaired, are classified as consumable, comprising nearly 700 thousand stock-keeping units. A large proportion of parts with erratic or slow-moving demand present particular problems as far as forecasting and inventory control are concerned. This research uses extensive demand and replenishment lead-time data to assess the practical value of models put forward in the academic literature for addressing these problems.

An analytical method for classifying parts by demand pattern is extended and applied to the RAF consumable inventory. This classification allows an evaluation of subsequent results across a range of identified demand patterns, including smooth, slow-moving and erratic. For a model to be considered useful it should measurably improve forecasting and inventory control and, given the large inventory, should not be overly complex as to require excessive processing. In addition, a model should not be too specialised in case it has a detrimental effect when demand does not adhere to a specific pattern.

Recent forecasting developments are compared against more commonly used, albeit less sophisticated, forecasting methods with the performance assessed using traditional measures of accuracy, such as MAD, RMSE and MAPE. The results are not considered ideal in this instance, as the measures themselves are open to questions of validity and different conclusions arise depending on which measure is utilised. As an alternative the implied stock-holdings, resulting from the use of each method, are compared. One recently developed method, a modification to Croston's method referred to as the approximation method, is observed to provide significant reductions in the value of the stock-holdings required to attain a specified service level for all demand patterns.

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# DECLARATION

I declare that this thesis comprises entirely my own work and has not been submitted for the award of a higher degree elsewhere, nor have any sections been published elsewhere.

Andrew Eaves

***"For want of a nail the shoe was lost;***
***for want of a shoe the horse was lost;***
***and for want of a horse the rider was lost."***

**Benjamin Franklin (1706-1790)**

# 1. INTRODUCTION

This introductory chapter describes the motivation behind my research by placing the study in a business context and continues by defining the purpose of the analysis with my aims and objectives. The third section describes the contribution of my research and the final section outlines the thesis structure.

## 1.1 Business Context

A modern military organisation like the Royal Air Force (RAF) is dependent on readily available spare parts for in-service aircraft and ground systems in order to maximise operational capability. The RAF has one of the largest and most diverse inventories in the western world, probably second only to the United States Air Force in size.

Within the RAF, those line items consumed in use or otherwise not economically repaired, such as resistors, screws and canopies, are classed as consumable, while the generally more expensive line items, such as airframes, panels and gearboxes, are classed as repairable. At the beginning of 2000 the RAF managed approximately 684,000 consumable line items or stock-keeping units, leading to 145 million units of stock with a total value of £2.1 billion.

With reductions in defence budgets and the necessity for cost-efficiencies as directed by measures such as the Strategic Defence Review, the large investment in consumable stock makes inventory management a prime candidate for perceived cost savings. Thus, there is a requirement for a reasoned and scientific analysis of the properties of the RAF consumable inventory as an aid to obtaining efficiencies in the supply environment.

Due to their requirement primarily as spare parts only 48 percent of consumable line items have been required in the previous 24-month period, and are therefore formally defined as active. Furthermore, a large proportion of the inventory is described as having an erratic or intermittent demand pattern, which is characterised by infrequent transactions with variable demand sizes. This erratic demand can create significant problems as far as forecasting and inventory control are concerned.

The RAF is fortunate in having long demand transaction histories for all line items in a readily accessible format. Each line has electronic records providing at least 8 years of individual demand transactions (though less if the line has been introduced more recently). Each record, along with the line identifier, provides the demand date, the quantity of units required, details of the RAF station placing the demand, a code for the urgency of the requirement, and a code as to how the demand was satisfied - whether it be off the shelf, a diversion order or an inability that must be back-ordered.

This extensive demand information, often uncommon in industry, provides an opportunity for a full and detailed analysis across a range of demand patterns. The primary focus of this study is the examination of forecasting methods suitable for the ordering and stock-holding of spare parts, with reference to the RAF consumable inventory. Particular emphasis is placed on line items with erratic demand.

## 1.2 Aims and Objectives

The aim of this research is to identify and assess the practical value of models designed to improve demand forecasting with regards to the ordering and stock-holding of consumable spare parts. This has led to a review of existing models put forward in the literature as suitable for the task, including some recent developments. The recent models are compared with more general models that have proved popular in the demand forecasting environment, including exponential smoothing. All appraisals are undertaken using actual demand data.

For the models to be considered useful they should measurably improve forecasting and inventory control, and satisfy two further criteria:

(i) They should not be overly complex and require unrealistic processing power. This is an important consideration when dealing with an inventory as large as that held by the RAF.

(ii) The models should not be too specialised that they have a detrimental effect when demand does not adhere to a specific pattern. Ideally the models would be applicable to a broad band of demand patterns across a range of industries.

The large number of line items with complete demand histories in the RAF inventory allows a detailed analysis across a range of demand patterns including smooth, slow-moving, irregular and erratic. Armstrong and Collopy [4] make a general observation that, "*the accuracy of various forecasting methods typically require comparisons across many time series. However, it is often difficult to obtain a large number of series. This is particularly a problem when trying to specify the best method for a well-defined set of conditions; the more specific the conditions, the greater the difficulty in obtaining many series*." Therefore, it is often the case that the small quantities of demand data available to the researcher are insufficient to yield conclusive results. This is particularly the case where the analysis is performed on a specific demand pattern, such as erratic or slow-moving demand. A major contribution of this research stems from the fact that a large quantity of realistic demand data has been used in the analysis and there is little need to generate simulated data.

A further beneficial aspect concerning the information held by the RAF lies in the disaggregated nature of the data. The recorded demand transactions allow a full analysis at the individual demand level, or at any aggregation level such as daily, weekly, monthly or quarterly. Comparing the models at differing levels of aggregation allows a comparison of their performance under various scenarios, which may assist other organisations in selecting the model that best suits the format of their data. Greater storage availability is allowing many organisations to maintain more detailed demand information, thus allowing more options for implementation.

One aim of my research is to produce results that are meaningful in the real world. Therefore, the effectiveness of the models is measured by a means appropriate to their actual implementation. For example, given that the purpose behind demand forecasting is to determine requirements over a replenishment lead-time, the performance of the various forecasting methods is measured over the lead-time period, alongside the more conventional one-period ahead comparison.

An over-riding aim, however, is to identify a means for increasing the performance of the RAF inventory at a constant or even reduced cost and in doing so provide an opportunity to increase aircraft availability. An important part of this research focuses on establishing the additional value of the implied stock-holding requirements under each forecasting model.

## 1.3 Contribution of Research

In the course of my research I have developed a number of techniques and models, some of which are applicable only to the RAF and others that have a wider application. The RAF has a unique operating environment, albeit with some similarities to other military organisations, and the large consumable inventory, both in terms of the number of stock-keeping units and the units of stock on-hand, provides scope for a valuable research contribution from a large scale analysis.

An over-riding contribution arises through the analysis of large quantities of real-world spare parts related data. The methodical collection and retention of data by the RAF allows a detailed analysis of the factors affecting the management of consumable line items. For instance, demand data is maintained at the individual transaction level, thereby allowing an analysis at any aggregation level including quarterly, monthly and weekly. As different demand aggregations lead to different performance results, the comparative performance is investigated and commented upon. Actual replenishment lead-time data, which is rarely available in significant quantities for spare parts, is analysed and incorporated within this research.

One model developed with widespread application is a modified chi-square goodness-of-fit testing method, called GOODFIT, within Microsoft Excel. Problems often arise with the standard chi-square test due to the requirement for data to be grouped to ensure each category contains at least five observations. As groupings are somewhat arbitrary it is frequently observed that one grouping methodology will accept the null hypothesis, whereas another grouping will not. GOODFIT differs in that boundaries are specified by forming categories with similar theoretical frequencies throughout, rather than combining groups just at the margins. Under the modified rules, the sum of the probabilities within each grouping will be equalised to the greatest extent possible. The aim is to provide a consistently fair method of automatically grouping observations across a range of probability distributions. Most published models for the management of spare parts assume each of the components of interest, including demand size, transaction interval and lead-time, follow specified probability distributions. The GOODFIT model allows complete testing of the validity of these assumptions using actual data.

Less frequent usage of erratic demand items means that, in general, few replenishment orders are placed and, combined with long lead-times in the defence industry, very little lead-time data is available. The few actual lead-time observations for each line item restricts the usefulness of the data on an individual item basis. To overcome this problem I have developed a methodology for combining line items likely to have a similar lead-time pattern and calculated aggregate statistics that apply to the entire grouping. ANOVA analysis was used initially to select three of the seven candidates identified as potential categorisation variables, a cluster analysis then placed the lead-time observations into six groupings for each variable. Any line item, regardless of whether or not it has a replenishment history, can now be assigned parameter values according to its lead-time grouping location.

A published analytical method for classifying demand as smooth, slow-moving, erratic or erratic with a highly variable lead-time has been tailored and extended for the RAF consumable inventory. The method decomposes the variance of the lead-time demand into its constituent causal parts and defines boundaries between the demand patterns. Classifying demand in this manner allows all further analysis to be compared between the identified demand patterns.

I have written a model called FORESTOC using SAS® software to compare the accuracy of various forecasting methods with RAF demand data. Individual demand transactions have been combined to give quarterly, monthly and weekly demand totals over a six-year period. A one year period is used to initialise the forecasts, beyond which point the forecast value is compared with the actual demand over a series of forward-looking lead-time periods. This is a methodology rarely used, with one-period ahead forecast comparisons being the norm. In a real setting it is the demand over a lead-time period that must be catered for, and therefore the forecast methods are measured against this.

One of the selected forecasting methods is the relatively well-known Croston's method, which separately applies exponential smoothing to the interval between demands and the size of the demands. All observed implementations to date use the same smoothing value for both series, although the two series themselves are assumed to be independent. This research identifies and uses two different smoothing values, which, in combination, provide optimal results across a hold-out sample. The effect of the selected smoothing constants is also examined.

A range of standard statistics for measuring forecast errors are calculated and contrasts are made between the identified demand patterns. Two methods of implementing the forecast measures are utilised:

(i) Measuring the errors observed at *every* point in time.

(ii) Only measuring immediately *after a demand* has occurred.

The first implementation is perhaps the more traditional measure, although the second implementation also has a case for consideration, as it is only after a demand has occurred that it would be necessary to initiate a new replenishment order. Thus, it is of greater importance that a particular forecasting method be accurate after a demand has occurred, rather than at every point in time and, therefore, the methods are assessed on this basis.

Weaknesses are identified in using the traditional measures of forecasting accuracy, such as Mean Absolute Deviation and Mean Absolute Percentage Error, and an alternative measure is investigated. The FORESTOC model has been extended to allow a comparison of the implied stock-holdings between the methods using back-simulation. A common basis is achieved by calculating the precise safety margin that provides a maximum stock-out quantity of zero for each method. The safety margin is calculated by iteratively adding the maximum stock-out quantity to the order-up-to level until no further stock-outs occur. Difficulties arise on occasions where the initial stock is too high, such that no reorders are required over the forecasting horizon, or the initial stock is not enough to prevent a stock-out before the first delivery, and restrictions are required. Again, the stock-holdings and safety margins can be compared across the previously identified range of demand patterns.

## 1.4 Thesis Structure

The thesis is divided into nine further chapters, namely:

(i) Demand for Consumable Spare Parts.

(ii) Review of the Literature.

(iii) Characteristics of the RAF Inventory.

(iv) Lead-Time Analysis.

(v) Demand Classification.

(vi) Forecasting Erratic Demand.

(vii) Alternative Forecasting Methods.

(viii) Forecast Performance by Implied Stock-Holding.

(ix) Conclusions.

Given the large quantity of data used throughout this research, Appendix A provides a data usage summary that describes the creation and usage of each dataset, together with the number of line items used for each part of the analysis.

### 1.4.1 Demand for Consumable Spare Parts

Demand for consumable spare parts tends to be divided into two categories, namely erratic and slow-moving. Both patterns are characterised by infrequent demand transactions. While an erratic demand pattern is determined by variable demand sizes, a slow-moving demand pattern is distinguished by low demand sizes.

This chapter introduces the notion of erratic demand and slow-moving demand and examines the causes behind these patterns. Formal means for identifying whether erratic and slow-moving demand patterns are present within an inventory are also investigated.

A published method for classifying demand as smooth, slow-moving, or sporadic (yet another term for erratic) is introduced in this chapter. The method decomposes the variance of the lead-time demand into its constituent causal parts and defines boundaries between the demand patterns. Classifying demand in this manner allows subsequent analyses to be compared between the identified patterns.

### 1.4.2 Review of the Literature

In this chapter a review of the academic literature focusing on the forecasting, ordering and stock-holding of consumable spare parts is undertaken. This review explores the development of techniques for managing erratic and slow-moving demand in particular, and examines the practical application of these techniques.

### 1.4.3 Characteristics of the RAF Inventory

This chapter describes the characteristics of the RAF inventory and introduces system parameters that affect the management of consumable spare parts. Like many organisations, the RAF operates a classical periodic review inventory management system, whereby the replenishment level and the replenishment quantity are calculated on a monthly basis using central system parameters for each line item.

However, there are a number of considerations within the RAF environment, such as the need to maintain adequate stock-holdings in case of war, which differentiate the supply system from most organisations. This chapter provides an outline of the RAF inventory characteristics as a means of scene-setting, thereby allowing subsequent analysis to be put into context and, perhaps more importantly, to provide a means for assessing whether assumptions made by the published models are appropriate.

Through an examination of actual demand data, an initial attempt is made to ascertain the extent to which erratic and slow-moving demand patterns exist within the RAF. As these demand patterns are characterised by infrequent transactions with either variable or low demand sizes, it is necessary to consider the transaction rate in unison with the demand size.

Most research on erratic demand assumes independence between successive demand sizes and successive demand intervals, and independence between sizes and intervals. A large-scale analysis of the demand size and the interval between transactions, including autocorrelations and crosscorrelations, is undertaken in this chapter using RAF data.

Identifying the demand pattern is most appropriately done using lead-time demand. Therefore, it is necessary to determine lead-time distributions and associated parameter values through a lead-time analysis, which becomes the focus of the next chapter.

### 1.4.4 Lead-Time Analysis

Although the replenishment lead-time is a fundamental component of any inventory management system, it often occurs that the lead-time distribution and associated parameter values have to be assumed due to a lack of observations. At first glance this would appear to be the case for the RAF where every line item in the inventory has a set lead-time value of a fixed and questionable nature. Fortunately another source of data is available to the RAF from which actual lead-time observations can be derived. This data is not currently used for setting the lead-time parameter values but it provides valuable information for the task of inventory management.

A detailed analysis of the actual lead-time observations is undertaken in this chapter and an initial attempt is made to fit distributions to these observations using a chi-square goodness-of-fit test.

The infrequent usage of many line items in the RAF inventory means that in general few replenishment orders are placed and, when combined with the long lead-times in the defence industry, in reality the lead-time data is incomplete. As a result, this chapter includes a methodology for grouping line items that are likely to have a similar lead-time distribution and calculates summary statistics that apply to the entire grouping. The first stage of this procedure is to identify predictors that provide the best means for grouping similar line items; the second stage groups the line items according to these predictors.

### 1.4.5 Demand Classification

The previously introduced method for classifying demand is tailored and extended for the RAF inventory in this chapter. All line items are classified through variance partition whereby the variance of the lead-time demand is decomposed into its constituent causal parts. Each line item is then assigned to one of the following demand patterns:

(i) Smooth.

(ii) Irregular.

(iii) Slow-moving.

(iv) Mildly Erratic.

(v) Highly Erratic.

An analytical classification of line items in this manner prompts an investigation of whether there are any shared characteristics between the line items within each classification. Such an investigation, termed demand fragmentation, is undertaken across a number of different factors; for example, by cluster grouping in accordance with each of the lead-time predictor variables, by transaction frequency and demand size, and by the level of autocorrelation and crosscorrelation.

The grouping of lead-time observations with the associated increase in sample size allows a more conclusive goodness-of-fit test on the lead-time observations over the one conducted in the previous chapter. In addition, goodness-of-fit tests are performed on the demand size distribution and the demand interval distribution in an attempt to determine whether assumptions made in the literature are valid.

### 1.4.6 Forecasting Erratic Demand

Traditional forecasting methods are often based on assumptions that are deemed inappropriate for items with an erratic demand pattern. This chapter introduces a forecasting model that compares Croston's method, which was developed specifically for forecasting erratic demand, with more conventional methods including exponential smoothing and moving average.

Optimal values for the smoothing parameters are determined using a hold-out sample of 500 RAF line items with the resultant parameters used across a larger sample of 18,750 line items. The selected line items comprise a random sample with equal representation between the five identified demand patterns. Forecasts are made using various demand aggregations with the measuring of forecast errors at every point in time as well as only after a demand has occurred. Analysing the demand for differing aggregation periods, or rebucketing, may lead to demand levelling over the longer periods and differences in relative forecasting performance may emerge.

A distinguishing feature of the comparisons is to recognise the purpose for which demand forecasts are made in reality and rather than just simply compare the forecast value with the actual one-period ahead value, the model compares the forecast value with the demand over a forward-looking lead-time period. If the purpose of the forecast is to supply data for inventory replenishment systems, consistency becomes more important, and accuracy at a single point is not so valuable.

### 1.4.7 Alternative Forecasting Methods

Alternatives to Croston's method have been proposed in the literature that seek to correct a mistake in the original derivation of the demand estimate and further improve forecasting performance. Results from these methods, collectively referred to as the modification methods, are compared with Croston's method in this chapter. Once again, the forecasting performance is analysed by demand pattern and aggregation level.

Also examined in this chapter is the effect of autocorrelation and crosscorrelation on the forecasting performance of exponential smoothing and Croston's method. Most research on erratic demand assumes independence between successive demand intervals and successive demand sizes, yet a growing body of research indicates such independence is not always present. Finally, consideration is given to the effect of smoothing parameters on forecasting performance by comparing optimal values across the identified demand patterns.

Comparing the performance of the forecasting methods using the traditional measures of accuracy is not considered ideal. The measures themselves are open to questions of validity and different conclusions can arise depending on which measure is utilised. As an alternative method for assessing forecasting performance, the implied stock reprovisioning performance for each method is examined in the next chapter.

### 1.4.8 Forecast Performance by Implied Stock-Holding

Stock reprovisioning performance is monitored for each forecasting method by an extension to the forecasting model. This extension allows a comparison of the implied stock-holdings for each method by calculating the exact safety margin that provides a maximum stock-out quantity of zero. The safety margin is calculated by iteratively adding the maximum stock-out quantity to the order-up-to level until no further stock-outs occur. In this manner the average stock-holdings for each method can be compared using a common service level of 100 percent.

A number of factors outside of the forecasting methods affect the stock-holding calculations, including the selected simulation period, the demand aggregation or periodicity of the data, the measurement interval, and the forecast and reorder update intervals. In order to ensure unbiased comparisons between the forecasting methods, each factor is examined in turn to assess the impact on the final calculations.

The selected smoothing parameters determine the forecast values and hence the order-up-to level, which in turn affects the implied stock-holdings. As an indication of the variability due to the smoothing parameter values, comparative stock-holdings are calculated for a sample of 500 line items using the range of optimal smoothing constants in terms of MAPE, as generated in a previous chapter. As it transpires, the optimal values for MAPE need not be the same as those which provide minimum stock-holdings overall, prompting the determination of a new set of optimal values.

Optimal smoothing parameter values that minimise the implied stock-holdings are calculated from the hold-out sample and applied to the same sample of 18,750 line items considered previously. Comparisons are made between the forecasting methods at different demand aggregations and between the five identified demand patterns. The monetary values of the additional stock-holdings above that of the best case are also determined.

### 1.4.9 Conclusions

In the final chapter of the thesis, the main conclusions are summarised, the contributions from this research are identified and areas for further research are outlined.

One of the main conclusions of the research is that a modification to Croston's method, known as the approximation method, offers a suitable alternative to exponential smoothing for demand forecasting. Using this method to provide more accurate forecasts, the RAF could realise substantial cost savings by reducing inventory levels while still maintaining the required service levels. Although RAF data is used exclusively in this research, there is little reason to believe the results would not be applicable to other holders and users of spare parts.

# 2. DEMAND FOR CONSUMABLE SPARE PARTS

Demand for consumable spare parts tends to be divided into two categories. Firstly, erratic or intermittent demand patterns are characterised by infrequent transactions with variable demand sizes, and secondly, slow-moving demand patterns which are also characterised by infrequent transactions but in this case the demand sizes are always low. It is generally accepted that slow-moving items differ from erratic items and a simplifying assumption has often been that the reorder quantity is equal to one for slow-moving items.

This chapter describes erratic demand and slow-moving demand in turn, and examines the processes that lead to such patterns. An analytical method for classifying demand as erratic or slow-moving is also introduced.

## 2.1 Erratic Demand

Under erratic demand, when a transaction occurs, the request may be for more than a single unit resulting in so-called lumpy demand. Such demand patterns frequently arise in parts and supplies inventory systems. Erratic demand can create significant problems in the manufacturing and supply environment as far as forecasting and inventory control are concerned. This section examines the causes of erratic demand, and the demand for a sample line item illustrates the consequences of one such cause. Alternatively, an actual occasion in which an erratic demand pattern need not be a problem is also considered. Finally, statistical means for assessing erratic demand are introduced.

### 2.1.1 Causes of Erratic Demand

The demand pattern for an erratic item has so much random variation that in general no trend or seasonal pattern can be discerned. Silver [66] identifies two factors leading to an erratic demand pattern:

(i) There may simply be a large number of small customers and a few large customers. Most of the transactions will be small in magnitude as they will be generated by the small customers, although occasionally one of the large customers will place a large demand.

(ii) In a multi-echelon system a non-erratic demand pattern at the consumer level may be transformed into a highly erratic demand pattern by inventory decisions made at higher levels. This phenomenon, known as the bull-whip effect, arises when small variations in demand are magnified along the supply chain.

Additional causes of erratic demand are identified by Bartezzaghi *et al*. [7]:

(iii) In considering the numerousness of potential customers, and in particular the frequency of customer requests, lumpiness increases as the frequency of each customer order decreases. In fact, the lower the frequency of orders, the lower the number of different customers placing an order in a given time period.

(iv) If there is correlation between customer requests lumpiness may occur even if there are a large number of customers. Correlation may be due to imitation and fashion, for example, which will lead to sudden peaks in demand. In the case of the RAF, periodic exercises and operations are often correlated between aircraft.

Through examining a forecasting system similar to that operated by the RAF, Foote [28] identifies a further cause of erratic demand:

(v) In large repair facilities there is a tendency to repair a component once a quarter or once a year owing to the long lead-times for spare parts or to reduce costs by minimising the number of set-ups.

In considering demand for spare parts as being generated by a failure process, Beckmann [9] suggests a further cause:

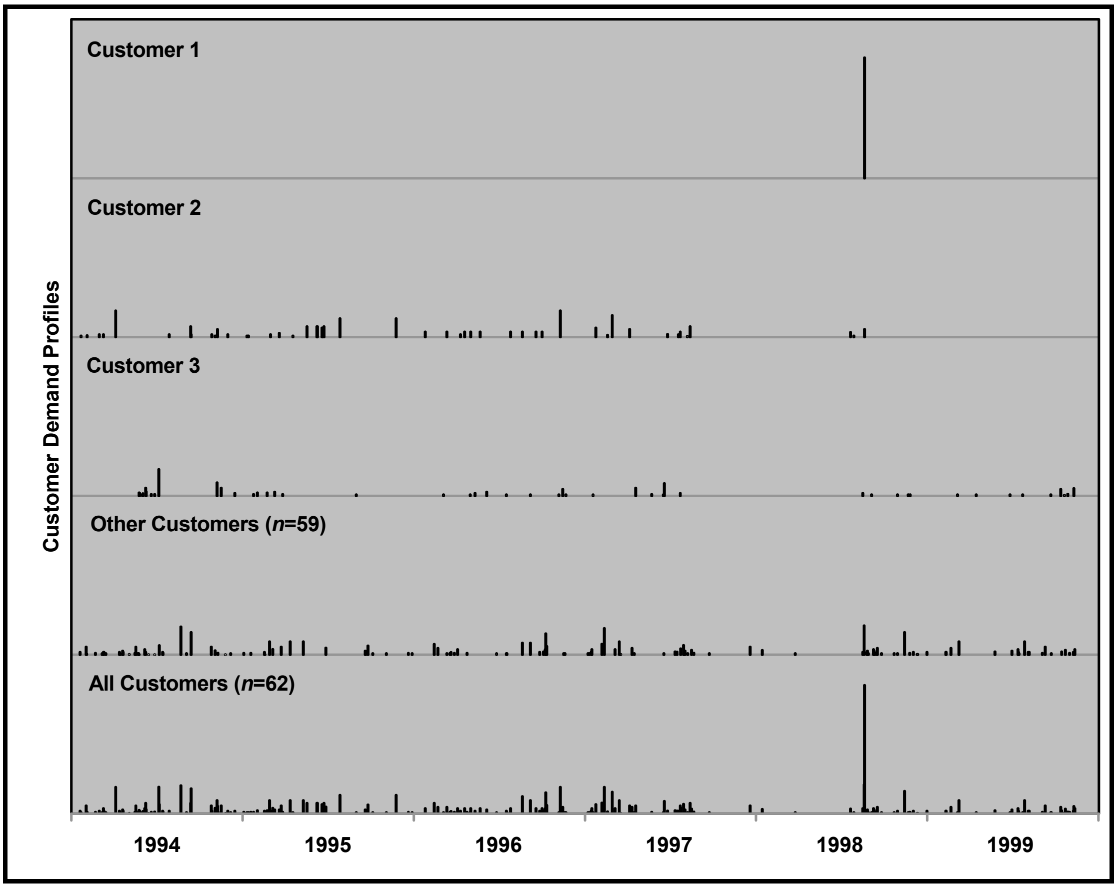
(vi) Multiple demands may occur through sympathetic replacement, whereby maintenance personnel discover a defective part on one aircraft and, as a result, inspect that item on other aircraft, replacing incipient failures.

Analysis undertaken in the course of this research has identified another situation in which an erratic demand pattern can emerge:

(vii) The aggregation of demand, or bucketing, pre-determines the level of intermittency in a given time series. What appears to be a smooth demand series at a quarterly aggregation may become decidedly erratic at a monthly or weekly aggregation.

An actual example of an erratic demand pattern generated by many customers placing small requests, along with a single customer placing a large request, is illustrated in Figure 2.1, where a customer is defined as a repair unit on an RAF station. A total of 62 customers have placed requests for this line item over a six year period. The first customer, shown in the top section, has placed a single request for 455 units. In comparison, the second customer has placed 48 requests with an average demand size of 25 units, while a third customer has placed 47 requests with an average demand size of 14 units. The remaining 59 customers have placed 155 requests between them for a total of 2,385 units, giving an average demand size of 15 units.

**Figure 2.1: Customer Demand Patterns.**




The bottom section presents the demand profile for all 62 customers combined. It is seen that the single large demand dominates the profile and what might otherwise be considered a smooth demand pattern is turned into an erratic demand pattern by the inclusion of this customer.

The demand profiles of Figure 2.1 may also show some correlation between customer requests which can contribute to an erratic demand pattern. No requests are observed by the first or third customers in the 12 months preceding the large single demand. The second customer and the remaining 59 customers also have few requests during this period, although many recommence their requests at a similar point to the large single request.

### 2.1.2 Alternative Handling of Erratic Demand

Some items that appear to have a lumpy history need not be treated as erratic. For example, the lumps may be due to occasional extraordinary requirements from customers, as Brown [14] discovered in the case of an O-ring used in the boiler tubes of an aircraft carrier in the US Navy. The demand history for the O-ring showed single digit demands with an occasional demand for over 300 units interspersed by zeros; a classic erratic demand pattern. However, a closer examination revealed that 307 units were required for overhauls carried out in shipyards and these were scheduled up to two years in advance.

Alternatively, a pre-determined array of spare parts may be required as *fly-away packs* to accompany a squadron of aircraft on planned exercises. In such cases it may be possible to include the requirements as scheduled demand, rather than having to forecast; all that would be necessary is an improvement in the flow of information.

### 2.1.3 Identifying Erratic Demand

An item is said to have an erratic demand pattern if the variability is large relative to the mean. After early research into erratic demand, Brown [14] suggested an item should be classed as erratic if the standard deviation of the errors from the best-fitted forecast model is greater than the standard deviation of the original series. Under such circumstances he recommends setting the forecast model as a simple average of the historic observations.

Straightforward statistical tests were more recently used by Willemain *et al*. [88] on actual data to gauge the level of intermittency, including:

(i) The mean interval between transactions, or equivalently, the percentage of periods with positive demand.

(ii) The degree of randomness in the data. Forecasting requirements are lowered if the demands are a fixed size or the transactions occur at fixed intervals. Thus, the coefficient of variation (CV), which expresses the standard deviation as a proportion of the mean, for the demand size and interval length are useful statistics.

(iii) Autocorrelations and crosscorrelations. Most research on erratic demand assumes independence between successive demand sizes and successive demand intervals, as well as independence between the sizes and intervals. In fact some substantial positive and negative autocorrelations and crosscorrelations were found in their data.

The authors commented that the sparseness of their data made it difficult to estimate correlations and only two out of their fifty-four demand series reached statistical significance at the 5 percent level. They considered the question of independence to be unanswered rather than interpreting their limited results as justifying the assumptions made in the literature. The large quantity of data available in this study allows a detailed analysis of autocorrelations and crosscorrelations and this is an area considered extensively in a later chapter.

The next section examines the second demand pattern often encountered in spare parts inventories, namely that of slow-moving spares.

## 2.2 Slow-Moving Demand

Slow-moving spares are mainly held as insurance against the very high costs that might otherwise be incurred if the item failed in use when a spare was not available. Any inventory control policy for slow-moving spares must take into account the run-out or shortage cost. The run-out cost for a particular spare is defined as the average difference between the cost of a breakdown where a spare is required but is not available and the cost of a similar breakdown when a spare is available.

Mitchell [53] indicates that a major problem associated with forecasting and inventory control of slow-moving spares is the lack of past records for giving reliable estimates of historic consumption and failure characteristics. Slow-moving spares often have zero consumption over a long period that would normally be more than adequate for analysis.

A further difficulty with slow-moving spares is their inflexibility regarding over-stocking. Whereas over-stocking of fast-moving spares is quickly remedied by natural consumption, this is not the case for slow-moving spares. Initial over-ordering is not the only cause of excess stock; an increase in stocks to allow for a short-term lengthening of lead-time may lead to serious over-stocking when the lead-time returns to normal. In any case, this leads to a greater chance that the over-stock will never be used and become obsolete.

A simplifying point for slow-moving spares is that the possible decisions are few in number. Rarely is it necessary to hold more than two spares so the decisions are essentially whether to hold zero, one or two. True insurance or stand-by spares are held because it is considered less costly to hold them rather than suffer the extra cost of a breakdown with them unavailable. However, in an analysis of slow-moving spares held by the National Coal Board, Mitchell [53] identified several groups, not peculiar to the coal industry, which could immediately be declared as excess stock:

(i) Overhaul items. These are bought for use on a specified date in preparation for a major overhaul, much like the O-rings used in the US Navy. Provided the manufacturer is given adequate notice of the requirement, there is no reason why overhaul items should be held in stock for longer than the time taken to examine them prior to use.

(ii) Adequate warning items. These either have a minor breakdown but can be economically patched-up for a period longer than the lead-time, or their wear indicates, by a period longer than the lead-time, their impending breakdown.

(iii) Wear items. These items wear out and have a failure rate that increases with their life. It may be economical to defer the purchase of a replacement spare until the current item is used.

Alternatively, the optimum stock-holding of random failure items does not vary, in which case the sole consideration is the run out cost. An examination of the value of slow-moving spares held at a particular colliery revealed overhaul items comprised 10 percent, adequate warning items 20 percent, wear items 20 percent and random failure items 50 percent. Thus, a significant portion of the stock can be considered as excess.

The causes behind both erratic and slow-moving demand patterns have been considered. Attention is now given to classifying line items according to their observed demand patterns. Such a process is necessary as the more commonly used methods for forecasting and stock-holding are not as effective if the demand is not smooth and continuous and alternative methods should be sought.

## 2.3 Demand Classifications

Williams [90] developed an analytical method for classifying demand into smooth, slow-moving or sporadic (erratic) by decomposing the variance of the lead-time demand (LTD) into constituent causal parts, known as variance partition. Assuming:

(i) The number of transactions per unit time are independent, identically distributed random variables (IIDRV) with mean $\bar{n}$ and variance $\mathrm{var}_n$.

(ii) The size of demands are IIDRV with mean $\bar{z}$ and variance $\mathrm{var}_z$.

(iii) The lead-times are IIDRV with mean $\bar{L}$ and variance $\mathrm{var}_L$.

(iv) The three sets of random variables are also independent of each other.

The underlying variance partition equation for the variable lead-time case is given as:

$$\mathrm{var}_{LTD} = \bar{z}^2 \bar{L}\ \mathrm{var}_n + \bar{n}\bar{L}\ \mathrm{var}_z + \bar{n}^2 \bar{z}^2\ \mathrm{var}_L \tag{1}$$

Equation (1) becomes dimensionless through:

$$C_{LTD}^2 = \frac{C_n^2}{\bar{L}} + \frac{C_z^2}{\bar{n}\bar{L}} + C_L^2 \tag{2}$$

where $C_z$ is the coefficient of variation for demand size, etc.

An evaluation of the three constituent parts of equation (2), translated as transaction variability, demand size variability and lead-time variability, leads to the classification of LTD for non-constant lead-times presented in Table 2.1.

**Table 2.1: Classification of Lead-Time Demand.**

| Lead-Time Demand Component | | | Type of Demand Pattern |
|---|---|---|---|
| Transaction Variability | Demand Size Variability | Lead-Time Variability | |
| Low | | | Smooth |
| High | Low | | Slow-moving |
| High | High | Low | Erratic |
| High | High | High | Erratic with highly variable lead-time |

It is observed that a slow-moving demand pattern is distinguished from an erratic demand pattern according to the level of demand size variability. The choice of boundaries between each of the categories is essentially a management decision. An item will change category as parameter values move from one region to another, therefore items may continually change if the parameters are borderline. This sensitivity is overcome by defining buffer zones around each boundary.

Classifying LTD by this means allows further analysis to be compared between the patterns and an investigation can be undertaken as to whether the models put forward have a detrimental effect when demand does not adhere to a specific pattern. This means of classifying demand is revisited and extended for RAF data in Chapter 6 once the constituent parts have been obtained and examined.

## 2.4 Concluding Remarks

Demand which is not smooth and continuous has traditionally presented difficulties in forecasting and inventory control. Two such demand patterns that tend to arise in a spare parts inventory are erratic demand and slow-moving demand. Both patterns are characterised by infrequent transactions, although while an erratic demand pattern has variable demand sizes, a slow-moving demand pattern always has low demand sizes.

Erratic demand occurs when there are a few large customers and many small customers, or when the frequency of many customer requests varies. Inventory control decisions made at progressively higher levels of a multi-echelon system can also transform a non-erratic demand pattern into a highly erratic demand pattern. In addition, correlation between customer requests can lead to an erratic pattern, which in the case of the RAF may occur through sympathetic replacement, periodic operations and exercises, or the tendency to repair a component infrequently in order to minimise the number of set-ups.

Alternatively, slow-moving demand occurs simply when there are few customers and little demand for an item. Such items in a spare parts inventory are mainly held as insurance against the immense costs that would otherwise occur if an item was required and a spare was not immediately available. The lack of past records of historic consumption and failure characteristics is a major problem associated with slow-moving spare parts.

The forecasting and holding requirement of erratic and slow-moving spare parts can be reduced if the items are required for a major overhaul or a planned exercise on a specified date, provided, of course, that the requirement is cascaded. Forecasting requirements are also lowered if the demands are fixed in size or the transactions occur at fixed intervals. Repairing a jet engine may require a fixed number of turbine blades, for example. Finally, an item may give warning longer than the lead-time of impending breakdown and spares can be procured at such time.

As the commonly used methods for forecasting and stock-holding are generally not as effective when demand is erratic or slow-moving, it is useful to classify line items according to their observed demand pattern. An analytical method described in the literature, which decomposes the variance of lead-time demand into constituent causal parts for classification purposes, has been introduced. Classification by this means will enable subsequent analysis to be assessed by demand pattern and allow an exploration of whether particular methods better suit specific patterns.

At this stage it is useful to review the large and growing body of academic literature that offer solutions to the problems faced by a spare parts inventory. The next chapter traces the development of techniques for managing line items with erratic and slow-moving demand in the form of a literature review.

# 3. REVIEW OF THE LITERATURE

This review of the academic literature focuses on considerations given to the forecasting, ordering and stock-holding for consumable spare parts. Such research tends to concentrate on erratic and slow-moving demand patterns as opposed to demand that is smooth and continuous. Widely published inventory control policies developed for smooth demand are considered inefficient when applied to items with erratic or slow-moving demand and alternatives are investigated.

The first section provides an historical summary of inventory policies put forward as suitable for managing erratic demand and slow-moving demand in turn. The second section examines more recent developments in demand forecasting, service level considerations, and the application of probability models. In each case an assumption of normally distributed lead-time demand is not suitable. The final section of this literature review provides examples of the practical application of the various methods.

## 3.1 Historical Summary

An inventory control policy for low demand was possibly considered first by Whitin and Youngs [86] in 1955, for a simple Poisson situation, and developed slightly by Heyvaert and Hurt [34] in 1956. Since this time, the study of erratic and slow-moving demand patterns have mostly progressed separately, and each will be reviewed in turn.

### 3.1.1 Erratic Demand

In cases of continuous review with convex holding costs and fixed replenishment costs, Beckmann [8] in 1962 proved the optimality of an $(s,S)$ inventory policy, whereby an order is placed to raise the available stock (on hand plus on order minus backorders) to an order-up-to level $S$ when the stock level falls to or below reorder point $s$. The model considered an arbitrary distribution for the intervals between demands and a distribution for the demand sizes that is independent of the previous demand size but may depend on the elapsed time since the last demand.

An important paper on erratic demand is the 1972 paper of Croston [19] who demonstrated that using simple exponential smoothing forecasts to set inventory levels could lead to excessive stock levels. He argues that exponential smoothing places most weight on the more recent data and therefore gives estimates that are highest just after a demand, and lowest just before a demand. The replenishment quantity is likely to be determined by the biased estimates that immediately follow a demand as a consequence. By way of solution, Croston suggested that unbiased forecasts are needed for stock replenishment decisions immediately after a transaction occurs, and should be based on separate forecasts of the demand size and the interval between transactions. The method proposed by Croston is seen to reduce the bias associated with exponential smoothing.

Other authors have assumed particular demand distributions, usually a compound distribution arising from combining distributions for transaction occurrence and demand size. In this manner, the total number of units demanded over a lead-time can be considered as the sum of a random number of demands, each generating a random demand size. The compound Poisson distribution where transactions are assumed to arrive in accordance with a stationary Poisson process, as developed by Adelson [1] in 1966, has frequently found favour in the literature.

An $(s,S)$ policy is normally superior to a $(Q,r)$ policy where a fixed quantity $Q$ is ordered when the stock level falls to or below reorder point $r$, in terms of reduced total holding and replenishment costs. In fact, as the demand pattern becomes more erratic in nature, an $(s,S)$ system increases in superiority and this tends to be the preferred method for consideration. An order-up-to level is intuitively appealing for an erratic demand item as the amount by which the reorder point is passed may vary widely between one replenishment requirement and the next. However, the computational complexity of determining the optimal order-up-to value sometimes restricts its use in favour of a fixed order quantity.

Recursive expressions for determining optimal parameters for an ($s$,$S$) policy under periodic review with discrete compound Poisson demand and constant lead-time were provided by Veinott and Wagner [81] in 1965 and improved by Bell [10] in 1970, while Archibald and Silver [3] consider the analogous case of continuous review in 1978.

A compound Poisson demand process with stochastic lead-time is considered in 1977 by Dirickx and Koevoets [22] who use Markov renewal theory to give very complex formulae for an ($s$,$S$) policy. Markov renewal theory was previously used in 1975 by Kao [42], although his methodology assumed zero lead-time while allowing arbitrary demand size and interval distributions.

Also in 1977, Bott [11] considered three compound Poisson distributions where the selected demand size distribution depended on the variability in the historical data. Bott suggests that as the negative binomial distribution has a variance-to-mean ratio (VMR) greater than unity for any choice of parameters, the demand size may be modelled by such a distribution if the sample data has a VMR greater than one. Similarly, the Poisson distribution may be suitable if the sample VMR is equal to one and the binomial distribution if it is less than one.

When combined with Poisson transaction arrivals, demand sizes with a geometric probability distribution provides a demand distribution referred to as stuttering Poisson (*sP*), as described by Sherbrooke [64] in 1966. As a special type of compound Poisson distribution the *sP* distribution has remained a popular choice in the erratic demand environment. Silver *et al*. [67] in their paper of 1971 considered an *sP* demand pattern for an (*s*,*S*) inventory policy with continuous review.

In his paper of 1978, Ward [82] used an approximate regression model to calculate reorder points based on a fixed service level, although no attempt is made to minimise the total operating cost. A (*Q*,*r*) inventory policy with continuous review is utilised. The model assumes constant lead-times and demand is modelled by the *sP* distribution. A regression model was also used by Mak and Hung [50] in 1986 for computing optimal (*s*,*S*) policies where the lead-time demand is modelled by an *sP* distribution and the lead-time itself is assumed constant.

In their paper of 1971, Foster *et al*. [30] studied the effect of demand distributions on optimal decisions and costs for a (*Q*,*r*) inventory policy. Using an (*s*,*S*) policy Naddor [56] in 1978 also examined how optimal decisions and costs are affected by different demand distributions, different shortage costs and different lead-times. Numerical solutions imply that the precise form of the distribution of demand is not essential for the determination of optimal decisions in the system. Where the standard deviation is relatively small compared to the mean, the decisions are hardly affected by the form of the distribution because of the relative flatness of the total cost around the optimum. However, when the standard deviation is relatively large compared to the mean, the decisions and costs are more sensitive to the form of the distribution.

### 3.1.2 Slow-Moving Demand

An (*S*-1,*S*) inventory policy is often adopted for slow-moving demand such that whenever the stock level falls less than or equal to *S*-1 an order is placed to raise the available stock to the order-up-to level *S*. Such a system is reasonable when the lead-times are small relative to the average interval between orders or the cost of reordering is small relative to the cost of holding stock.

Karush [43] in 1957 utilised a queueing model where customer arrivals are given by a Poisson process with demand sizes of a single unit. Replenishment is initiated upon a demand request leading to an ($S$-1,$S$) policy and the lead-time is treated as random. The problem of allocating inventory investment among competing line items to minimise the value of total lost sales is considered.

Under an ($S$-1,$S$) policy with continuous review, whenever a demand for an arbitrary number of units occurs, a reorder is placed immediately for that number of units. This observation allows Feeney and Sherbrooke [25] in their paper of 1966 to utilise Palm's theorem which states, in inventory terms, that if demand is Poisson then in the steady state the number of units on order is also Poisson for any lead-time distribution. The authors show Palm's theorem can be generalised to any compound Poisson distribution. Steady state probability distributions for stock on-hand and backorders are obtained from the steady state distribution for the number of units on order by translating the origin of the distribution $S$ units.

Sivazlian [70] in 1974 considered a continuous review inventory system operating under an ($s$,$S$) policy where demand occurs as one unit at a time. With instantaneous delivery of orders, the model is equivalent to a piece of equipment consisting of $Q$=$S$-$s$ elements subject to failure, where, upon failure of the last element, all $Q$ elements are immediately replaced. Silver and Smith [69] in their paper of 1977 provided a methodology for constructing indifference curves that determine the optimal inventory level.

Again, an important paper in this area is by Croston [20] in 1974 discussing whether or not to stock a product. Significant savings can be obtained by deciding not to stock particular items, but to either provide an acceptable substitute, for which there is higher total demand, or to supply to order only. Tavares and Almeida [79] in 1983 also consider whether it is economic to have zero or one item in stock.

A frequent problem with slow-moving spares is that there is little or no historic data for forecasting demand. One approach is Bayesian forecasting; this was proposed by Silver [65] in 1965 for selecting a reorder point based on a stock-out probability or service level with uncertain lead-time. Smith and Vemuganti [71] in 1969 also use a Bayesian approach to update the demand distribution parameters as the available information increases over time.

Brown and Rogers [15] in 1973 also considered a Bayesian approach that incorporates usage estimates developed at initial provisioning and provides for progressive updating as data becomes available. The authors indicate that an adequate spare parts inventory for ensuring high system reliability early in a system life will be very costly and extremely wasteful. The inventory required will be large but very little of it will actually be used. Therefore, one course of action is to accept low reliability in the early life of the system and procure spares as needed until sufficient data allows a more economical inventory policy. Similar ideas are used by Burton and Jaquette [16], also in 1973, to establish initial provisioning procedures for slow-moving repair items.

Through the consideration of spare parts usage on submarine systems, Haber and Sitgreaves [32] in 1970 pool usage data for common design items. They assume demand for each product is Poisson distributed, while mean demand for the items in a particular class have a gamma distribution. The model allows estimates for spare parts usage regardless of whether or not a particular spare has been used in the past, while also making possible the estimation of usage rates for new items.

Williams [89] in 1982 derived approximate expressions for the optimum reorder point $r$ in a $(Q,r)$ model while assuming the lead-time is short enough that only zero or one demand can occur in a lead-time. In the case of slow-moving demand this is frequently a reasonable assumption to make. The method uses an iterative procedure with stochastic transaction arrivals and gamma-distributed demand sizes.

Schultz [62] in 1987 considered an $(S-1,S)$ policy with periodic review, constant lead-time and arbitrary demand size and interval distributions. The usual implementation places an order immediately after each transaction and thus ignores the frequency with which transactions occur. It is shown by Schultz that introducing a delay in placing the order can lead to significant holding cost reductions with little additional risk or cost of stock-outs if the average number of periods between demands is large relative to the lead-time plus the delay.

## 3.2 Further Developments

Methods that assume lead-time demand can adequately be approximated by the normal distribution, in general, cannot be utilised for erratic and slow-moving line items and alternatives are required. A particular problem in the case of erratic demand is that the actual stock level when reordering takes place will not be $r$ but some level below $r$ as one transaction may cause the stock level to fall significantly.

### 3.2.1 Forecasting Demand

In 1972 Croston [19] demonstrated his forecasting method to be superior to exponential smoothing (ES) when assuming the intervals between transactions follow the geometric distribution (demand occurs as a Bernoulli process), their size is normally distributed, and the intervals and sizes are independent of each other. Willemain *et al*. [88] in 1994 violated these assumptions in generating a comparative evaluation between Croston's method and ES. Various simulated scenarios covered a log normal distribution of demand size, and both positive and negative autocorrelations and crosscorrelations in the intervals and sizes. Through making comparisons only at times of positive demand, in all cases Croston's method was found to provide more accurate estimates of the true demand. The authors concluded that Croston's method is quite robust and has practical value beyond that claimed in Croston's original paper. However, an important observation was the fact that results from industrial data showed very modest benefits as compared to the simulation results.

The usefulness of Croston's method was also investigated by Johnston and Boylan [39,40] in 1996. A simulation analysis was conducted to determine the minimum interval between transactions that was required for a modification of Croston's method to outperform ES. Using a Poisson arrival process and a number of demand size distributions, comparisons were made between the errors observed at every point in time and only after a demand occurred. It was observed that the modified method outperformed ES when the average interval between demands is greater than 1.25 periods and the greater the interval the more marked the improvement. In addition, longer forecasting horizons were seen to improve the relative performance of the method while any variability in the demand size has only a small effect on the improvement.

Syntetos and Boylan [76] in 1998 quantified the bias associated with Croston's method through simulation, while in a second paper [77] of 1998 the same authors provided three modifications to Croston's method that attempt to give unbiased estimates of the demand per period. They indicate that Croston's estimates of the demand size and the interval between transactions are determined to be correct; it is an error in their combining which fails to produce accurate estimates of the demand per period.

Wright [91] in 1986 provided an extension to Holt's two-parameter smoothing method for the case of intermittent data. Consideration is given to time series which naturally occur at irregular time intervals, such as the inventory applications covered in this research, as well as cases where the frequency of reporting changes from annual to quarterly, for example, or where occasional data observations are simply unavailable in an otherwise regularly spaced series. In many applications the extended procedure requires only about twice the resources of the regular Holt's method.

Sani and Kingsman [60] in 1997 compared periodic inventory control policies and demand forecasting methods in an attempt to determine which are best for slow-moving and erratic demand items. Periodic systems are put forward as preferred by stock controllers due to the convenience of regular ordering days for the stockist, as well as for the supplier who can plan efficient delivery routes. Ten periodic inventory policies are compared using real-world data from a spare parts depot and in each case five demand forecasting methods are used to determine values for $s$ and $S$. The comparisons include simple rules developed by practising stock controllers which relate alternative sets of $(s,S)$ values to ranges of annual demands and the value or criticality of the item. Using two performance measures, namely annual inventory cost and the proportion of demands satisfied immediately from stock, the authors conclude that a 52-week moving average forecasting method is best, followed closely by Croston's method.

A bootstrap approach as a means of forecasting lead-time demand is considered by Willemain *et al*. [87] in 2000. The bootstrap method utilises samples of historic demand data to repeatedly create realistic scenarios that show the evolution of the lead-time demand distribution.

With respect solely to slow-moving line items, Ritchie and Wilcox [59] in 1977 considered renewal theory from the point of view of the supplier to forecast all-time future demand for spares where demand is declining or is about to decline in the immediate future due to the phasing out of the equipment to which the spares are fitted. At a point in time the spares will not be produced as part of a normal production run but must instead be produced during special production runs and will therefore be expensive to produce. For some items the costs of setting up a special run may be so great that it is worthwhile producing enough spares during the last production run to satisfy all future demand.

In their paper of 1997, Bradford and Sugrue [13] presented a methodology for estimating the demand pattern of slow-moving line items using an aggregation-by-items approach, assuming demand follows an arbitrarily mixed, heterogeneous Poisson distribution. The consideration of demand heterogeneity arises due to the low annual usage-value of some slow-moving items and hence, for planning and control purposes, it is often expedient to treat them as a group and estimate the aggregate distribution of demand.

### 3.2.2 Service Level Considerations

Silver [66] suggests there are a number of different ways of measuring customer service, although none is universally acceptable. Four commonly used measures are:

(i) Item availability as the fraction of time stock on-hand is greater than zero.

(ii) Fraction of demand satisfied without backorder.

(iii) Average amount on backorder, which may be time-weighted or at a random point in time.

(iv) Delay time as the average number of days that a customer request is backordered before it can be delivered.

Stock replenishment rules based on customer service levels should be linked to the current estimates of demand along with a measure of the forecast variability. Although demand forecasting is a prerequisite to inventory control decisions, in practice forecasting and ordering systems have traditionally been considered independent of each other.

A variance calculation given by Croston [19] in 1972 was only an approximation to the variance of the demand per period and the suggested replenishment rules did not consider the observed demand pattern. Such a stocking methodology would guard against only one transaction during a lead-time and lead to exceptionally low replenishments. In the study of a spare parts depot conducted by Sani and Kingsman [60] in 1997, Croston's suggestion was, not surprisingly, seen to result in poor service levels.

More recent authors have considered the joint impact of forecasting and ordering systems on customer service levels. In unison with Croston's method, Silver and Peterson [68] in 1985 use *MAD*(z), the mean absolute deviation of non-zero sized transactions, to establish safety stock parameters. Alternative methods for directly estimating the variance of the demand were suggested by Johnston and Boylan [39] in 1996, and also by Sani and Kingsman [60].

A simple model to include the service level effect from the reorder quantity in both smooth and erratic demand contexts was developed by Etienne [24] in 1987. He argued safety stock decisions should not be made independently of reorder quantity decisions, as substantial reductions in safety stock are permitted while still achieving target service levels. A reorder is observed to act as a buffer against variations in demand for every period except the last.

Also in 1987, the interactions between forecasting and reordering with respect to service level and inventory cost were examined by Watson [83]. Large fluctuations in the forecast parameters for line items with erratic demand are shown by simulation to lead to a significant discrepancy between the target service level and that actually achieved. Assuming a stuttering Poisson demand pattern, the discrepancies were found to vary between positive and negative, depending on the parameters of the line item and the reorder model. It was concluded that attempts to accurately achieve a target service level by using an elaborate reorder formula when forecast fluctuations are large might be futile.

In discussing the presence of forecast errors in an ordering system not solely limited to erratic or slow-moving demand, Wemmerlöv [84] in 1989 examined the effect of introducing adequate safety stocks to counter the effects of demand uncertainty. In order to avoid estimating the cost of stock-outs, safety stocks were introduced so that the service levels from each simulation run were the same and gave a common service level of 100 percent. Thus, once the safety stocks necessary to completely eliminate stock-outs were determined, performance could be calculated as the sum of the holding and ordering costs only. By enforcing a service level of 100 percent the methodology of Wemmerlöv's is removed from reality but it does give a relatively impartial means for comparing simulated ordering and stock-holding results.

Dunsmuir and Snyder [23] in 1989 emphasised that erratic demand items typically possess positively skewed frequency profiles with a large spike at zero. By including a component to explicitly model the chance of positive demand, and assuming a gamma distribution for demand size, they attempted to determine a reorder level consistent with a specified customer service level under a periodic review system. Snyder [73] in 1999 replaced the gamma probability distribution with simulated lead-time demand parameters using a parametric bootstrap approach.

The method presented by Dunsmuir and Snyder is extended by Janssen *et al*. [36] in 1998 to allow for an undershoot with a periodic ($R$,$s$,$Q$) inventory model. Under this policy the inventory position is monitored every $R$ time units and when the inventory position falls below $s$, a quantity of $Q$ units are ordered such that the inventory position is raised to a value between $s$ and $s$+$Q$. When demand is not unit sized, and particularly when there is a high probability that demand is zero during the lead-time, the undershoot of the inventory position below the reorder point can have a significant impact on service levels.

Janssen *et al*. [37] consider an ($R$,$s$,$Q$) model again in 1999, when they seek to determine the reorder point $s$ given a service level constraint. An approximation method is derived for calculating the reorder point such that a target service level is achieved. The underlying demand process is assumed to be a compound renewal process where data is not collected per unit of time, but instead interarrival times and demand sizes are collected for individual customers.

Gardner [31] in 1990 developed trade-off curves between inventory investment and customer service levels for alternative forecasting models. As forecast errors are the primary determinant of safety stock calculations, the better the forecast accuracy, the smaller the inventory investment required to achieve a specified customer service level. Alternatively, increased customer service can be achieved from a fixed inventory investment.

### 3.2.3 Probability Models

Many inventory models rely upon the probability distribution of lead-time demand (LTD) as this knowledge is essential for determining inventory decision variables, such as expected backorders, lost sales, and stock-out risk. Determining the LTD distribution requires taking the distributions of both the demand per unit time and the lead-time into account. A common assumption is that the LTD is normal, although in circumstances of erratic or slow-moving demand the normality assumption may be inappropriate. In 1970, Silver [66] indicated that the majority of inventory control methods were based upon assumptions about the demand distribution that were invalid for items with erratic demand. In such cases, "*the procedures tend to be computationally intractable*".

Empirical evidence that the normal distribution does not provide a reasonable model for LTD for erratic demand items is provided by Mitchell *et al*. [54] in 1983. Since LTD is generally the sum of several demands, the normal distribution would only be suitable when a large number of transactions occur during the lead-time. In the case of erratic demand, the central limit theorem cannot reasonably be expected to apply.

With the normal distribution not normally able to satisfactorily model LTD for an erratic demand item, alternative distributions are required. Often compound distributions are chosen as they allow the total demand over a lead-time to be considered as the sum of a random number of transactions, each generating a random demand. The compound Poisson distribution has frequently found favour. Under this distribution, the transactions arrive in accordance with a stationary Poisson process and to adequately represent demand, the distribution for demand size will depend on the variability in the historical data.

Although the uncertainty of the lead-time in practical settings is well documented, the scarcity of lead-time data often restricts the modelling to constant lead-times. In any case, realistic constant lead-time models can be obtained from both a constant-Poisson distribution and a stuttering Poisson (*sP*) distribution. The constant-Poisson distribution models the situation where each demand has a fixed quantity and the number of transactions arriving within any interval of time follows a Poisson distribution. Alternatively, under the stuttering Poisson, or geometric-Poisson distribution, when a transaction occurs, the request is for one or more units of the item, with the quantity given by the geometric distribution.

During an analysis of USAF data, Mitchell *et al.* [54] used the stuttering Poisson and constant-Poisson distributions to describe the demand patterns with constant lead-time. Demand histories were analysed for 6,529 line items, arranged in six categories ranging from inexpensive nuts and bolts to expensive electronic parts. To assess the appropriateness of the Poisson distribution the authors compared the observations to the expectations and employed the Poisson dispersion goodness-of-fit test due to ease of implementation; the test statistic involves only the sample variance to mean ratio. The Poisson distribution was seen to provide a reasonable fit to almost all the weekly arrival patterns at the 5 percent significance level. When testing the geometric distribution against the demand sizes, the small number of customer demands precluded an objective goodness-of-fit test. However, other characteristics of the data, such as constant demand size for a large percentage of items, were found to be consistent with the geometric assumption. A decisive test of how well the geometric-Poisson model works in an operational sense was conducted by measuring its effectiveness in predicting demand during a lead-time using a hold-out sample.

Variable lead-time models also appear in the literature. In 1982 Nahmias and Demmy [57] considered an inventory system in which demand occurrences arise according to a Poisson process, demand sizes follow a logarithmic distribution, and lead-times are gamma distributed. Both the exact and approximate distributions for lead-time demand are derived. Bagchi *et al*. [6] in 1983 examined the Poisson-like Hermite distribution, where demand per unit time is Poisson and lead-time is normally distributed, as a model for slow-moving line items. Bagchi [5] in 1987 introduced two analytical lead-time demand models for stuttering Poisson demand and variable lead-time, with the first model utilising the normal distribution and the second the gamma distribution to represent the lead-time distribution.

Probability models tend to be complicated and not very useful for determining demand in a practical setting. The models require a large number of recursive calculations in order to obtain the probability density function of lead-time demand. As a result, models of this type contravene the requirement of this research that they should not be overly complex and require unrealistic processing power. The large inventory held by the RAF dictates such a requirement.

## 3.3 Practical Applications

Schuster and Finch [63] in their paper of 1990 provided a non-technical description of their implementation of a spreadsheet model for production scheduling at Welch's, a US producer of fruit juices, drinks and spreads, when demand is erratic. Demand for Welch's products is not typically seasonal, but product promotions lead to pronounced peaks and demand during a promotion period can be several times greater than non-promotional demand. The model uses the forecast of demand during a lead-time in a reorder point calculation and safety stock is based on the forecast mean absolute deviation. It was found that the forecasts were often negatively biased such that actual demand was less than forecast demand resulting in excess inventory. By way of solution the authors utilise a suppression factor to link the safety stock level to the forecast bias in recent periods.

Material requirements planning (MRP) for production scheduling under the situation of erratic demand is considered by Ho [35] in 1995. The author examines the impact of various degrees of demand lumpiness on the performance of MRP systems through a simulation study. Although it is intuitively expected that the MRP performance will deteriorate as the demand pattern becomes more erratic, it was observed that the system actually improves to a certain extent. This is because the number of set-ups, and hence the set-up costs, decrease while the carrying cost increases to a lesser extent within certain ranges.

As an extension to a continuous review ($s$,$S$) inventory policy with $sP$ demand, Mak [49] in 1996 incorporated a cut-off point such that customer orders with transaction sizes greater than cut-off point $w$ are satisfied by placing a special replenishment order rather than satisfying from stock. In addition, it is specified that if the stock level is below order-up-to level $S$ when a special replenishment order is placed, the stock will be replenished jointly to raise the available stock to $S$. In 1999 Mak *et al*. [51] extended the model to include a genetic search algorithm for determining the optimal solution. The concept of a cut-off point was suggested by Silver [66] in 1970 as a means of improving the performance of the inventory system.

Coordination of replenishment for slow-moving items is considered by Thompstone and Silver [80] in 1975. An ($S$,$c$,$s$) inventory policy with compound Poisson demand, continuous review and zero lead-time is utilised. Under such a policy, whenever available stock for item $i$ falls to or below $s_i$ an order is placed to raise it to $S_i$. At the same time any other item $j$ within the associated family with available stock at or below the *can-order* point $c_j$ is included in the order to raise the level to $S_j$. Cost savings are realised if the cost of replenishing two or more items at the same time is less than the total cost of replenishing each of the items separately.

Bartezzaghi *et al*. [7] in 1999 examined the behaviour of forecasting methods when dealing with erratic demand at the master production scheduling level. They argue demand lumpiness is a consequence of different market dynamics, including numerousness of customers, heterogeneity of customers, frequency and variety of customer requests, and correlations in customer behaviour. Subsequently, they use simulation experiments to compare the performance of three forecasting methods with differing parameter values for the sources of lumpiness.

In their paper of 1980, Muckstadt and Thomas [55] examined a two-echelon inventory system where most line items are slow-moving. Using an ($S$-1,$S$) policy with continuous review the authors found that a multi-echelon inventory method designed to take advantage of the system structure was able to achieve the same average level of performance as an individually stocked single-location method with a significantly lower investment. Results indicated it was efficient to centralise safety stock for relatively high cost, slow-moving items and the larger the number of slow-moving items the more important a multi-echelon system will be.

In an examination of a spare parts system in a production plant of Mars confectionary, Strijbosch *et al*. [75] in 2000 compared the performance of a simple ($Q$,$r$) inventory model, used to approximate the current system which is based on intuition and experience, to one which is more advanced and likely to provide a better performance in terms of service and costs. Demand for spare parts is found to be slow-moving with 40 percent of all spares having no demand over a three year period. The simple model does not take the undershoot of the reorder level into account and the normal distribution is used as the distribution of demand during lead-time. On the other hand, the advanced model takes undershoots into account and utilises the gamma distribution for lead-time demand. The authors show the advanced approach yields service levels close to the desired level under most circumstances, and even with an increase in computer time and less intuitive decision rules for the users, the manufacturer has decided to implement this new approach.

Fortuin and Martin [29] in 1999 observed that the electronic industry, the automotive industry and airline operators tend to increasingly outsource spare parts management. In addition, in these industries there is a tendency towards co-operation, such as the joint exploitation of spare parts inventories, together with joint acquisition and collective maintenance. Such actions serve to reduce the problems associated with the management of spare parts for the individual organisations. For the most part, the RAF is precluded from these actions due to reasons of security or being sole users of parts that prohibits such co-operation. However, the joint development and operation of an aircraft type, such as the tri-national *Eurofighter*, allows co-operation in the management of spare parts. The current system of parts identification by NATO Stock Number (NSN) assists in this respect.

## 3.4 Concluding Remarks

Forecasting and inventory control policies developed for smooth and continuous demand are generally inefficient when dealing with a spare parts inventory. Instead, the academic research has concentrated on developing alternative models suitable for erratic and slow-moving demand patterns.

Many authors have assumed specific demand distributions when considering erratic and slow-moving demand, often incorporating compound distributions obtained from combining distributions for the interval between demands and the size of the demands. The usage of a compound Poisson distribution as a model for demand per unit time often finds favour in the literature. The selected models have strived to become more realistic over the years and lead-times that were first restricted to being constant, and then approximated by the normal distribution, have since taken the form of a gamma distribution among others. Probability models for lead-time demand have commonly been constructed without published justification for the choice of distributions, nor have practicality issues been considered.

Mitchell *et al*. [54] do, however, examine the appropriateness of compound Poisson distributions for describing demand within the United States Air Force, which would be expected to have similar demand patterns to the RAF. It is most likely that the lack of actual data available to other researchers has prevented the validation of their selected distributions. The large quantity of actual data available in this instance could make a valuable contribution as far as the reasonableness of fit is concerned.

Irrespective of the appropriateness of the distributions that have been put forward by authors such as Bagchi *et al*. [5] and Nahmias and Demmy [57], my preliminary analysis with spreadsheet models has shown probability models are not very useful in practice. Large numbers of increasingly more complicated recursive calculations are required to obtain probability density functions, and the large RAF inventory discourages their actual usage.

An alternative avenue for investigation, and the focus selected for this research, is the performance of forecasting methods put forward as suitable for implementation in a spare parts environment. A paper on forecasting erratic demand written by Croston [19] in 1972 has received widespread recognition. The idea of separately forecasting the interval between demands and the size of the demands when demand is erratic or slow-moving holds intuitive appeal.

Croston theoretically demonstrated the superiority of his method over exponential smoothing when assuming particular distributions for the arrival and size of demands. Willemain *et al*. [88] subsequently reported the forecasting method proposed by Croston offered significant improvements in performance even when these assumptions were violated. The authors created simulated demand data and also used real-world data from industrial sources for their comparisons with smoothing constants ranging between 0.01 and 0.9. Interestingly they remarked that they placed their emphasis "*on the MAPE for one-step-ahead forecasts, comparing forecasted values per period with actual values (both zero and nonzero)*" although they do not specify how they managed to calculate MAPE when the actual values are zero.

Despite the initial praise given to Croston's method, the results from the industrial data showed only modest benefits compared to the results from the simulation data. An analysis using large quantities of actual data would provide a useful comparison as far as the practitioner is concerned. In an inventory modelling context it would also be useful to make the analysis over a lead-time period rather than just the one-period ahead, which is reported in the vast majority of the literature.

The modest performance of Croston's method with real data recently prompted Syntetos and Boylan [76,77] to examine Croston's methodology. They found a mistake was made in Croston's mathematical derivation and as a result the method fails to produce accurate estimates of the demand per period. After quantifying the bias associated with Croston's method the authors provide three modifications which attempt to improve the performance. The three modified methods are only slightly more complex than Croston's method and may provide useful alternatives should the original prove unsatisfactory with actual RAF data.

An investigation of the forecasting literature reveals that there is no general agreement on the best measure of forecasting accuracy. Each measure has its advantages as well as disadvantages and it is necessary to examine the forecasting context in order to determine which measure is the most suitable. In an inventory context the methodology of Wemmerlöv's [84] which compares stock-holding and ordering costs given a 100 percent service level, may provide a useful means for comparing forecast performance and is therefore worthy of further consideration.

In my view there is a requirement for a reasoned assessment as to which forecasting method offers the best all-round performance in a spare parts inventory setting, or indeed, in any environment where slow-moving and erratic demand is observed. Exponential smoothing is probably the most commonly used method in practice despite its widely publicised short-comings in the academic literature. In the thirty years since first being published, Croston's method has attained a lot of recognition although it has not attained a matching level of implementation. Despite the theoretical attractiveness of Croston's method it would appear the method does not provide an improvement in forecasting performance that warrants its implementation in reality. Recent researchers have identified an error in Croston's formulation and alternative methods have been proposed although their comparative performance with sufficient real demand data has not yet been examined.

The research that follows aims to make a contribution by using large quantities of actual demand data to compare commonly used forecasting models against those more recently proposed in the literature. Wemmerlöv's methodology will be developed as an additional performance measure which is more appropriate in an inventory context, thereby alleviating weaknesses of the traditional measures of accuracy.

The next chapter outlines some of the inventory characteristics facing the RAF. An examination of these characteristics allows the subsequent analysis to be put into context while also providing a means for assessing whether assumptions made by the published models are appropriate.

# 4. CHARACTERISTICS OF THE RAF INVENTORY

The RAF maintains central system parameters that combine to form a unique forecasting, ordering and stock-holding system. A classical periodic review inventory management system is operated, whereby replenishment levels and replenishment quantities are calculated for each line item based upon item parameters. A monthly review period is operated and the demand forecast is provided by the exponential smoothing of historical demand using aircraft fleet sizes as a driving factor. The replenishment level is based on a fixed lead-time parameter and safety stock is directly proportional to the mean level of the forecast demand. The replenishment quantity is calculated as a constrained economic order quantity (EOQ) which may be modified to accommodate policy-based minimum and maximum replenishment quantities. In effect, the RAF operates an $(s,S,T)$ policy which is approximated by a $(Q+r,r,T)$ policy where $T$ is the review period of one month in both cases. Thus, at the time of review if the stock level for a line is less than or equal to $Q + r$ - *minimum replenishment* then a quantity is ordered; otherwise no order is placed.

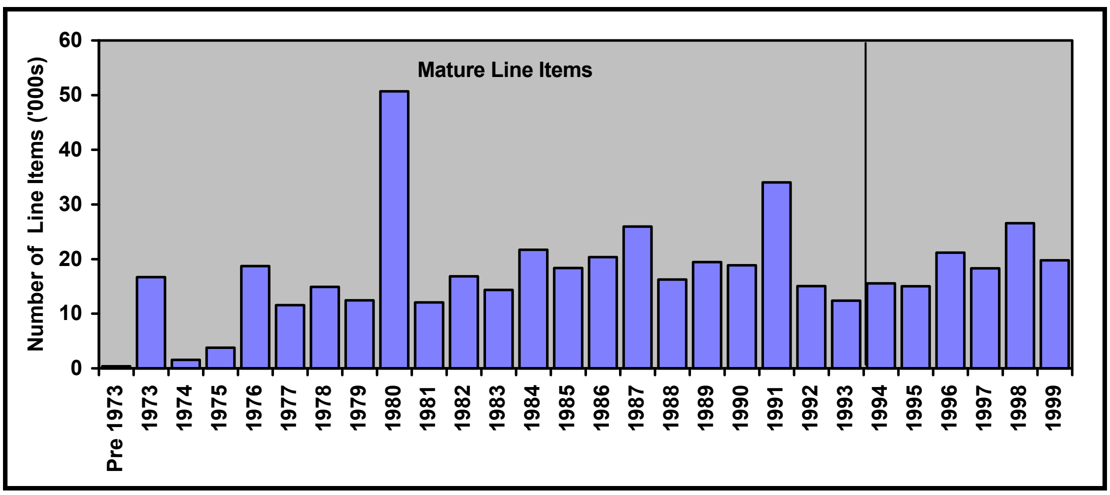
Each line item has up to 60 fields of information that affect the manner in which stock is held and reprovisioned. These fields, for example, determine whether stock can be reprovisioned automatically, whether the line item is obsolete, the minimum stocks to be held in case of war, as well as several parameters for calculating the replenishment level and replenishment quantity.

This chapter describes those parameters that affect the consumable line items considered by this study. Not only is it important to know which parameters play a role but also the scale of their involvement. In this manner it is possible to assess whether assumptions made by the published models are appropriate in the RAF context.

## 4.1 Initial Provisioning Date

Within the RAF individual demand histories will only extend as far back as the date of initial provisioning (IP) for each line item. An IP date generally comes into effect when the aircraft, or major assembly to which the item is fitted, comes into service. Although some 28 percent of current line items do not have a recorded IP date, the rate at which the remaining line items have entered service over the years is illustrated in Figure 4.1.

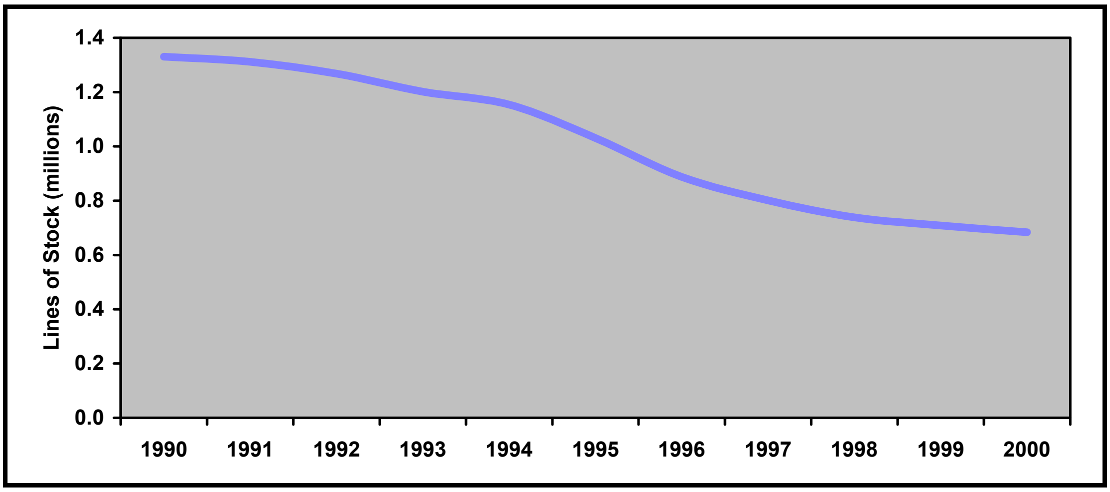
**Figure 4.1: Initial Provisioning Date for Consumable Line Items.**




An initial surge in IP requirements can be seen in 1973 with the introduction of the *Jaguar*, the RAF's longest serving combat aircraft. A larger surge in IP requirements followed in 1980 with the introduction of the *Tornado*, another combat aircraft. The date of introduction of other aircraft can also be identified, such as *AWACS*, an airborne command and control system, in 1991.

As new aircraft types enter service, others reach the end of their planned life cycle and each year a number of line items become obsolete. The RAF consumable inventory has undergone significant reductions in recent years as illustrated by the stock quantities of Figure 4.2, where on average there is a net decrease of 62,000 line items each year.

**Figure 4.2: Total RAF Consumable Inventory.**




Given the continual evolution of the RAF inventory, any demand analysis will need to consider whether a line item is mature, whereby the IP date is greater than six years, so as to provide sufficient data. The removal of non-mature line items from the current inventory leaves some 376,000 for which complete demand histories are available.

## 4.2 Replenishment Order Quantity

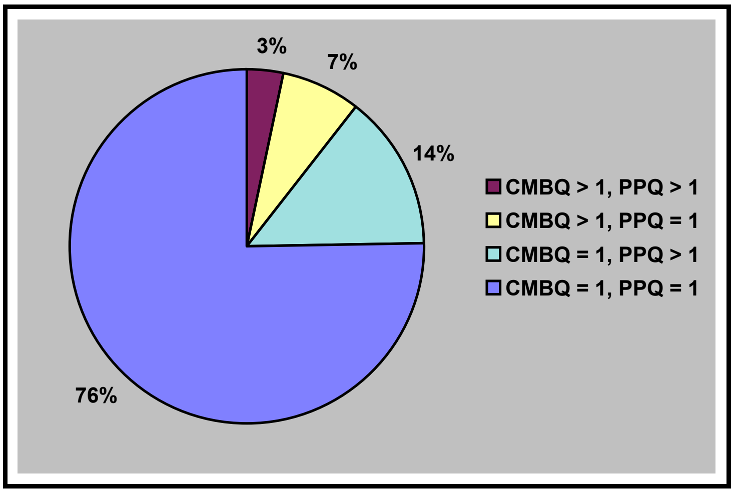
A replenishment order quantity placed by the RAF may be constrained by the supplier in two ways:

(i) The contractor's minimum batch quantity (CMBQ). The CMBQ is the minimum quantity of a line item that a manufacturer is prepared to supply and is therefore the minimum size of a replenishment order.

(ii) A primary packaged quantity (PPQ). The PPQ indicates the quantity of a line item contained within a single package and therefore the replenishment order must be a multiple of this.

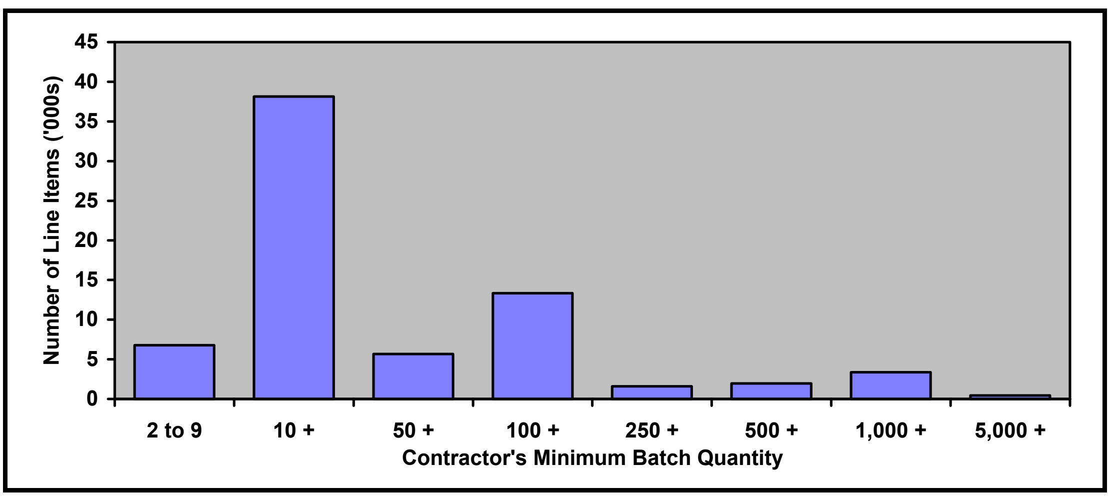
Figure 4.3 presents the frequency in which CMBQ and PPQ restrictions apply to consumable line items. The majority do not have any restrictions, although 24 percent of line items are constrained by a CMBQ and/or a PPQ.

**Figure 4.3: Assigned CMBQ and PPQ Frequencies.**



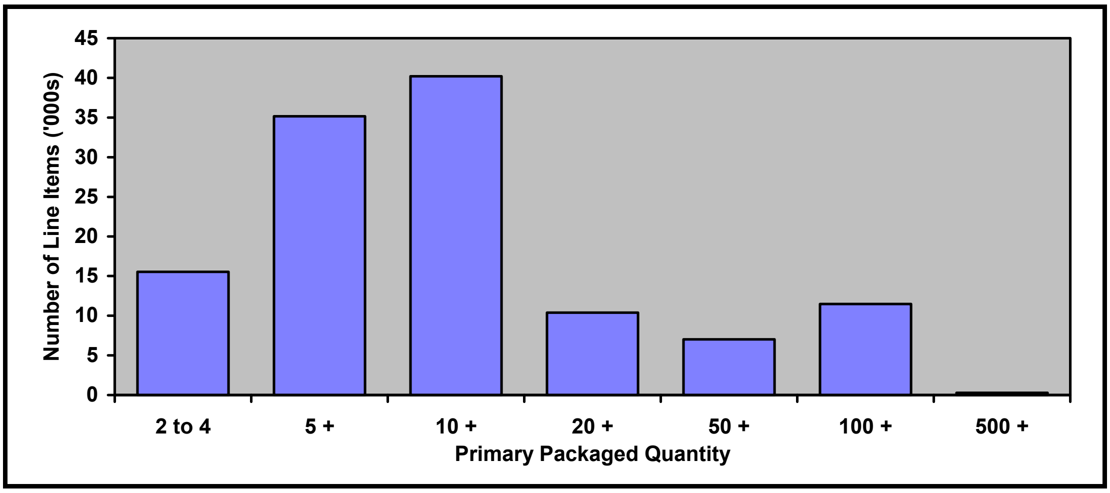

For those line items where a CMBQ acts as a constraint on the minimum replenishment order quantity the effect is likely to be significant. Figure 4.4 presents the frequency at which a CMBQ value is active.

**Figure 4.4: Assigned Contractor's Minimum Batch Quantity (CMBQ) Values.**



Similarly, the PPQ can have a significant influence on the size of the replenishment quantity as illustrated in Figure 4.5, which presents the frequency at which a PPQ value is active.

**Figure 4.5: Assigned Primary Packaged Quantity (PPQ) Values.**




As a result, the CMBQ and PPQ quantities need to be considered when developing replenishment parameters.

## 4.3 Replenishment Lead-Time

The replenishment lead-time is a fundamental component of any inventory management system. Each line item in the RAF inventory is assigned two lead-time parameter values which combine to give a total lead-time value. The two components of the lead-time are:

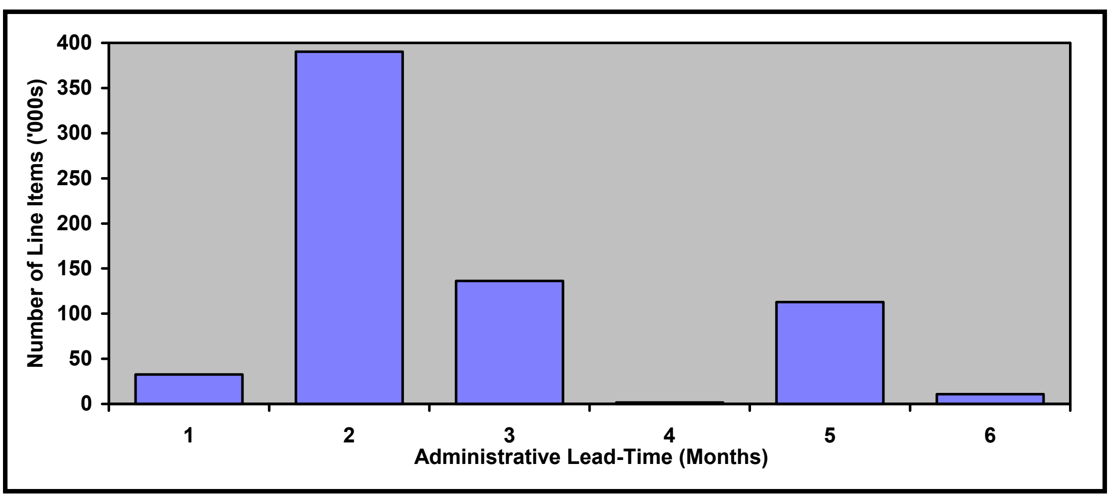
(i) The administrative lead-time (ALT). The ALT is the number of months taken to process a stock replenishment request through to the point where a contract is placed with the manufacturer.

(ii) The purchasing lead-time (PLT). The PLT is the number of months taken by the manufacturer to produce and deliver a replenishment order.

Both the set ALT and the set PLT are recognised to be poor reflections of the true lead-time as they are rarely updated on the central computer system. Furthermore, many line items do not require reprovisioning beyond their initial provisioning and therefore no steady state lead-time data is available, prompting the use of default values.

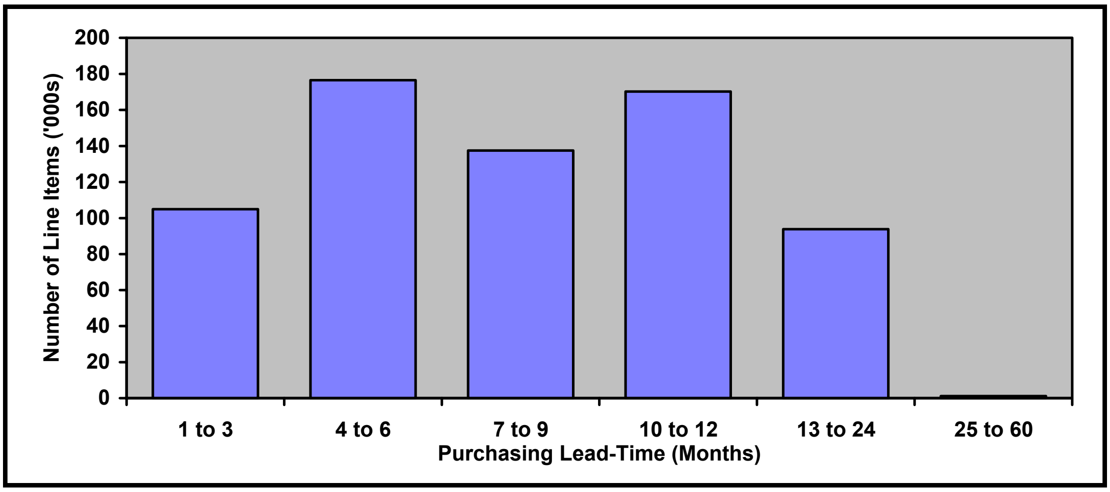
The ALT can contribute a surprisingly significant portion to the total lead-time as illustrated by Figure 4.6, which shows that the set value can be anything up to six months. The average ALT is 2.7 months with a standard deviation of 1.2 months.

**Figure 4.6: Administrative Lead-Time (ALT).**




With the PLT component taking up to 60 months as illustrated in Figure 4.7, the total lead-time can be quite significant. The average PLT is 8.9 months with a standard deviation of 4.4 months.

**Figure 4.7: Purchasing Lead-Time (PLT).**




Combining ALT and PLT gives an average and standard deviation for the total lead-time of 11.6 and 4.7 months respectively.

## 4.4 Stock-Holdings

As a matter of policy, and due to difficulties in forecasting actual requirements, the RAF has traditionally maintained stock-holdings that may be considered excessive in other industries. Reasons behind the large stock-holdings can be summarised as:

(i) The necessity to maintain adequate stocks in case of war and the likely increase in usage combined with a disruption to the supply chain.

(ii) The long and variable replenishment lead-times in the defence industry and the subsequent high cost and risk of stock-out.

(iii) The relatively high cost of procurement beyond initial provisioning due to the RAF being the sole user of many components. This tends to lead to large quantities of stock being procured at the time of initial provisioning.

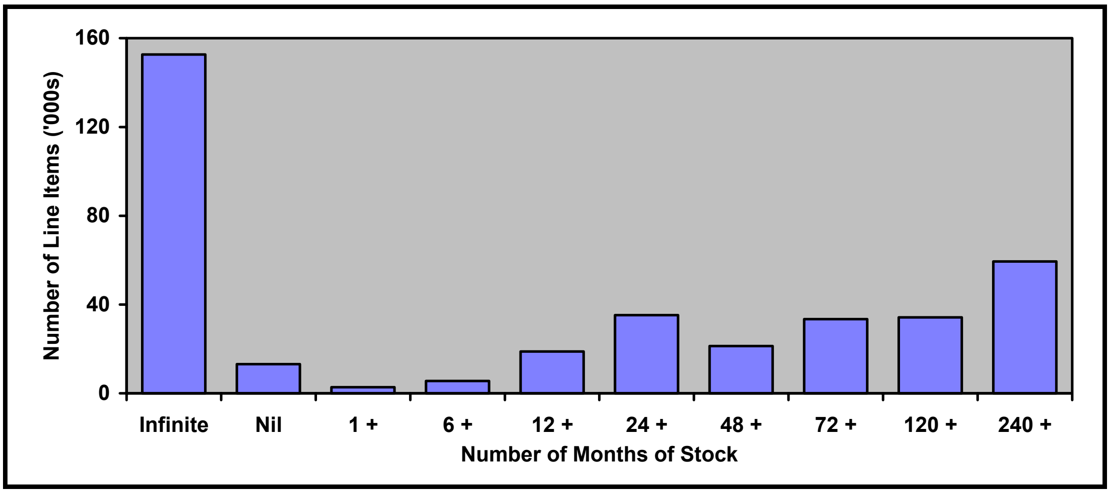
(iv) The exceptionally high cost of procuring new stock as opposed to holding stock, meaning near-obsolete line items are retained rather than disposed of.

(v) The fact that any item can be crucial and a stock-out may ground an aircraft.

(vi) The wide geographic dispersal of RAF stations and the necessity for each to maintain stocks of what are in fact common user spares.

Figure 4.8 summarises the stock-holding status of current line items that have been in existence for six years or more. The unit of measurement is the number of months of stock remaining (MOSR) in the system, assuming future usage is equal to the historic average monthly usage.

**Figure 4.8: Months of Stock Remaining (MOSR).**




The first bar represents those line items for which there have been no demand requirements in the past six years and therefore an infinite number of months of stock remains, while the second bar represents those line items for which total demand requirements have exceeded the available stock. The remaining bars show the number of months of stock that remain for all other line items.

With a median MOSR of 85 months, or about 7 years, the implication for any stock replenishment policy based upon stocks currently held is that many line items will not require a reprovisioning action for a substantial period.

## 4.5 RAF Demand Analysis

The large quantity of unprocessed demand data available to the RAF allows a detailed analysis of the demand patterns for consumable spare parts. This section initially examines the annual usage-value of the consumable inventory, and continues with a large scale analysis of both the demand sizes and the intervals between transactions in the RAF data.

### 4.5.1 Annual Usage-Value

In terms of practical inventory control, the potential monetary savings that can be achieved by the use of mathematical models on an individual line item basis are relatively small. For slow-moving or low value items the net benefits of a sophisticated inventory control model may in fact be negative and it is therefore important to identify a suitable level of control. One commonly used classification method is the *usage-value* criterion, which in this case utilises the annual monetary value of demand.

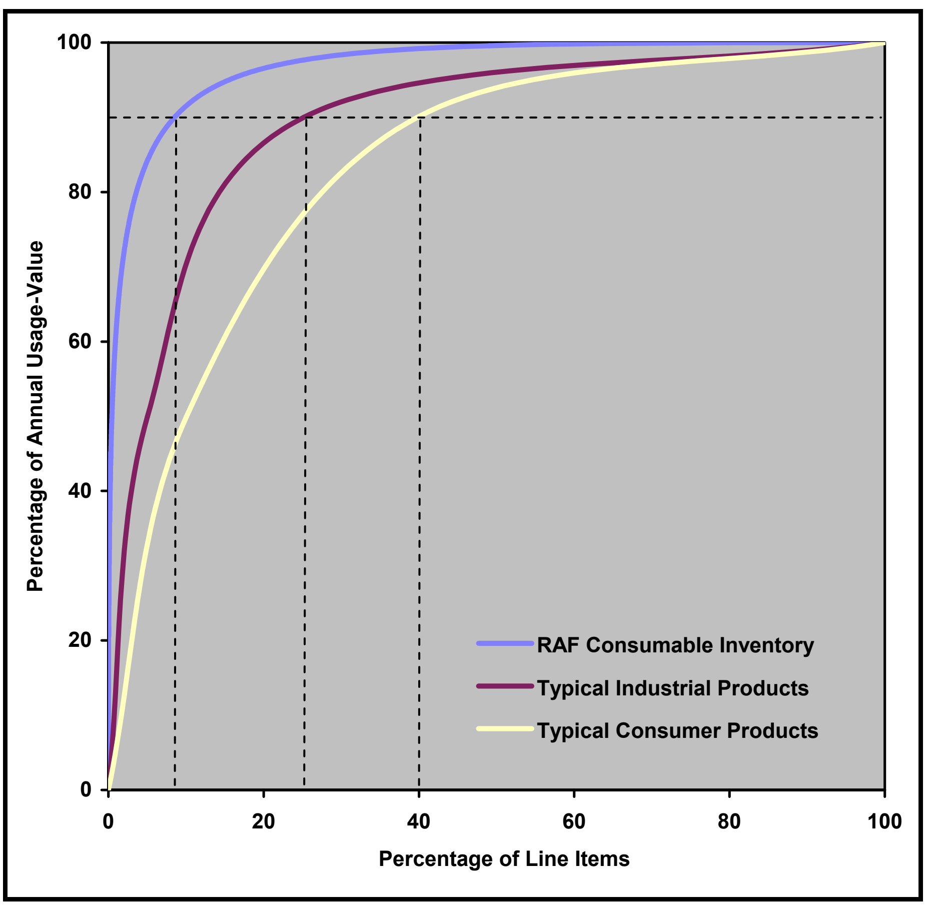
Over the 72 month period from January 1994 to December 1999 some 59.5 percent of RAF line items with an IP date greater than six years recorded positive demand. Table 4.1 provides a summary of the unit value and annual demand for these line items, as well as the annual usage-value calculated as the average annual demand multiplied by the unit value on an individual line item basis. The RAF records unit values to the nearest penny as seen in the table.

**Table 4.1: Summary Statistics for Unit Value and Demand.**

| Statistic | Unit Value (£) | Annual Demand (Units) | Annual Usage-Value (£) |
|---|---|---|---|
| Mean | 246.07 | 90.7 | 2,120.97 |
| Maximum | 654,938.27 | 380,955.5 | 7,830,902.81 |
| Upper Quartile | 112.99 | 12.0 | 371.07 |
| Median | 22.09 | 2.5 | 62.22 |
| Lower Quartile | 3.53 | 0.6 | 10.39 |
| Minimum | 0.01 | 0.2 | 0.02 |

It is observed that all three data series are heavily skewed to the right as the means far exceed their respective medians. This suggests a large number of line items with a low usage-value, and that is without even considering the 40.5 percent of line items with zero usage-value.

**Figure 4.9: Distribution of Annual Usage-Value.**



The suggestion of a large number of line items with a low usage-value is confirmed by the plot of Figure 4.9. This figure presents the cumulative percentage of the annual value of demand in descending rank against the cumulative percentage of line items from actual RAF data. Once again, the 40.5 percent of line items with zero usage-value are excluded. Also shown in the figure are similar plots for typical industrial products and typical consumer products.

In the case of the RAF consumable inventory it is seen that just 8.5 percent of line items account for 90 percent of the annual value of demand. By way of contrast, Daellenbach *et al*. [21] suggest that in the case of typical industrial products some 25 percent of line items account for 90 percent of the annual value, and, in the case of typical consumer products, some 40 percent of line items account for 90 percent of the annual value. The RAF usage-value is therefore considered far from typical.

An analysis of an inventory by usage-value can help identify a suitable level and type of control for each line item stocked. Potential savings that can be achieved with inventory control models are relatively low on an individual item basis, while the cost of such control models, including demand forecasting, may be relatively high. The net benefits of a sophisticated inventory control system may actually be negative for slow-moving or low value items, while the control offered by a computerised system may not be tight enough for expensive or high volume items.

Classifying line items based on their usage-value into at least three groups, referred to as the *ABC classification*, may be desirable. Daellenbach *et al*. [21] suggest the exact percentage breakdown appropriate for a given organisation will vary anywhere between 5-10-85 and 10-30-60. Thus, the first 5 to 10 percent of line items (as ranked by their usage-value) are designated as A items, accounting for about 50 percent of the total value; the next 10 to 30 percent of the ranked line items, accounting for about 40 percent of the total value, are designated as B items; while the remaining 60 to 85 percent of line items, accounting for the remaining 10 percent of the total value, form the C class of items.

The degree of control exercised over each line item is tailored according to the ABC classification, with class A items receiving the highest degree of individual attention. Demand for class A items is forecasted for each line item individually, considerable effort is made to keep tight control over replenishment lead-times, and safety stocks can be kept relatively small, thereby allowing savings in inventory investment since stock control is tight. Since they cover no more than 10 percent of all line items, the cost of the extra effort in maintaining tight control is kept within strict limits.

At the other end of the scale are the C items where the objective is to maintain adequate control in an inexpensive manner. C items are often group controlled with items classified into subgroups with similar characteristics. It should be borne in mind that any one of the large number of C class items has the potential to have serious consequences for aircraft availability if shortages occur. For instance, stock-outs of inexpensive rivets or diodes may keep an aircraft grounded just the same as a more expensive electronic item, and therefore, large safety stocks are kept to ensure minimal shortages.

### 4.5.2 Demand Patterns

A glance at Figure 4.10, which presents the frequency of demand transactions over a six-year period, suggests that a large proportion of line items in the RAF inventory are slow-moving; approximately 40.5 percent of line items had no demand over six years, while 37.3 percent experienced fewer than 10 transactions.

**Figure 4.10: Number of Demand Transactions over a Six-Year Period.**

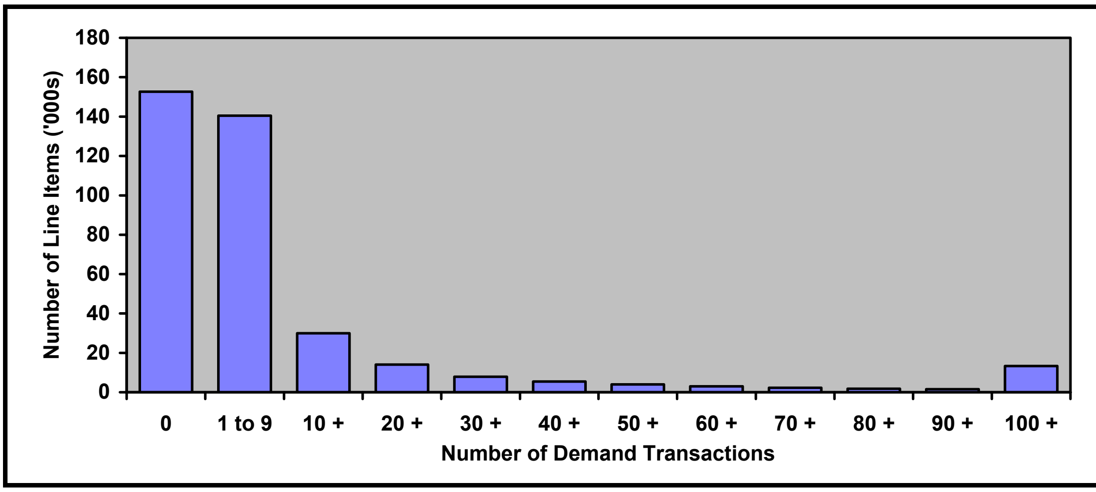



Table 4.2 presents demand size statistics for specific transaction frequency groupings. Line items experiencing between 1 and 9 transactions over the six-year period have an average demand size of 7.6 units. At the other end of the scale, 3.5 percent of line items experienced 100 or more transactions with an average demand size of 19.6 units. Such line items are more likely to have a smooth demand pattern.

**Table 4.2: Demand Statistics by Transaction Frequency.**

| Number of Demand Transactions | Percentage of Line Items | Demand Size | |
|---|---|---|---|
| | | Average | Average CV |
| 0 | 40.5% | - | - |
| 1 to 9 | 37.3% | 7.6 | 44.0% |
| 10 to 99 | 18.6% | 10.9 | 77.2% |
| 100 + | 3.5% | 19.6 | 127.9% |

In between the slow-moving line items, and the line items considered to have a smooth demand, lie a significant number that have infrequent transactions and are therefore likely to have an erratic demand pattern. Some 70,000 line items, or 18.6 percent, recorded between 10 and 99 demand transactions with an average demand size of 10.9 units.

In determining whether these line items do in fact experience erratic demand it is also necessary to examine the variability of the demand size. The average coefficient of variation (CV) across each line item is 77.2 percent in this case. Such a high value appears to confirm the notion of erratic demand in the RAF inventory.

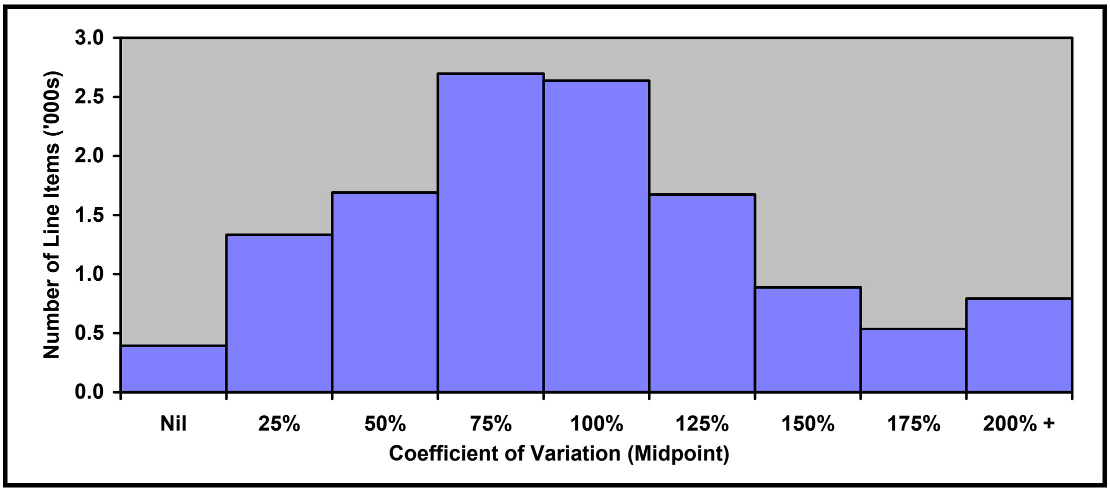
Over a 72 month period some 12,644 line items in the RAF inventory recorded between 50 and 99 demand transactions, or equivalently, between 0.69 and 1.38 transactions per month on average. These are considered arbitrary boundaries for erratic demand at this stage. It is likely that the lower bound is too high for capturing erratic demand in this instance, but an important consideration is that there are enough observations for the analysis that follows.

The next section involves a more detailed examination of the demand size and interval between transactions for those line items initially considered to have an erratic demand pattern. In all cases individual transactions are analysed such that there is no aggregation of demand.

### 4.5.3 Demand Size

The CV of the demand size has been calculated as a measure of randomness and the plot of Figure 4.11 suggests the demand sizes are moderately variable with the majority of line items having a CV greater than 50 percent and a significant proportion having a CV greater than 100 percent. Of particular interest is the fact that nearly 400 line items had a constant demand size, and in all such cases the demand was for one unit. The average CV is 96.8 percent for line items with a non-constant demand size.

**Figure 4.11: Coefficient of Variation - Demand Size.**



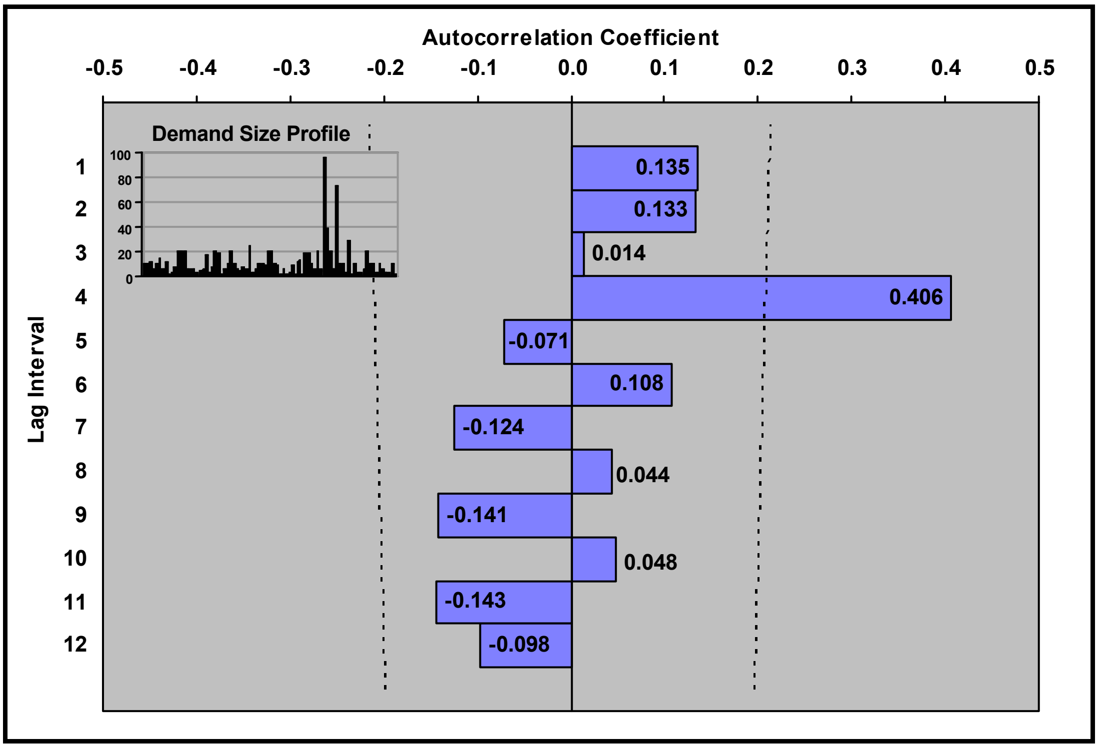

Autocorrelation can be used to determine whether a given series is random by measuring the extent to which a variable measured over time is correlated with itself when lagged by one or more periods. Pearson's autocorrelation sample estimate at lag $k$ is calculated as:

$$r_k = \frac{\sum_{n=k+1}^{N}(x_n - \bar{x})(x_{n-k} - \bar{x})}{\sum_{n=1}^{N}(x_n - \bar{x})^2} \tag{3}$$

where $N$ is the total number of observations.

A plot of autocorrelations against their lags, referred to as a correlogram, is illustrated in Figure 4.12 for a sample line item. In the case of demand size, positive autocorrelations occur when a high demand is matched with another high demand or a low demand is matched with a low demand. On the other hand, negative autocorrelations occur when a high demand is matched with a low demand or vice-versa.

**Figure 4.12: Sample Correlogram - Demand Size.**




If the series is random, Box and Pierce [12] show that autocorrelation coefficients from equation (3) are approximately normal with mean zero and variance $(n-k)/(n(n+2))$ at lag $k$. The coefficients are examined to see if any are significantly different from zero using an acceptance region of:

$$\pm Z\left(\sqrt{\frac{n-k}{n(n+2)}}\right) \tag{4}$$

For the sample data with $n = 82$, the confidence limits from equation (4) at the 5 percent significance level are shown in Figure 4.12 as the two near-vertical dotted lines. Twelve lags are examined and one autocorrelation, $r_4 = 0.406$, is identified as significant, indicating a pattern in the demand size based on lags of four. The interpretation of the significance level for hypothesis testing is presented in Appendix B.

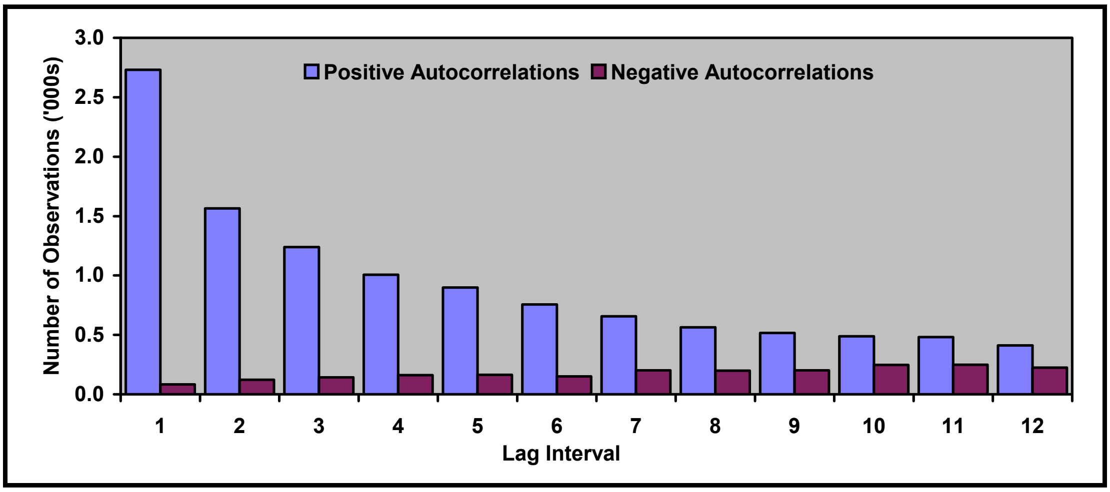
Inset in Figure 4.12 is the demand profile for the sample line item showing the size of successive demand requests over a 72 month period. The plot is dominated by two large requests that are four transactions apart, strongly contributing to the significant autocorrelation at lag 4. This observation illustrates a weakness of Pearson's method for determining the presence of autocorrelation in demand data; the method assumes the data is normally distributed and is therefore not a practical measure when the data is skewed, as in this example.

An investigation of alternative methods for determining whether there is autocorrelation in demand data that is more than likely skewed is presented in Appendix C. For purposes of convenience it is necessary to perform the autocorrelation analysis using only one of the identified methods. A natural logarithm transformation method was selected mainly because it examines the demand sizes rather than ranks or comparisons against the median. The method was also found to be effective in removing the variability from skewed data while being easy to implement.

Using a logarithm transformation on successive demand sizes, it was observed in Appendix C that 3,129 or 25.5 percent of the 12,251 selected line items with a non-constant demand size are significantly autocorrelated as a whole. Some 9.2 percent of autocorrelation coefficients are individually significant, of which 84.0 are positively correlated and the remaining 16.0 percent are negatively correlated.

Figure 4.13 summarises the lag interval in which individually significant autocorrelation coefficients occur. It is seen that positive autocorrelations dominate the results, with the early lags arising more frequently. On the other hand, in the case of significant negative autocorrelations, there is a pattern of increasing frequency as the lag interval increases.

**Figure 4.13: Lag of Significant Autocorrelation Coefficients - Demand Size.**




The next section provides a similar analysis of the interval between transactions, prior to a combined analysis of demand sizes and intervals.

### 4.5.4 Interval Between Transactions

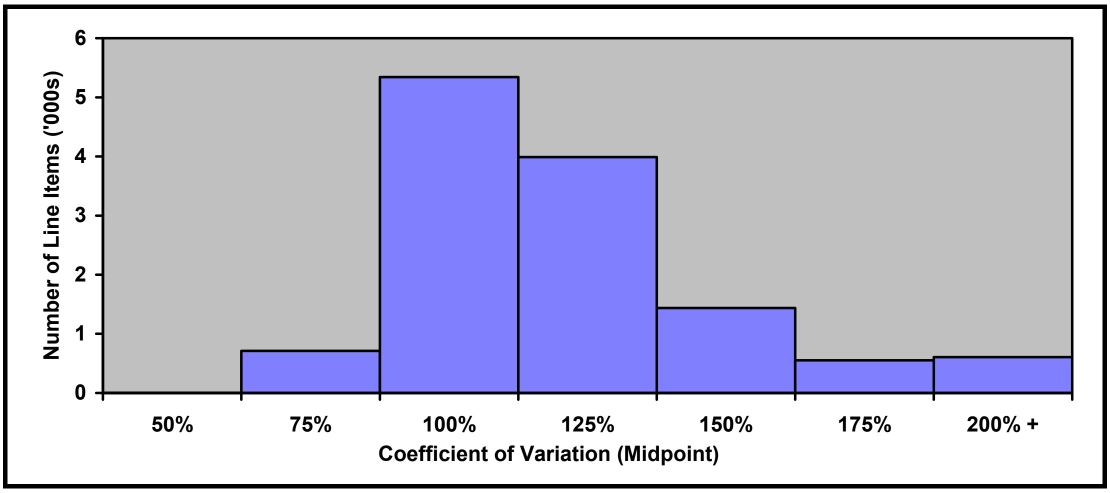
Although defence is quite obviously a 7-day a week operation, the spread of demand transactions during the week for the line items under consideration, as presented in Table 4.3, suggests transactions should be reassigned to weekdays only. Thus, in this analysis the few demand transactions placed on a Sunday have been moved forward by one day and those placed on a Saturday have been moved back by one day.

**Table 4.3: Transactions by Weekday.**

| Day of Week | Percentage of Transactions |
|---|---|
| Sunday | 0.65% |
| Monday | 19.56% |
| Tuesday | 21.41% |
| Wednesday | 21.63% |
| Thursday | 20.55% |
| Friday | 14.53% |
| Saturday | 1.67% |

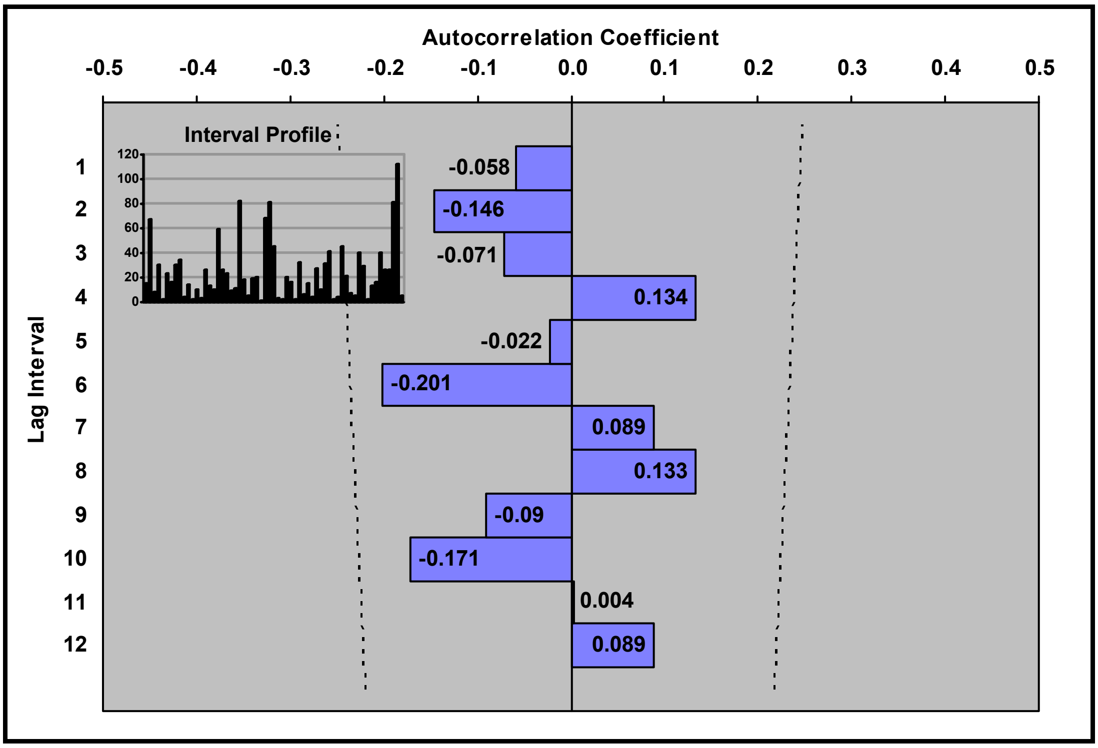
The CV for the interval between transactions reveals the interval is highly variable, as seen in the plot of Figure 4.14, where the majority of line items have a CV greater than 100 percent. The average CV is 121.9 percent in this instance.

**Figure 4.14: Coefficient of Variation - Interval Between Transactions.**




A correlogram for the interval between transactions for the previously considered sample line item with a logarithm transformation is presented in Figure 4.15, while the profile inset presents the non-transformed series. As logarithms are undefined for zero values, it was necessary to transform the data into daily demand to eliminate intervals of zero duration. In this case, positive autocorrelations occur when a long interval is matched with another long interval or a short interval is matched with a short interval. On the other hand, negative autocorrelations occur when a long interval is matched with a short interval or vice-versa. In this instance, none of the autocorrelation coefficients are significant at the 5 percent significance level. The calculated $Q$-statistic of 11.287 is less than the tabulated $\chi^2$ value of 18.307 and therefore the intervals between transactions are considered random as a whole.

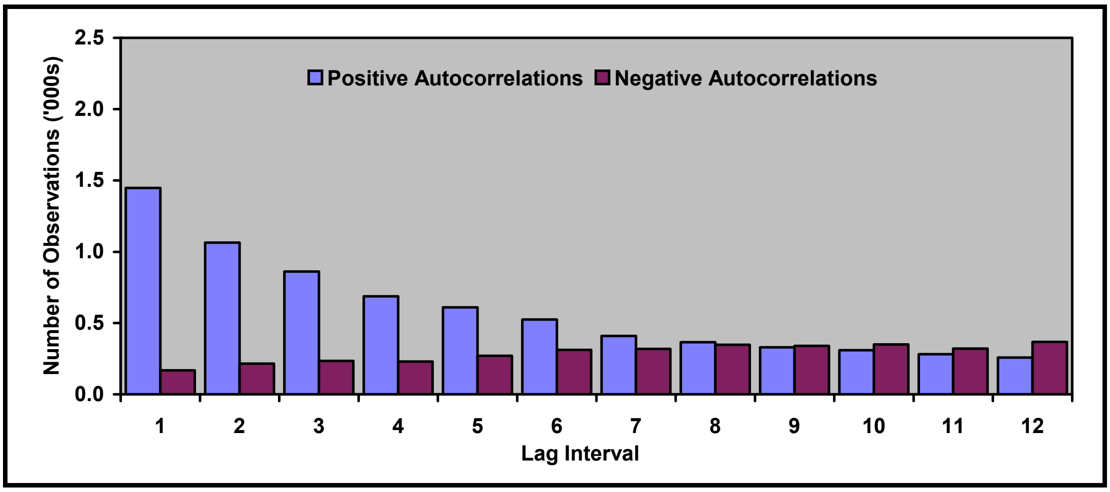
**Figure 4.15: Sample Correlogram - Interval Between Transactions (Log Transform).**




An examination of the autocorrelation coefficients for the interval between transactions for all sample line items reveals that 7.0 percent of coefficients are individually significant, of which 67.3 percent are positively correlated with the remaining 32.7 percent negatively correlated. Some 49.0 percent of line items contain individually significant autocorrelation coefficients, although only 19.2 percent of line items are significantly autocorrelated as a whole at the 5 percent level.

The lag intervals in which the individually significant autocorrelation coefficients occur are summarised in Figure 4.16. The results show that positive autocorrelations again dominate, with the early lags occurring more frequently. The significant negative autocorrelations again show a pattern of increasing frequency as the lag interval increases, and in this case the negative autocorrelations are more frequent than the positive autocorrelations at the higher lags.

**Figure 4.16: Lag of Significant Autoc'n Coef's - Interval Between Transactions.**




The next section examines crosscorrelations between the demand sizes and the intervals between transactions.

### 4.5.5 Demand Size and Interval Between Transactions

In order to obtain the demand size, and the appropriate transaction interval for this part of the analysis, each transaction interval is matched with the demand that occurs after, rather than before, the interval. The interpretation is that the demand size is a function of the time since the previous demand, rather than the time until the next transaction being a function of the current demand size.

The manner in which the demand interval is matched with a demand size is demonstrated in Table 4.4 for another selected line item. As a first step, any demands placed on a non-weekday are reassigned to the closest weekday, as shown by the reassignment of the demand placed on the Saturday to the preceding Friday, and the interval is then calculated as the number of weekdays since the previous transaction. The first demand that does not have an assigned transaction interval is discarded.

**Table 4.4: Example Interval Between Transaction Calculations.**

| Actual Demand Date | Weekday Assigned Demand Date | Transaction Interval (days) | Demand Size |
|---|---|---|---|
| Wednesday March 20, 1996 | Wednesday March 20, 1996 | - | 44 |
| Wednesday March 20, 1996 | Wednesday March 20, 1996 | 0 | 16 |
| Tuesday March 26, 1996 | Tuesday March 26, 1996 | 4 | 100 |
| Saturday March 29, 1996 | Friday March 28, 1996 | 3 | 20 |
| Wednesday April 3, 1996 | Wednesday April 3, 1996 | 3 | 14 |
| ⋮ | ⋮ | ⋮ | ⋮ |

Crosscorrelation measures the extent to which a variable measured over time is correlated with another variable when lagged by one or more periods. The sample crosscorrelation between $x$ and $y$ at lag $k$ is given by:

$$r_{xy}(k) = \frac{c_{xy}(k)}{\sqrt{\sigma_x^2 \sigma_y^2}}$$

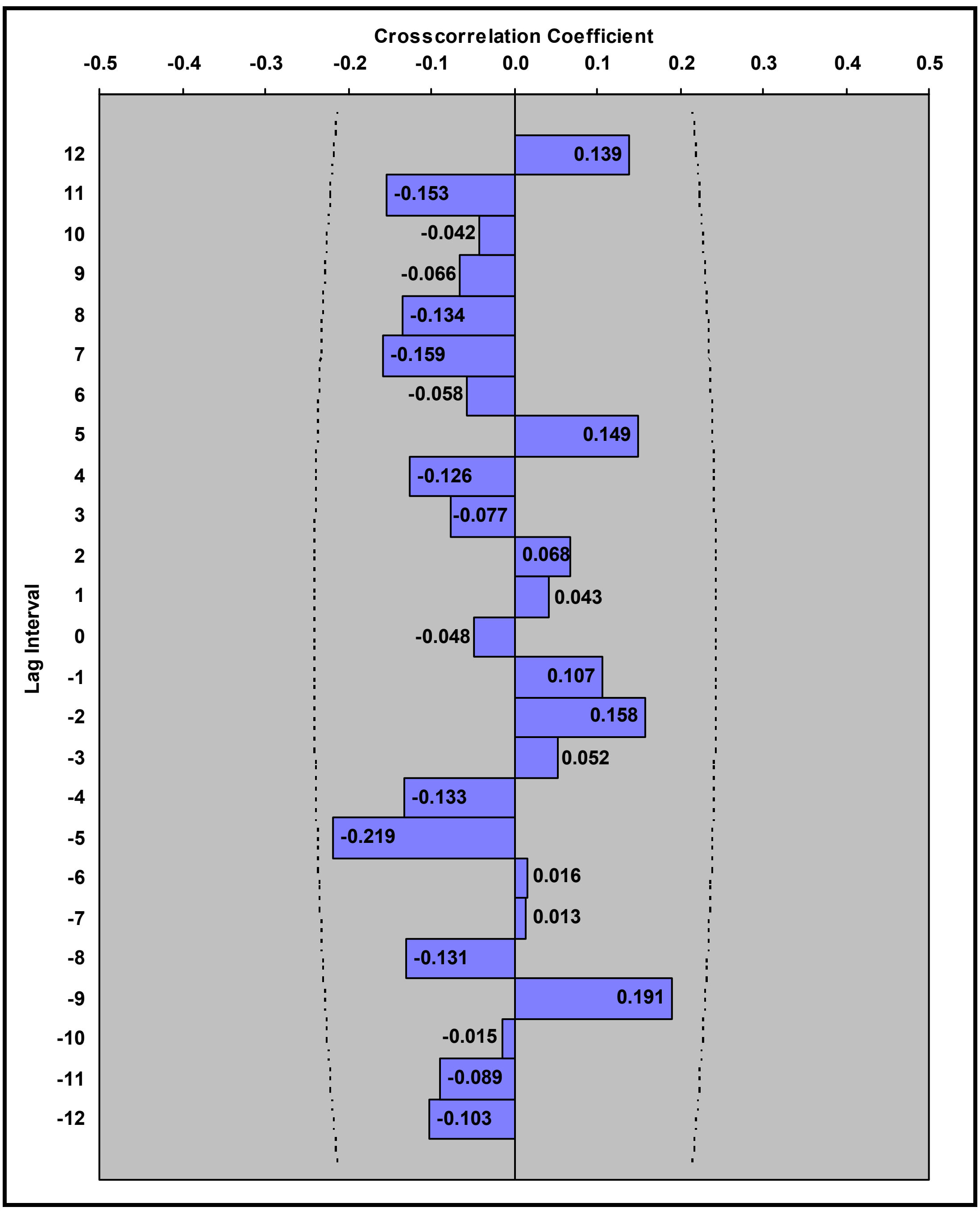
$$\text{where } c_{xy}(k) = \begin{cases} \dfrac{\sum_{n=1}^{N-k} (x_n - \bar{x})(y_{n+k} - \bar{y})}{N} & k = 0,1,\ldots,(N-1) \\[2ex] \dfrac{\sum_{n=1-k}^{N} (x_n - \bar{x})(y_{n+k} - \bar{y})}{N} & k = -1,-2,\ldots,-(N-1) \end{cases}$$

is the sample cross covariance function, and $\sigma_x^2$ and $\sigma_y^2$ are the population variances for observations on $x_n$ and $y_n$ respectively.

Positive crosscorrelations occur when a high demand is matched with a long interval or a low demand is matched with a short interval. On the other hand, negative crosscorrelations occur when a high demand is matched with a short interval or a low demand is matched with a long interval.

Crosscorrelation coefficients for the logarithms of demand size and intervals between transactions are illustrated in Figure 4.17. The correlogram presents the negative lags (demand size leading), the positive lags (transaction interval leading) and the zero lag for pairwise combinations. No coefficients are deemed individually significant at the 5 percent level. In considering the crosscorrelations as a whole, the calculated $Q$-statistic of 23.052 is less than the tabulated $\chi^2$ value of 35.173 at the 5 percent level with 23 degrees of freedom, therefore the two series are considered independent of each other.

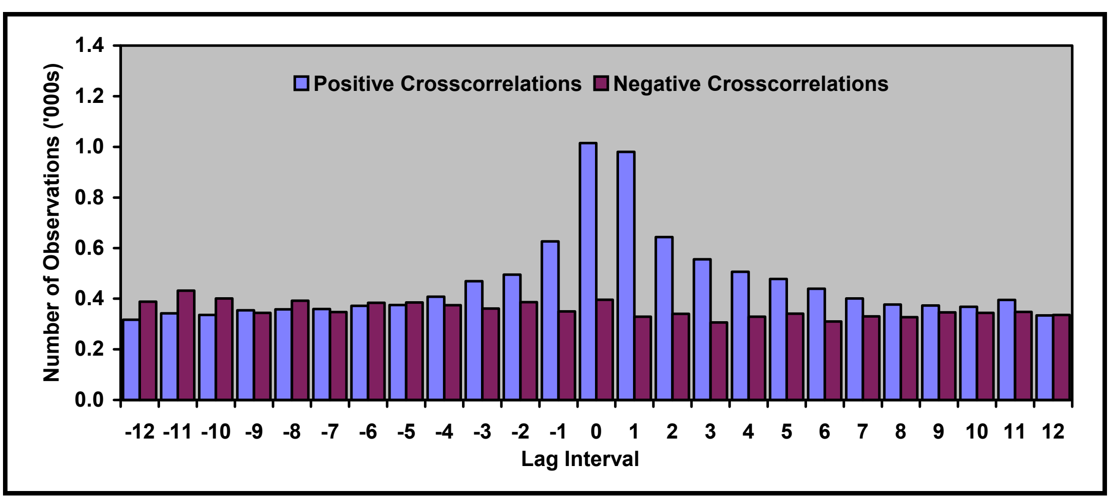
**Figure 4.17: Sample Correlogram - Demand Size and Interval Crosscorrelation.**



Examining the crosscorrelations for all sample line items reveals 6.5 percent of coefficients are individually significant, of which 56.7 percent are positive with the remaining 43.3 percent negative. Some 76.0 percent of line items contain individually significant crosscorrelation coefficients, although only 17.8 percent of line items are significantly crosscorrelated as a whole at the 5 percent level.

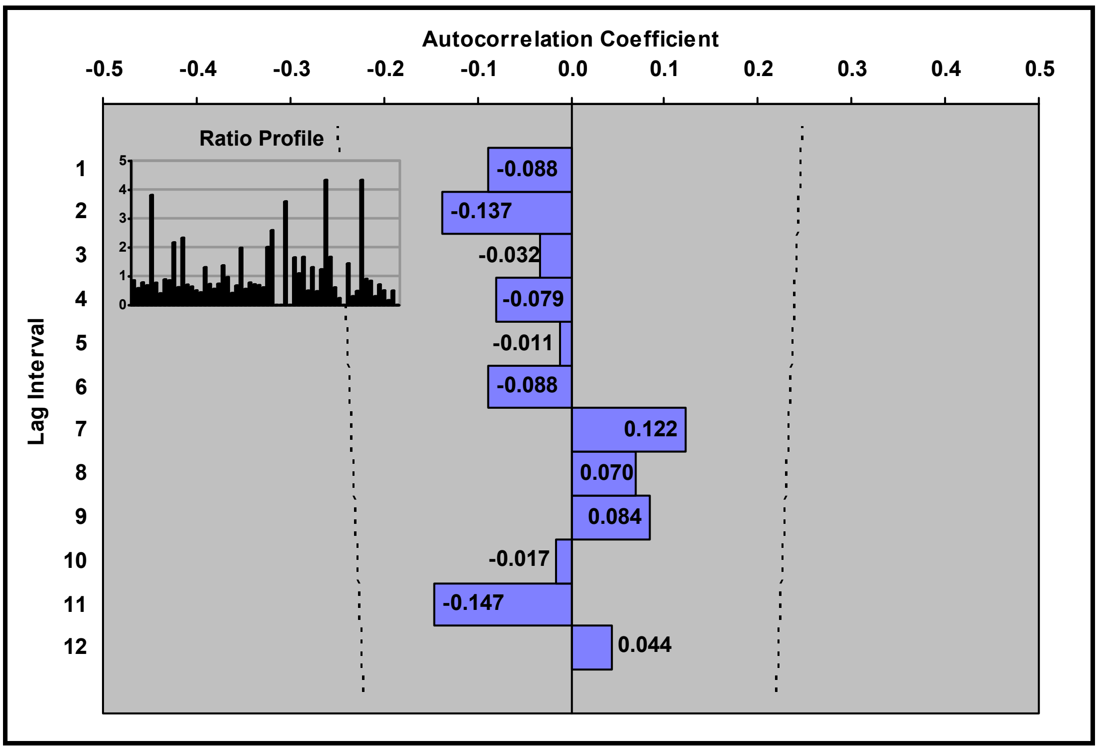
Figure 4.18 summarises the lags and leads, as negative (demand size leading), zero, and positive (transaction interval leading), in which individually significant crosscorrelation coefficients occur. Significant positive crosscorrelations are more frequent than negative crosscorrelations overall and show a clear dominance at the shorter lags. A small majority of positive crosscorrelations lie to the right of the peak at lag zero, indicating a tendency for shorter intervals to be followed by a lower demand or longer intervals to be followed by a higher demand. On the other hand, negative crosscorrelations tend to be more evenly spread across the range of lag intervals.

**Figure 4.18: Lag of Significant Crosscorrelation Coefficients.**




An alternative means of examining the relationship between the demand size and the interval between demands is to consider the autocorrelation of their ratio. A ratio can only be calculated when the logarithm of the interval between demands is greater than zero, implying an interval greater than one, otherwise remaining undefined. Figure 4.19 presents the correlogram for the sample line item for the ratio of the demand size and interval. Again, none of the autocorrelation coefficients are deemed significant at the 5 percent level. The calculated $Q$-statistic of 6.451 is less than the tabulated $\chi^2$ value, therefore successive demand size and interval ratio values are considered random.

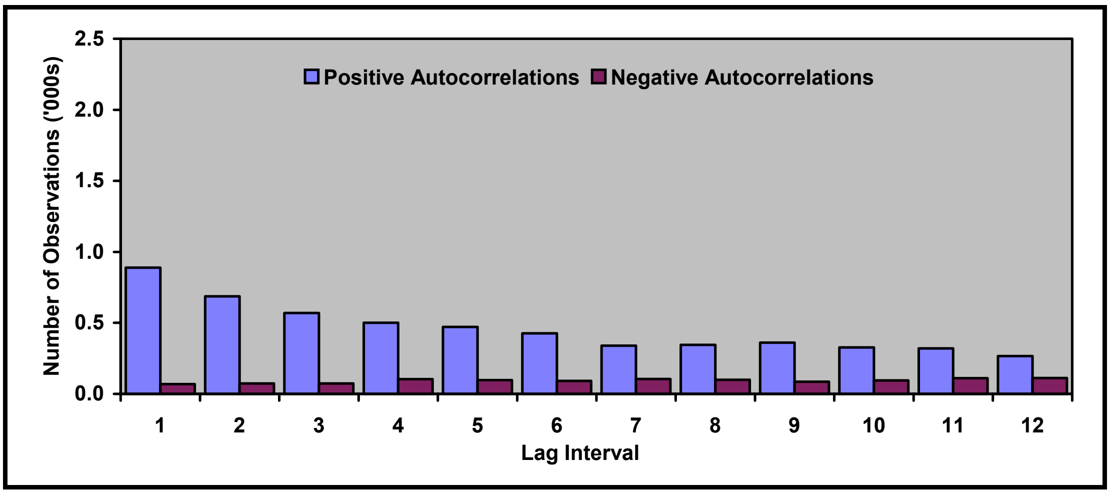
**Figure 4.19: Sample Correlogram - Demand Size and Interval Ratio.**




An examination of the autocorrelation coefficients for the demand size and interval ratio for all sample line items, indicates that 4.4 percent of coefficients are individually significant, of which 83.1 percent are positively correlated with the remaining 16.9 percent negatively correlated. Some 35.4 percent of line items contain individually significant autocorrelation coefficients, although only 10.1 percent of line items are significantly autocorrelated as a whole at the 5 percent level.

Figure 4.20 summarises the lag of significant autocorrelation ratio coefficients. Once again positive autocorrelations dominate the plot and show a decline in frequency as the lag interval increases. The significant negative autocorrelations marginally increase in frequency as the lag interval increases.

**Figure 4.20: Lag of Significant Autocorrelation Ratio Coefficients.**




The observed autocorrelation and crosscorrelation results from the RAF sample data are summarised in the next section.

### 4.5.6 Autocorrelation and Crosscorrelation Summary

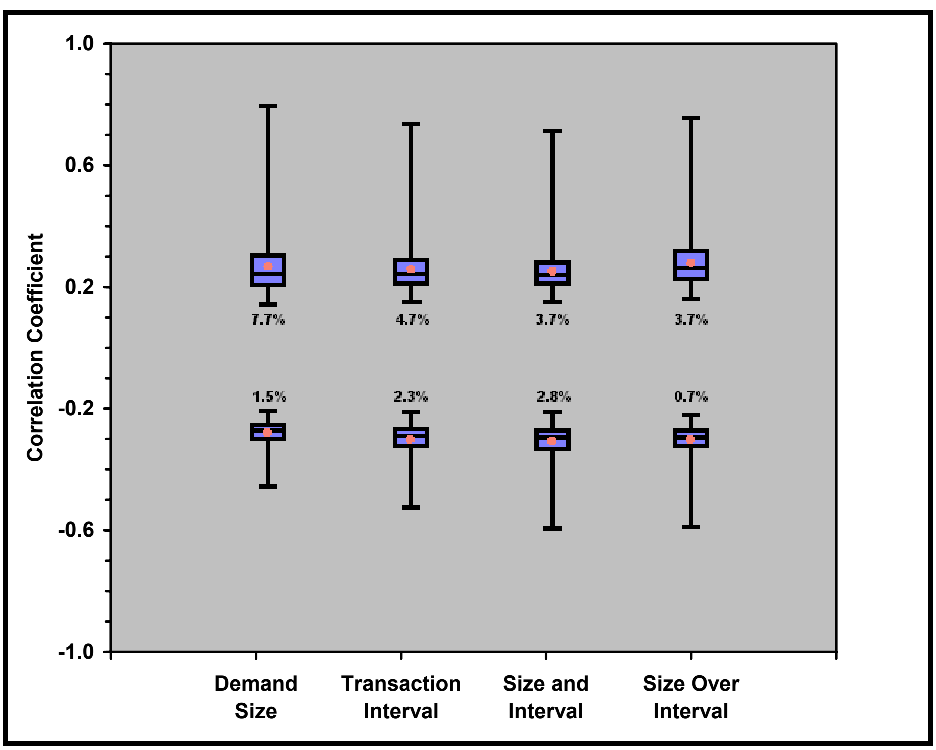
To summarise the autocorrelation and crosscorrelation coefficients for the logarithm-transformed series considered in this chapter, Table 4.5 presents statistics for individually significant lags, as well as line items that have significant correlations as a whole. The percentages of individually significant lags are only slightly more than would arise naturally given a significance level of 5 percent. However, more compelling evidence of the presence of autocorrelation is the percentage of line items significant as a whole. This measure determines whether a cluster of coefficients are significantly different from zero and therefore provides a more powerful test. A moderate proportion of line items are observed to be autocorrelated via this means.

**Table 4.5: Autocorrelation Summary Statistics (Log Transform Series).**

| Autocorrelation / Crosscorrelation | Percentage of Individually Significant Lags | | | Percentage of Line Items (*n* = 12,251) | |
|---|---|---|---|---|---|
| | Total | Proportion | | With Signif Lags | Signif as a Whole |
| | | Negative | Positive | | |
| Demand Size | 9.2 | 16.0 | 84.0 | 55.1 | 25.5 |
| Interval Between Demands | 7.0 | 32.7 | 67.3 | 49.0 | 19.2 |
| Demand Size and Interval | 6.5 | 43.3 | 56.7 | 76.0 | 17.8 |
| Demand Size and Interval Ratio | 4.4 | 16.9 | 83.1 | 35.4 | 10.1 |

Figure 4.21 presents the size of individually significant coefficients as box and whisker plots as produced by SAS/STAT software [61] using the BOXPLOT procedure.

**Figure 4.21: Box and Whisker Plots for Significant Autocorrelations.**



Two plots are presented, one for significant positive correlation coefficients and one for significant negative correlation coefficients, for the demand size, the interval between transactions, the demand size and interval crosscorrelations, and the ratio of demand size and interval autocorrelations. The bottom and top edges of the boxes are located at the sample twenty-fifth and seventy-fifth percentiles. The centre horizontal line is drawn at the median, while the whiskers extend from each box as far as the maximum and minimum values, and the mean for each series is displayed by a marker symbol. The percentages assigned to each plot indicate the overall proportion of coefficients that are significant.

The plots show some large positive and negative autocorrelation and crosscorrelation coefficients in the data. There is more variability in the positive correlation coefficients with the greater length of the boxes indicating a higher interquartile range and the greater length of the whiskers also indicating a higher range. As was previously observed, a higher proportion of significant correlation coefficients are positive in nature as indicated by the percentage values. Although the total percentage of significant coefficients appears low, they are spread among the line items such that up to a quarter are significantly autocorrelated and/or crosscorrelated.

## 4.6 Concluding Remarks

The particular manner in which the RAF operates leads to unique characteristics in the management of the inventory. A large inventory of reserve stocks is maintained in case of war when stock-outs can have severe repercussions. Replenishment lead-times are considerably longer than those faced by most organisations. Also contributing to the high stock-holdings is the fact that the RAF is the sole user of many parts and it is more economical to procure large quantities of stock at the time of initial provisioning. Other factors also lead to stock-holdings that would be considered excessive in other industries, such as the wide geographic dispersal of RAF stations.

An analysis of the annual usage-value indicates that the RAF has a large number of line items with low usage and/or low value. Just 8.5 percent of line items account for 90 percent of the annual value of demand, whereas with typical industrial products it is more like some 25 percent of line items that account for 90 percent of the annual value. Given the large number of C class items, it is sensible to maintain adequate control at the lowest possible cost, which, for the most part, entails keeping large safety stocks to ensure minimal stock-outs.

An ABC inventory analysis generally receives widespread support in the academic literature. However, the costs of maintaining tailored levels of control for the different classifications can prove prohibitive in reality. With a large inventory such as that maintained by the RAF, it is more realistic to use universal policies which provide good results in an uncomplicated manner for all line items. This is particularly the case where the people tasked with monitoring and procuring a specific range of line items are in a post for a period often less than two years.

The large amount of demand data maintained by the RAF has allowed thorough testing for the presence of autocorrelation in both the demand size and the interval between transactions. For most organisations the sparseness of data does not allow a detailed estimate of the level of autocorrelation and questions of independence are left unanswered. The results from the large-scale analysis undertaken in this chapter should have wide application and they raise questions about the validity of published models for erratic and slow-moving demand that assume independence between successive demand sizes, independence between successive demand intervals, and independence between the demand sizes and intervals.

The results have shown some large positive and negative autocorrelation and cross-correlation coefficients in the data. A higher proportion of significant correlation coefficients were positive in nature and they tended to have more variability and higher values than the negative coefficients. Approximately a quarter of line items were found to be significantly autocorrelated and/or crosscorrelated, which suggests many models in the literature are too simplistic with their assumptions. A later chapter will investigate the effect of this on forecasting performance.

An important characteristic of the RAF inventory that has only been touched upon in this chapter is the consideration of the replenishment lead-time. As a fundamental component of any inventory management system, the next chapter provides an in-depth lead-time analysis.

# 5. LEAD-TIME ANALYSIS

The replenishment lead-time is the elapsed time between placing a replenishment order and receiving the items in stock. If the lead-times and demands during these periods are both known with certainty, then inventory replenishments can be timed such that items arrive in stock at the exact time that the last unit is withdrawn. The critical inventory level that triggers a replenishment order will be equal to the lead-time demand. If the replenishment order is placed earlier, some items in stock will not be used; if placed later, some demand requests will go unsatisfied until new stock arrives.

For the vast majority of items in the RAF inventory, the demand is subject to a high degree of uncertainty, and similarly the lead-times are variable. In such cases, the demand during the lead-time can no longer be predicted exactly, and it becomes impossible to time replenishments such that idle stock or stock shortages are avoided. A detailed analysis of actual RAF lead-time observations is undertaken in this chapter.

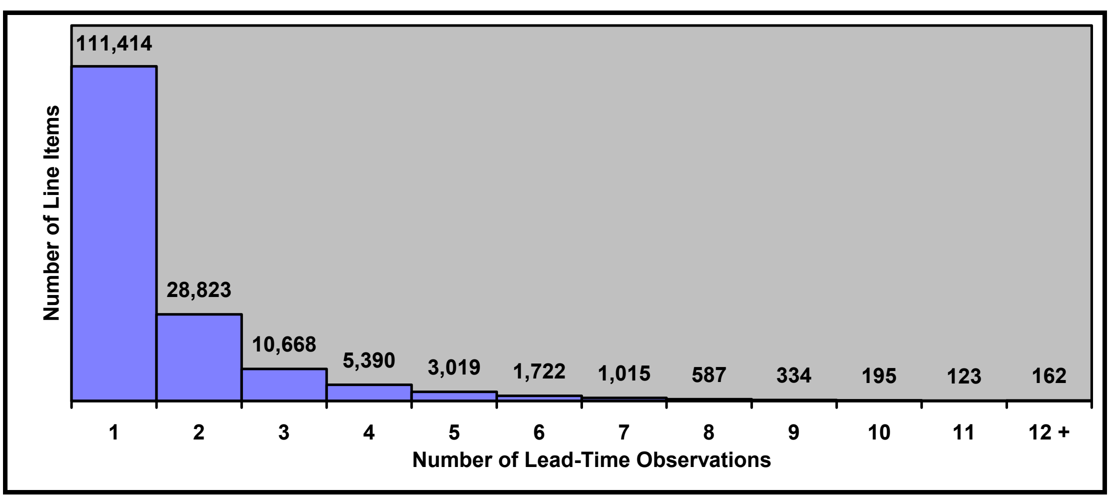
## 5.1 RAF Lead-Time Analysis

Every line item in the RAF inventory has a set administrative lead-time (ALT) and a set purchasing lead-time (PLT) which combine to provide an overall lead-time value. Unfortunately, the fixed nature of this value along with the recognised inaccuracies in both parameters prevents a complete analysis of lead-time demand and another source of lead-time data has been investigated. This secondary data contains monthly updates of the quantity outstanding against individual procurement contracts. The replenishment lead-time can be calculated as the elapsed time between the date a contract is placed with a manufacturer and the date on which the delivery against that contract is received. In this instance, the lead-time only comprises the PLT component and does not include the time to recognise the requirement for replenishment nor set up the required contract.

The less frequent usage of erratic and slow-moving demand items means that in general few replenishment orders are placed, and combined with the long lead-times in the defence industry, very little lead-time data is available to the RAF. This is observed with the actual data where 268,330 contracts for 163,452 unique consumable line items were placed and completed between September 1993 and October 1999. Thus, each line item that has been replenished has on average only 1.6 lead-time observations over a six-year period.

Figure 5.1 presents the frequency of lead-time observations in the RAF data. Some 68 percent of line items recorded only one lead-time observation over the period. At the other end of the scale, 162 line items recorded 12 or more lead-time observations, with the maximum being 23.

**Figure 5.1: Lead-Time Observations for All Consumable Line Items.**




The overall mean replenishment lead-time value is 7.1 months and the standard deviation is 6.1 months. In comparison, the set PLT values are 8.9 and 4.4 months respectively. Thus, the set value overestimates the actual mean lead-time value but it underestimates the spread of the values.

By their very nature, the 162 line items with 12 or more observations are likely to be the higher usage items with a smoother demand pattern. In fact, the average number of demand transactions for these items over the period from January 1994 to December 1999 is 396 and the average demand quantity is 25 units. In comparison, the average number of demand transactions and the average demand quantity for line items with fewer than 12 lead-time observations is only 76 and 20 respectively. This suggests that the line items for which a full lead-time analysis is possible may not provide a truly representative picture.

Although most models in the literature assume lead-times are normally distributed, data available to the RAF indicates that the geometric or negative exponential distributions provide a better lead-time representation. The following section provides details of a modified chi-square goodness-of-fit test that has been used for this analysis.

### 5.1.1 Goodness-of-Fit Test

A goodness-of-fit test tests the null hypothesis that a random sample was drawn from a population with a specified distribution. A common form of this test is the chi-square goodness-of-fit test which measures the degree of fit between observed ($O_i$) and expected ($E_i$) frequencies within each category. A chi-square statistic is calculated as:

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \qquad (5)$$

where $k$ is the final number of categories.

A calculated chi-square statistic less than a tabulated value at the appropriate degrees of freedom indicates a good fit. The general rule for finding the number of degrees of freedom when using this test is to subtract the number of estimated parameters, including the total expected frequency, from the total number of categories. Thus, the number of degrees of freedom for the Poisson distribution would be the final number of categories less two, one for the total expected frequency and one for the sample mean.

Problems often arise with the standard chi-square test through the requirement for data to be grouped together to ensure that each group has an expected frequency of at least five observations. However, there is not complete agreement on this matter; Allen [2] indicates that, "*It has been found, empirically, that the chi-square test works best when all the* $E_i$ *are at least 5, for if* $E_i$ *is small, the division by* $E_i$ *in the term* $(O_i - E_i)^2 / E_i$ *can cause a large error in the value of* $\chi^2$", whereas Snedecor and Cochran [72] are less strict in suggesting that, "*A working rule is that no class expectation should be below 1; two extreme expectations may be close to 1 provided that most of the other expected values exceed 5*", and Chatfield [17] goes further by suggesting that, "*If the expected number in any category is too small, the category should be combined with one or more neighbouring categories. (If there are more than about ten categories, then the approximation is valid provided that less than 20 per cent of the values of* $E_i$ *are less than five, and provided that none is less than one)*".

Groupings are therefore somewhat arbitrary and questions may arise as to what constitutes suitable boundaries for combining neighbouring categories. It is frequently observed that one grouping will accept the null hypothesis whereas another grouping will not, as illustrated by the following example.

Monthly demands over a 72-month period are compiled in a frequency table for a single line item as presented in the first two columns of Table 5.1.

**Table 5.1: Monthly Demand Frequencies.**

| Demands Per Month | Frequency | |
|---|---|---|
| | Observed | Expected |
| 0 | 2 | 1.17 |
| 1 | 8 | 4.80 |
| 2 | 12 | 9.90 |
| 3 | 12 | 13.62 |
| 4 | 10 | 14.04 |
| 5 | 5 | 11.58 |
| 6 | 8 | 7.96 |
| 7 | 8 | 4.69 |
| 8 | 3 | 2.42 |
| 9 | 4 | 1.82 |
| **Total** | **72** | **72.00** |

We want to know whether the monthly demand arrivals follow a Poisson distribution with a mean of $\lambda$. First, we determine an estimate of the monthly arrival rate, which turns out to be 4.13 demands per month. Expected frequencies for this distribution can then be determined as shown in the third column. Now the question is how to combine those categories with the low expected frequencies.

As it turns out, if the demands are grouped into eight categories as {0-1, 2, 3, 4, 5, 6, 7, 8-9} giving a minimum expected category frequency of 4.24, then the null hypothesis is not rejected at the 5 percent significance level as the computed $\chi^2$ of 12.394 is less than the tabulated value of 12.592 with 6 degrees of freedom. However, if the demands are grouped into seven categories as {0-1, 2, 3, 4, 5, 6, 7-9} giving a minimum expected category frequency of 5.97, then the null hypothesis is rejected with a computed $\chi^2$ of 12.387 and a tabulated value of 11.070 with 5 degrees of freedom.

The first grouping method marginally falls short of the five observations rule, although it meets the requirements specified by Snedecor and Cochran as well as Chatfield. On the other hand, the second method meets all requirements for the number of observations but it might be considered unfair to reject the null hypothesis in this instance. We require a methodology that can be applied automatically to large quantities of data while providing consistently fair groupings of observations across a range of probability distributions.

### 5.1.2 Modified Chi-Square Goodness-of-Fit Test

In the course of my research I have developed a modified chi-square goodness-of-fit testing method called GOODFIT within Microsoft Excel. GOODFIT differs in that boundaries are specified by forming categories with similar theoretical frequencies throughout, rather than combining groups just at the margins. Under the modified rules the sum of the probabilities within each grouping will be equalised to a high degree. Continuing with the sample data presented previously, cumulative Poisson probabilities for $\lambda = 4.13$ are presented in Table 5.2.

**Table 5.2: Cumulative Poisson Probabilities.**

| Demands Per Month (*i*) | Cumulative Probability | Demands Per Month (*i*) | Cumulative Probability |
|---|---|---|---|
| 0 | 0.01616 | 9 | 0.99008 |
| 1 | 0.08284 | 10 | 0.99643 |
| 2 | 0.22035 | 11 | 0.99882 |
| 3 | 0.40944 | 12 | 0.99964 |
| 4 | 0.60443 | 13 | 0.99990 |
| 5 | 0.76530 | 14 | 0.99997 |
| 6 | 0.87590 | 15 | 0.99999 |
| 7 | 0.94107 | 16 | 1.00000 |
| 8 | 0.97468 | | |

The upper boundary for each category is determined as the maximum $i$ with a cumulative probability less than or equal to the category number divided by the number of categories. The upper bound for the last category is the maximum $i$. Thus, if there are five categories the groupings will be calculated as:

Category 1 - $\max\{i\}$ where $F(i) \le 1/5$ $\Rightarrow$ 0, 1

Category 2 - $\max\{i\}$ where $F(i) \le 2/5$ $\Rightarrow$ 2

Category 3 - $\max\{i\}$ where $F(i) \le 3/5$ $\Rightarrow$ 3

Category 4 - $\max\{i\}$ where $F(i) \le 4/5$ $\Rightarrow$ 4, 5

Category 5 - $\max\{i\}$ $\Rightarrow$ 6 - 16

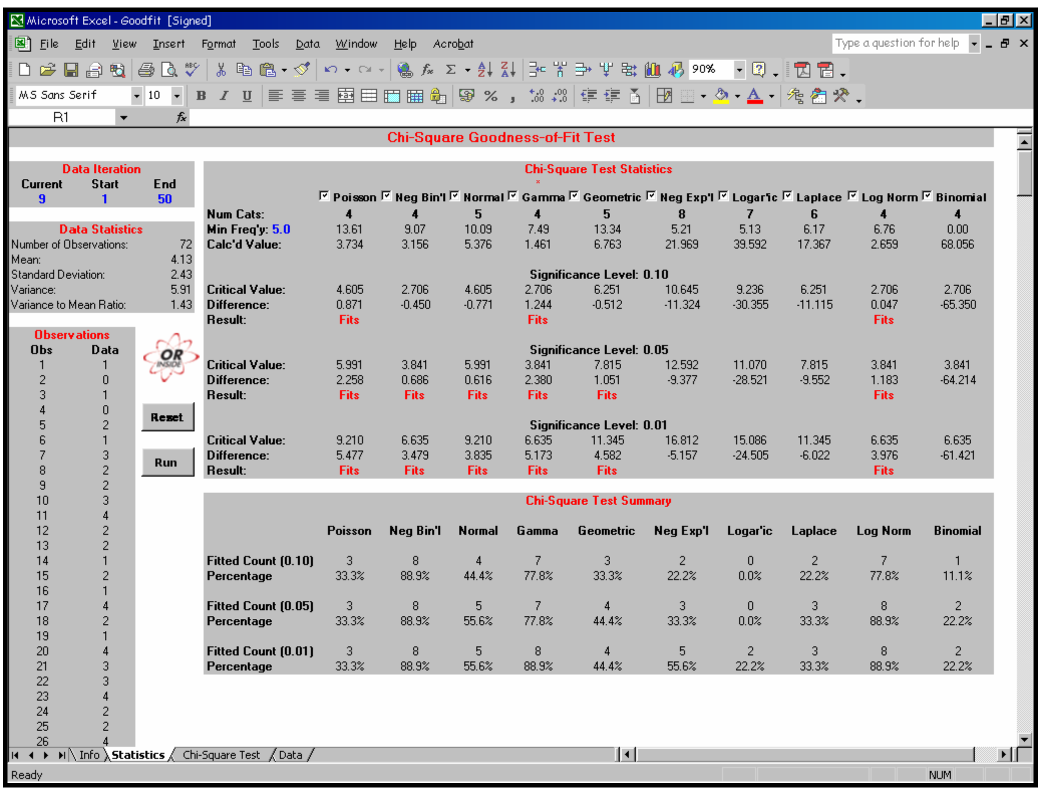
In this case the computed $\chi^2$ of 9.974 exceeds the tabulated value of 7.815 at the 5 percent significance level and the null hypothesis of fitting a Poisson distribution is rejected. The null hypothesis fluctuates between being rejected or accepted depending on the number of categories considered, as illustrated in Table 5.3. The minimum number of categories for the Poisson distribution is three, allowing for two degrees of freedom. The null hypothesis is accepted for 3 and 4 categories and also for 8, 9 and 10 categories, although the five observations rule is broken for the latter three results.

**Table 5.3: Sample Goodness-of-Fit Test Results.**

| Number of Categories (*n*) | Computed Chi-Square Value | Tabulated $\chi^2_{0.05,n-2}$ | Minimum Expected Frequency in a Category | Accept or Reject Hypothesis |
|---|---|---|---|---|
| 3 | 3.536 | 3.841 | 15.87 | Accept |
| 4 | 3.374 | 5.991 | 13.61 | Accept |
| 5 | 9.974 | 7.815 | 5.97 | Reject |
| 6 | 10.473 | 9.488 | 5.97 | Reject |
| 7 | 12.387 | 11.070 | 5.97 | Reject |
| 8 | 12.394 | 12.592 | 4.24 | Accept |
| 9 | 13.340 | 14.067 | 1.82 | Accept |
| 10 | 13.342 | 15.507 | 1.17 | Accept |

For each distribution being tested, GOODFIT will iterate the number of categories from the minimum determined by the degrees of freedom up to a maximum of either a set number or when the theoretical frequency in a group drops below a specified limit. If for any number of categories the null hypothesis is not rejected, the data is assumed to fit the distribution being tested and another distribution can then be considered. Figure 5.2 presents a sample of the GOODFIT statistics output screen.

**Figure 5.2: GOODFIT Statistics Screen.**




The GOODFIT methodology could be further enhanced by offering softer rules on the boundaries such that the sum of the probabilities within each grouping would be equalised to the greatest extent possible. Thus, in the case of the sample line item the upper bound for the first category need not be *exactly* 1/5 but rather *approximately* 1/5. Looking at Table 5.2, this would allow an initial grouping of {0-2} with an expected probability of 0.22035, which is close to 1/5. Continuing in this manner the five categories would be defined as $\{0\text{-}2,3,4,5,6\text{-}16\}$. In this case the computed $\chi^2$ of 9.670 still exceeds the tabulated value of 7.815 at the 5 percent significance level and the null hypothesis is again rejected.

A modified goodness-of-fit test without the soft boundary enhancement was conducted on all 162 individual line items with 12 or more lead-time observations (less one with a constant lead-time). As shown in Table 5.4, the low number of lead-time observations prevents a complete description of the lead-time distribution and a range of theoretical distributions are candidates given their high fitment rate. The sample sizes available at this stage of the analysis are too small for determining which distribution best fits the lead-time data. It is seen that each of the geometric, negative exponential, negative binomial and gamma distributions all fit over 85 percent of the lead-time data at the 5 percent significance level.

**Table 5.4: Goodness-of-Fit Test Results - Individual Lead-Time Observations.**

| Probability Distribution | Goodness-of-Fit Statistics ($n$ = 161) | | | | | |
|---|---|---|---|---|---|---|
| | Alpha 0.10 | | Alpha 0.05 | | Alpha 0.01 | |
| | Count | Percent | Count | Percent | Count | Percent |
| Geometric | 138 | 85.7% | 150 | 93.2% | 158 | 98.1% |
| Negative Exponential | 131 | 81.4% | 149 | 92.5% | 155 | 96.3% |
| Negative Binomial | 131 | 81.4% | 140 | 87.0% | 147 | 91.3% |
| Gamma | 127 | 78.9% | 137 | 85.1% | 145 | 90.1% |
| Poisson | 119 | 73.9% | 132 | 82.0% | 152 | 94.4% |
| Normal | 120 | 74.5% | 128 | 79.5% | 143 | 88.8% |
| Laplace | 112 | 69.6% | 126 | 78.3% | 139 | 86.3% |
| Log Normal | 115 | 71.4% | 121 | 75.2% | 133 | 82.6% |
| Logarithmic | 89 | 55.3% | 101 | 62.7% | 136 | 84.5% |

The hypothesis testing description in Appendix B indicates the alpha value, also known as the significance level of the test, determines the probability of rejecting the null hypothesis ($H_0$) when it is true, thus committing a Type I error. As alpha moves from 0.10 to 0.05 and on to 0.01, the probability of committing a Type I error decreases, although the probability of committing a Type II error, whereby a false $H_0$ is not rejected, increases. The risk of committing either type of error can be reduced by increasing the number of lead-time observations for each line item, a consideration investigated in a later chapter.

## 5.2 Lead-Time Grouping Analysis

The low number of actual lead-time observations for each line item, which more often than not equals zero, restricts the usefulness of this data on an individual item basis. Therefore, it is necessary to group line items that may have a similar lead-time pattern and calculate summary statistics that apply to the entire grouping. Groupings could be based on the following predictors:

(i) Item price - The higher the price, the greater the complexity of the item and possibly the longer the lead-time.

(ii) Item activity - There may be a relationship between the lead-time and the frequency of reprovisioning. In the absence of comprehensive reprovisioning data, an alternative is to examine the frequency with which demand occurs.

(iii) Set PLT - The set parameter value may in fact bear some resemblance to the true lead-time value.

(iv) The manufacturer - A given manufacturer is likely to produce line items with similar characteristics and apply similar manufacturing procedures.

(v) Supply Management Branch (SMB) - The SMB has two hierarchical levels:

(a) SMB Level 1 (SMB L1) - The management team responsible for a defined range of equipment comprising an aircraft-based, a technology-based, or a commodity-based range of items. For example, SMB21 is responsible for the avionics, electrical and weapons systems for the *Harrier*.

(b) SMB Level 2 (SMB L2) - Identifies range managers (RM) within an SMB tasked with monitoring and procuring specific line items. RM21b is responsible for mission display components for the *Harrier*, for example

(vi) NATO Supply Classification (NSC) - The NSC defines the commodity grouping of a line item and has three hierarchical levels:

(a) NSC Level 1 (NSC L1) - Classifies commodity groupings under a broad categorisation such as weapons.

(b) NSC Level 2 (NSC L2) - A refinement of NSC L1 into additional commodity groupings. For example, weapons at NSC L2 consist of missiles, guns, armament equipment, ammunition and explosives.

(c) NSC Level 3 (NSC L3) - A further refinement upon NSC L2.

SAS software has been used to measure the degree to which each predictor, or independent variable, explains the variation in the dependent variable, that is the variation in the replenishment lead-time across the line items. By matching the lead-time observations with the mature line items currently in the RAF inventory a total of 72,712 unique line items are available for a lead-time analysis, representing 19.3 percent of the inventory. In this analysis the replenishment lead-time for an individual line item is taken as the average of all lead-time observations for that item.

### 5.2.1 Analysis of Variation

Three separate SAS procedures have been used in an exploratory analysis of the variation in the lead-time, namely PROC REG, PROC ANOVA and PROC GLM, depending on the nature of the predictor.

PROC REG is a general-purpose regression procedure suitable for identifying a linear relationship between the dependent variable and an independent variable with a numeric measurement, such as item price in this instance. The procedure determines the linear equation that minimises the sum of the squared errors (SSE) between the actual lead-time values and the values predicted by the equation. In effect, the SSE is a measure of the amount of variation in the independent variable that remains *unexplained* by the variation in the dependent variable. Alternatively, the sum of squares for regression (SSR) measures the amount of *explained* variation.

An indication of how much variation is explained by the fitted model is provided by the coefficient of determination, denoted $r^2$ and defined as:

$$r^2 = 1 - \frac{SSE}{TSS}$$

where $TSS$ is the total sum of squares which measures the variation in the dependent variable.

It follows that $r^2$ measures the proportion of the variation in the replenishment lead-time that is explained by the variation in the dependent variable. Whether a given $r^2$ is considered large or small depends on the context of the analysis. As the replenishment lead-time is likely to depend on a range of factors and be quite unpredictable, an $r^2$ of say 0.3 may be considered large in this instance.

Provided the underlying population of the independent variable is normally distributed, the F-test identifies whether the model is significant as a whole. The test initially determines the mean squares of SSE and SSR, denoted MSE and MSR respectively, whereby each sum of squares is divided by the applicable degrees of freedom. Finally, the test statistic is defined as the ratio of the two mean squares:

$$F = \frac{MSR}{MSE}$$

which is F-distributed with 1 and $n-1$ degrees of freedom for the simple linear regression model.

A large value for $F$ indicates that most of the variation in the lead-time is explained by the regression equation and that the model is useful. On the other hand, a small value for $F$ indicates that most of the variation is unexplained. With a null hypothesis that the model is not significant, the rejection region for determining whether $F$ is large enough to justify rejecting the null hypothesis at significance level $\alpha$ is:

reject $H_0$ if $F > F_{\alpha,1,n-1}$

PROC ANOVA performs an analysis of variance whereby the dependent variable is measured against the classifications provided by a categorical independent variable, such as the manufacturer in this case. An analysis of variance test measures the variation due to effects in the classification. The SAS ANOVA procedure is suitable when the data is balanced whereby each cell in the classification has an equal number of observations. Alternatively, in the case of unbalanced data it is necessary to use the GLM (General Linear Models) procedure for the ANOVA test. If the data is balanced, the arithmetic for calculating sums of squares is greatly simplified and the ANOVA procedure is therefore more efficient than the GLM procedure.

The coefficient of determination ($r^2$) has the same interpretation for the ANOVA and GLM procedures as it does for the REG procedure, in that it measures how much variation in the dependent variable can be accounted for by the model. Similarly, the $F$ ratio has the same interpretation, although the test statistic is F-distributed with $k-1$ and $n-k$ degrees of freedom, where $k$ is the number of categories, provided the independent variable is normally distributed and population variances are equal. The F-test is a test of the null hypothesis that there are no differences among the population means.

Results from individual models for ascertaining how well the predictor variables explain the variation in the actual lead-times are presented in Table 5.5. Assuming each predictor variable is normally distributed, the F-test suggests that the models are significant overall. The possibility of violating the assumptions is examined in a later section. Suffice to say the $r^2$ value will still be legitimate.

**Table 5.5: Analysis of Actual Lead-Time Data.**

| Indep't Variable | SAS® Test Applied | Test Statistics | | | | $F_{0.01}$ Value | Model Significant |
|---|---|---|---|---|---|---|---|
| | | $r^2$ | $F$ Ratio | $df_n$ | $df_d$ | | |
| Price | Reg | 0.017 | 1,233.87 | 1 | 72,710 | 6.64 | Yes |
| Activity | Reg | 0.000 | 12.45 | 1 | 68,571 | 6.64 | Yes |
| Set PLT | Reg | 0.092 | 7,319.82 | 1 | 72,710 | 6.64 | Yes |
| Manuf'r | GLM | 0.240 | 11.86 | 1,593 | 59,750 | 1.09 | Yes |
| SMB L1 | GLM | 0.160 | 130.78 | 106 | 72,605 | 1.35 | Yes |
| SMB L2 | GLM | 0.196 | 43.75 | 403 | 72,308 | 1.17 | Yes |
| NSC L1 | GLM | 0.034 | 506.54 | 5 | 72,706 | 3.02 | Yes |
| NSC L2 | GLM | 0.050 | 212.28 | 18 | 72,693 | 1.93 | Yes |
| NSC L3 | GLM | 0.093 | 61.25 | 122 | 72,589 | 1.32 | Yes |

The analysis of the variation in the actual lead-times indicates that the manufacturer with the highest $r^2$ explains the variation to the greatest extent. Next are the range managers at SMB L2 who, with additional information, explain more variation than SMB L1. At the other end of the scale the item price and item activity explain very little of the lead-time variation.

### 5.2.2 Diagnostics - Item Price

The results of the analysis suggest there is virtually no linear relationship between the price of an item and the lead-time as indicated by the low $r^2$ value. This unexpectedly poor result prompted further analysis of the price data using PROC UNIVARIATE to generate descriptive statistics. Selected sample statistics, including the mean, median and standard deviation, the skewness and kurtosis as measures of shape, and the Kolmogorov-Smirnov (K-S) $D$ statistic as a test for normality, are shown in Table 5.6.

**Table 5.6: Analysis of Item Price Data.**

| *n* | Mean ($\bar{x}$) | Median | Std Dev (*s*) | Skewness | Kurtosis | *D* Statistic |
|---|---|---|---|---|---|---|
| 72,712 | 226.82 | 24.21 | 900.15 | 14.52 | 372.93 | 0.401 |

Skewness is a measure of the tendency of the deviations from the mean to be larger in one direction than the other, calculated as:

$$\frac{n}{(n-1)(n-2)}\sum_{i=1}^{n}\left(\frac{x_i-\bar{x}}{s}\right)^3$$

If the sample comes from a normal population and $n$ exceeds 150, the skewness is approximately normally distributed with mean zero and a standard deviation of $\sqrt{6/n}$ or 0.009 in this case (Snedecor and Cochran [72]). Since skewness is positive and far in excess of its standard deviation, the value indicates the distribution is heavily skewed to the right.

In addition, the heaviness of the tail can be measured by the kurtosis calculated as:

$$\frac{n(n+1)}{(n-1)(n-2)(n-3)}\sum_{i=1}^{n}\left(\frac{x_i-\bar{x}}{s}\right)^4-\frac{3(n-1)^2}{(n-2)(n-3)}$$

For sample sizes from the normal distribution in excess of 1000, kurtosis is normally distributed with mean zero and a standard deviation of $\sqrt{24/n}$, or 0.018 in this case. Again, the kurtosis value is far in excess of its standard deviation indicating a very heavy tail.

With the measures of skewness and kurtosis suggesting the presence of a small proportion of relatively large prices, the population distribution is likely to be far from normal. Confirmation of a rejection of the null hypothesis that the price data was drawn from a population with a normal distribution is provided by the K-S goodness-of-fit test. The $D$ statistic (obtained from PROC UNIVARIATE using the NORMAL option) is calculated as the maximum absolute difference between the cumulative distribution function (CDF) of the hypothesised distribution $F(x)$ and the CDF of the sample $S_n(x)$, as:

$$D_n = \max\left(\max_j\left|\frac{j}{n}-F(t_j)\right|,\left|\frac{j-1}{n}-F(t_j)\right|\right)$$

where $t_j$ is the $j^{th}$ smallest observation.

Critical regions for determining whether $D$ is large enough to justify rejecting the null hypothesis are of the form:

$$D_n \geq \mathit{Tabulated\ Value}$$

where the tabulated value comes from a table such as Lilliefors [47].

In this instance, the sample size is far beyond the range of any tabulated values and an extrapolated critical value is obtained from Table 5.7. This table caters for situations where the parameters of the hypothesised distribution are unknown and have to be estimated from the sample.

**Table 5.7: Critical Values for the Kolmogorov-Smirnov Test for Normality.**

| Sample Size ($n$) | Level of Significance | | | |
|---|---|---|---|---|
| | **0.10** | **0.05** | **0.02** | **0.01** |
| Over 100 | $\frac{0.8255}{\sqrt{n}}$ | $\frac{0.8993}{\sqrt{n}}$ | $\frac{0.9885}{\sqrt{n}}$ | $\frac{1.0500}{\sqrt{n}}$ |

The maximum observed deviation of $D_{72,712} = 0.401$ is far in excess of the critical value of 0.004 at the 1 percent significance level, providing strong evidence that price is not normally distributed. Such an observation suggests transforming the price data in an attempt to remedy the non-normality and improve upon the observed coefficient of determination.

The type of transformation depends on the form of the error values from the regression equation. The most commonly used transformations include:

(i) The log transformation - used when either (a) the variance of the error increases as the predicted value increases, or (b) the distribution of the errors is positively skewed.

(ii) The square transformation - used when either (a) the variance is proportional to the predicted value, or (b) the error distribution is negatively skewed.

(iii) A square root transformation. This may be useful when the variance is proportional to the predicted value.

(iv) The reciprocal transformation. This is recommended if the variance appears to significantly increase when the predicted value increases beyond some critical value.

As the type of transformation depends on the form of the error values from the regression equation it is useful to examine the errors. Statistics from an analysis of the error values for the price data are shown in Table 5.8. The errors are skewed to the right and as $D_{72,712}$ is greater than the K-S tabulated value at any significance level, the errors are not normally distributed.

**Table 5.8: Analysis of Item Price Regression Errors.**

| *n* | Mean | Median | Std Dev | Skewness | Kurtosis | *D* Statistic |
|---|---|---|---|---|---|---|
| 72,712 | 0.00 | -1.61 | 6.56 | 2.28 | 8.61 | 0.147 |

It is also necessary to examine the variance of the errors, for which a useful diagnostic is to test for heteroscedasticity, which exists when the error values do not have a constant variance. One method for determining the absence of homoscedasticity is through testing that the first and second moments of the model are correctly specified as described by White [85], obtainable from SAS by requesting the SPEC option under the REG procedure. In this instance, the null hypothesis for homoscedasticity is certainly rejected with a computed $\chi^2$ of 178.661 compared to a tabulated value of 5.991 at the 5 percent significance level with 2 degrees of freedom.

The results suggest a possible remedy through a log transformation or a square root transformation, both representing relatively simple transformations. An alternative means for transforming data is available through the TRANSREG (transformation regression) procedure. This procedure iteratively derives an optimal variable transformation using the method of alternating least squares.

An ordinary regression model assumes the variables are measured on an equal interval scale and can therefore be geometrically represented as vectors in an $n$ dimensional space. On the other hand, the alternating least squares algorithm allows variables whose full representation is a matrix consisting of more than one vector to be represented by a single vector, which is an optimal linear combination of the columns of the matrix.

As a rule, the transformed data from PROC TRANSREG should not be used in hypothesis testing but rather as an exploratory data analysis procedure because the usual assumptions are violated when variables are optimally transformed by this means. The test statistics reported by the REG and GLM procedures are no longer truly valid, except possibly $r^2$. However, in this study the sample size is so large that the fact that degrees of freedom are lost by the transformation procedure bears little relevance.

The impact upon the regression analysis from the transformations to item price is illustrated in Table 5.9. The log transformation improves the $r^2$ value over the default value to a greater extent than the square root transformation, although the result is still somewhat poor in both cases. An optimal cubic spline transformation with fifty knots was applied separately to the item price and the log of the item price. Splines are defined as piecewise polynomials of degree $n$ whose function values and $n-1$ derivatives agree at the points where they join. Knots give the fitted curve freedom to bend and are the abscissa values, or vertical distance to the y-axis, of the join points. Again, both transformations produce an improved, but still somewhat poor, result according to the $r^2$ value.

**Table 5.9: Item Price Regression Analysis.**

| Item Price Transformation | Test Statistics | | | | $F_{0.01}$ Value | Model Significant |
|---|---|---|---|---|---|---|
| | $r^2$ | $F$ Ratio | $df_n$ | $df_d$ | | |
| Default | 0.017 | 1,233.87 | 1 | 72,710 | 6.64 | Yes |
| Log | 0.055 | 4,192.98 | 1 | 72,710 | 6.64 | Yes |
| Square Root | 0.046 | 3,519.85 | 1 | 72,710 | 6.64 | Yes |
| Cubic Spline | 0.061 | 4,701.86 | 1 | 72,710 | 6.64 | Yes |
| Log C. Spline | 0.061 | 4,702.39 | 1 | 72,710 | 6.64 | Yes |

Descriptive statistics for the transformed prices and the resultant regression errors are shown in Table 5.10. It is seen from the K-S $D$ statistic that none of the transformed series or their regression errors attain normality and in all cases the error values maintain a high degree of heteroscedasticity as shown by the last column.

**Table 5.10: Analysis of Transformed Item Price.**

| Transf'n | Series | $n$ | Mean | Std Dev | Skewness | Kurtosis | $D$ Stat | H Stat |
|---|---|---|---|---|---|---|---|---|
| Default | Actual | 72,712 | 226.82 | 900.15 | 14.52 | 372.93 | 0.401 | - |
| | Errors | 72,712 | 0.00 | 6.56 | 2.28 | 8.61 | 0.147 | 178.7 |
| Log | Actual | 72,712 | 3.04 | 2.47 | -0.26 | -0.17 | 0.026 | - |
| | Errors | 72,712 | 0.00 | 6.43 | 2.36 | 9.30 | 0.127 | 216.2 |
| Square Root | Actual | 72,712 | 9.00 | 12.08 | 3.72 | 22.78 | 0.230 | - |
| | Errors | 72,712 | 0.00 | 6.46 | 2.32 | 9.04 | 0.127 | 290.7 |
| Cubic Spline | Actual | 72,712 | 226.82 | 900.15 | 0.82 | 0.30 | 0.098 | - |
| | Errors | 72,712 | 0.00 | 6.41 | 2.35 | 9.31 | 0.128 | 246.2 |
| Log C. Spline | Actual | 72,712 | 3.04 | 2.47 | 0.82 | 0.29 | 0.103 | - |
| | Errors | 72,712 | 0.00 | 6.41 | 2.35 | 9.31 | 0.128 | 246.7 |

All of these diagnostics suggest the item price represents a poor predictor and should be discarded from further consideration.

### 5.2.3 Diagnostics - Item Activity

Overall results from Table 5.5 indicated that an insignificant portion of the variation in the actual lead-time could be accounted for by item activity, in fact $r^2$ equals 0.000 to three decimal places. Further descriptive statistics for the activity data, as generated by PROC UNIVARIATE, are shown in Table 5.11.

**Table 5.11: Analysis of Item Activity Data.**

| Series | *n* | Mean | Median | Std Dev | Skewness | Kurtosis | *D* Stat | H Stat |
|---|---|---|---|---|---|---|---|---|
| Actual | 68,573 | 47.11 | 13.00 | 161.76 | 28.54 | 1,895.20 | 0.388 | - |
| Errors | 68,573 | 0.00 | -1.52 | 6.49 | 2.29 | 8.44 | 0.153 | 20.9 |

The item activity statistics are widely dispersed about the mean, as demonstrated by the high standard deviation. The high values for skewness, kurtosis and the K-S $D$ statistic all suggest the data points are not normally distributed. The impact upon the regression analysis from various transformations to item activity is shown in Table 5.12. None of the transformations improve the $r^2$ value to any great extent.

**Table 5.12: Item Activity Regression Analysis.**

| Item Activity Transformation | Test Statistics | | | | $F_{0.01}$ Value | Model Significant |
|---|---|---|---|---|---|---|
| | $r^2$ | $F$ Ratio | $df_n$ | $df_d$ | | |
| Default | 0.000 | 12.45 | 1 | 68,571 | 6.64 | Yes |
| Log | 0.003 | 174.23 | 1 | 68,571 | 6.64 | Yes |
| Square Root | 0.002 | 111.34 | 1 | 68,571 | 6.64 | Yes |
| Cubic Spline | 0.095 | 7,597.91 | 1 | 72,710 | 6.64 | Yes |
| Log C. Spline | 0.095 | 7,597.26 | 1 | 72,710 | 6.64 | Yes |

Table 5.13 presents some descriptive statistics for each of the transformed series and the regression errors that result. None of the transformations have achieved normality and the error values remain highly heteroscedastic.

**Table 5.13: Analysis of Transformed Item Activity.**

| Transf'n | Series | *n* | Mean | Std Dev | Skewness | Kurtosis | *D* Stat | H Stat |
|---|---|---|---|---|---|---|---|---|
| Default | Actual | 68,573 | 47.11 | 161.76 | 28.54 | 1,895.20 | 0.388 | - |
| | Errors | 68,573 | 0.00 | 6.49 | 2.29 | 8.44 | 0.153 | 20.9 |
| Log | Actual | 68,573 | 2.60 | 1.52 | 0.29 | -0.29 | 0.044 | - |
| | Errors | 68,573 | 0.00 | 6.49 | 2.31 | 8.57 | 0.139 | 39.2 |
| Square Root | Actual | 68,573 | 4.97 | 4.74 | 3.73 | 29.93 | 0.201 | - |
| | Errors | 68,573 | 0.00 | 6.49 | 2.30 | 8.52 | 0.144 | 38.5 |
| Cubic Spline | Actual | 72,712 | 47.11 | 161.76 | 9.49 | 157.60 | 0.366 | - |
| | Errors | 72,712 | 0.00 | 6.30 | 2.38 | 9.27 | 0.148 | 471.2 |
| Log C. Spline | Actual | 72,712 | 2.60 | 1.52 | 9.50 | 157.62 | 0.366 | - |
| | Errors | 72,712 | 0.00 | 6.30 | 2.38 | 9.27 | 0.148 | 470.9 |

All the above diagnostics suggest that item activity is a poor predictor and should also be discarded from any further consideration.

### 5.2.4 Diagnostics - Purchasing Lead-Time

With a coefficient of determination of only 0.092, the regression analysis also suggests there is virtually no linear relationship between the set PLT and the replenishment lead-time. Additional descriptive statistics for the set PLT and regression errors are presented in Table 5.14.

**Table 5.14: Analysis of Set Purchasing Lead-Time (PLT) Data.**

| Series | *n* | Mean | Median | Std Dev | Skewness | Kurtosis | *D* Stat | H Stat |
|---|---|---|---|---|---|---|---|---|
| Actual | 72,712 | 8.33 | 8.00 | 4.30 | 0.92 | 1.39 | 0.166 | - |
| Errors | 72,712 | 0.00 | -1.49 | 6.31 | 2.41 | 9.87 | 0.137 | 384.8 |

With relatively low values for both the skewness and kurtosis and the median close to the mean, the actual series would at first glance appear somewhat normal. However, as the K-S *D* statistic is greater than the critical value of 0.003 at the 1 percent significance level, the null hypothesis is rejected. The error values from the regression model are also not normally distributed and are skewed to the right. The computed $\chi^2$ of 384.8 far exceeds the tabulated value of 5.991 at the 5 percent significance level indicating heteroscedasticity.

Transforming the set PLT data with a log transformation, a square root transformation, and a cubic spline on both the actual and the log of the series, produces the regression statistics presented in Table 5.15. Each of the transformations failed to significantly improve upon the coefficient of determination, with the log transformation actually reducing the value of $r^2$.

**Table 5.15: Set PLT Regression Analysis.**

| Set PLT Transformation | Test Statistics | | | | $F_{0.01}$ Value | Model Significant |
|---|---|---|---|---|---|---|
| | $r^2$ | $F$ Ratio | $df_n$ | $df_d$ | | |
| Default | 0.092 | 7,319.82 | 1 | 72,710 | 6.64 | Yes |
| Log | 0.085 | 6,788.82 | 1 | 72,710 | 6.64 | Yes |
| Square Root | 0.092 | 7,329.73 | 1 | 72,710 | 6.64 | Yes |
| Cubic Spline | 0.099 | 7,954.22 | 1 | 72,710 | 6.64 | Yes |
| Log C. Spline | 0.099 | 7,953.97 | 1 | 72,710 | 6.64 | Yes |

Table 5.16 presents some descriptive statistics for each of the transformed series and the regression errors that result. Not surprisingly, none of the transformations achieved normality and the errors remain highly heteroscedastic.

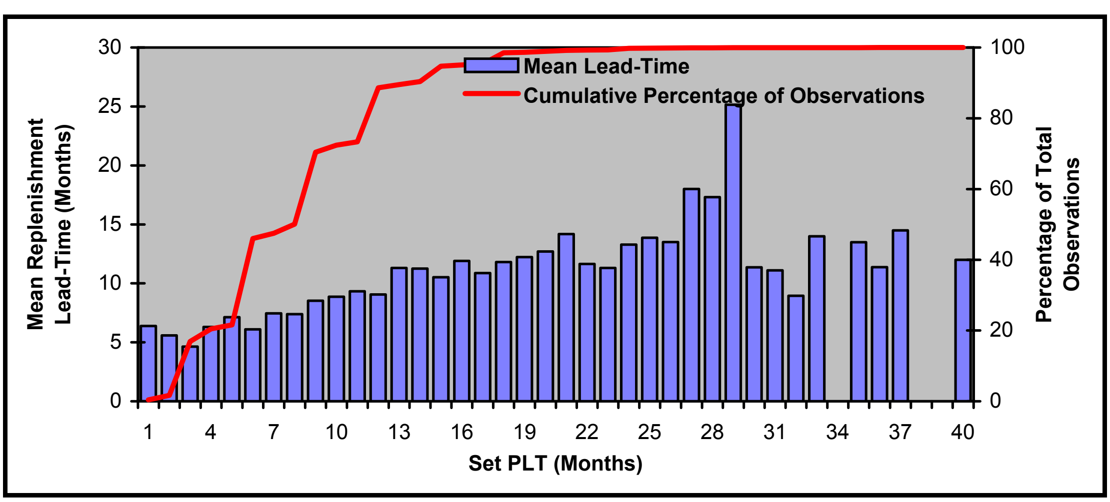
**Table 5.16: Analysis of Transformed Set PLT.**

| Transf'n | Series | *n* | Mean | Std Dev | Skewness | Kurtosis | *D* Stat | H Stat |
|---|---|---|---|---|---|---|---|---|
| Default | Actual | 72,712 | 8.33 | 4.30 | 0.92 | 1.39 | 0.166 | - |
| | Errors | 72,712 | 0.00 | 6.31 | 2.41 | 9.87 | 0.137 | 384.8 |
| Log | Actual | 72,712 | 1.98 | 0.56 | -0.44 | -0.19 | 0.154 | - |
| | Errors | 72,712 | 0.00 | 6.33 | 2.41 | 9.71 | 0.142 | 338.1 |
| Square Root | Actual | 72,712 | 2.79 | 0.74 | 0.22 | -0.24 | 0.137 | - |
| | Errors | 72,712 | 0.00 | 6.31 | 2.41 | 9.84 | 0.141 | 350.0 |
| Cubic Spline | Actual | 72,712 | 8.33 | 4.30 | 0.40 | -0.37 | 0.175 | - |
| | Errors | 72,712 | 0.00 | 6.28 | 2.44 | 10.03 | 0.143 | 368.7 |
| Log C. Spline | Actual | 72,712 | 1.98 | 0.56 | 0.40 | -0.37 | 0.175 | - |
| | Errors | 72,712 | 0.00 | 6.28 | 2.44 | 10.03 | 0.143 | 368.7 |

Despite the previous diagnosis suggesting the set PLT would also make a poor predictor for the replenishment lead-time, it would be premature to dismiss such a prospective variable out of hand. After all, the set PLT is such a fundamental component of the RAF inventory management system that one would expect some conscious effort to attain a degree of accuracy.

Further analysis of the set PLT is illustrated in Figure 5.3. The vertical bars of this chart show the average replenishment lead-time for each set PLT value.

**Figure 5.3: Comparing the Replenishment Lead-Time and the Set PLT.**



It is observed that there is initially some measure of a linear relationship between the two variables; in fact the coefficient of determination is equal to 0.93 for PLT values up to 20 months, dropping to 0.73 at 30 months, and 0.28 at 35 months.

The result of a $t$-test for the significance of a linear trend in the set PLT data up to 20 months is shown in Table 5.17. As the calculated $t$ statistic of 14.89 is greater than the tabulated value of 2.10 at the 95 percent confidence level with $n-2$ degrees of freedom, the slope of the line up to 20 months is considered significant.

**Table 5.17: Test for Significant Trend.**

| Regression Statistics | |
|---|---|
| R Square | 0.93 |
| Standard Error | 0.71 |
| Observations | 20 |
| Intercept | 4.70 |
| Slope | 0.41 |
| **Significance Test** | |
| $t$ Statistic | 14.89 |
| Tabulated Value ($t_{0.05,18}$) | 2.10 |
| Conclusion | Trend Significant |

In addition, over 98.5 percent of the observations have a set PLT of no more than 20 months as shown by the plotted cumulative percentage of observations. This suggests the vast majority of set PLT values do in fact have a strong correlation with the replenishment lead-time.

However, the set PLT does not appear to be an accurate predictor of the true value of the actual lead-time. The line of best fit for the first 20 observations is determined as:

$$\textit{Mean Replenishment Lead-Time} = 0.41 \times \textit{Set PLT} + 4.70$$

The mean lead-time is seen to be in excess of 5 months when the set PLT is actually only one month, therefore the set PLT initially under-estimates the lead-time values. As the average lead-time increases at less than half the rate of the set PLT there soon comes a point where the lead-time is severely over-estimated. A glance at Figure 5.3 suggests that beyond 20 months the small sample sizes prevent the formation of a definitive relationship and no significant pattern emerges.

The conclusion drawn from this part of the analysis is that the set PLT may be an acceptable predictor on condition that a PLT value in the proximity of 20 months becomes the cut-off, whereby values below are either utilised as predictors in their own right or combined, but all values above are certainly part of one group.

### 5.2.5 Diagnostics - Manufacturer

The remaining variables, including the manufacturer, the SMB and the NSC, represent categorical factors and it was therefore necessary to use an analysis of variance test (ANOVA) to measure the variation due to effects in the classification.

There are some 1,594 unique manufacturers with recorded replenishment lead-time values. As the manufacturers supply the RAF with differing numbers of line items and therefore have differing numbers of lead-time observations, the manufacturer presents unbalanced data classifications for which the SAS general linear models (GLM) procedure is appropriate.

The highest resultant $r^2$ value of 0.240 suggests the variation in the lead-times is moderately explained by the manufacturer and with the $F$ ratio exceeding the tabulated value the model is considered significant as a whole. The manufacturer is therefore considered a prime candidate to act as a predictor.

### 5.2.6 Diagnostics - Supply Management Branch

With SMB Level 1 selected as the first factor, SMB Level 2 is a second factor nested within the first. The RAF data identifies 107 supply management teams at SMB L1, comprising 404 range managers at SMB L2 with recorded replenishment lead-times. The number of range managers within an SMB varies from as low as one to as high as fifteen. In addition, the range managers are responsible for differing numbers of line items, leading to differing numbers of lead-time observations at both levels. Thus the classification of the data is far from balanced prompting the use of the GLM procedure.

It was seen in Table 5.5 of Section 5.2.1 that, as one-way classifications, SMB L1 and SMB L2 have coefficients of determination of 0.160 and 0.196 respectively. In considering some of the very low $r^2$ values shown in the table, these results would appear to be quite important to this analysis.

The hierarchical classification of the SMB predictors provides scope for additional ANOVA analysis through the specification of nested effects, as demonstrated by Steel and Torrie [74]. The results from a two-factor nested model are shown in Table 5.18, where the notation **b(a)** indicates predictor **b** is nested within predictor **a**.

**Table 5.18: Supply Management Branch (SMB) Nested Effects.**

| Source of Variation | Type I SS Test Statistics | | | $F_{0.01}$ Value | Factor Significant |
|---|---|---|---|---|---|
| | *F* Ratio | $df_n$ | $df_d$ | | |
| SMB L1 | 136.03 | 106 | 72,308 | 1.35 | Yes |
| SMB L2(SMB L1) | 10.82 | 297 | 72,308 | 1.20 | Yes |

Type I SS, referred to as sequential sum of squares, is used to measure the incremental sum of squares as each source of variation is added to the model. Test statistics are determined such that the preceding predictor variables are already in the model. This means the variance common to both the main effect and the nested effect is attributed to the first predictor, namely SMB Level 1. The results indicate there is significant variation between each supply management team at SMB Level 1, as well as between range managers within a supply management team.

A two-factor SMB model without nesting has also been considered and the Type III SS, or partial sums of squares, whereby the test statistics are corrected for all other variables in the model, have been examined. The $F$ Ratio for SMB Level 1 is observed to be undefined as all the explained variation is attributed to SMB Level 2. This means it is only necessary to select one of the SMB levels as a predictor. SMB L2 explains more of the variation than SMB L1, while only adding slightly to the complexity, and is therefore selected to act as a predictor.

### 5.2.7 Diagnostics - NATO Supply Classification

In a similar manner to the SMB, the NSC presents a hierarchical structure with NSC Level 3 nested within NSC Level 2, which is itself nested within NSC Level 1. There are 6 categories at NSC L1, 19 categories at NSC L2 and 123 categories at NSC L3. As one-way classifications, NSC L1, NSC L2 and NSC L3 have coefficients of determination of 0.034, 0.050 and 0.093 respectively. On the face of this analysis, the NSC at all levels would appear to be a poor predictor.

For completeness, the Type I results from a three-factor nested model for the NSC are presented in Table 5.19. The large $F$ Ratios indicate there is significant variation between the NSC at Level 1, also at Level 2 within the higher level NSC, and between the NSC at Level 3.

**Table 5.19: NATO Supply Classification (NSC) Nested Effects.**

| Source of Variation | Type I SS Test Statistics | | | $F_{0.01}$ Value | Factor Significant |
|---|---|---|---|---|---|
| | *F* Ratio | $df_n$ | $df_d$ | | |
| NSC L1 | 539.01 | 5 | 72,589 | 3.02 | Yes |
| NSC L2(NSC L1) | 100.24 | 13 | 72,589 | 2.13 | Yes |
| NSC L3(NSC L2)(NSC L1) | 33.40 | 104 | 72,589 | 1.35 | Yes |

Furthermore, the Type III SS from a three-factor model without nesting indicates all the explainable variation is attributed to NSC Level 3 and the $F$ Ratios for Level 1 and Level 2 are undefined. However, this observation is purely academic as the NSC remains a poor predictor at all levels and is therefore discarded.

### 5.2.8 Specification of Effects

In considering the previous results, the lead-time observations are best grouped on the basis of the manufacturer, the range manager (RM) at SMB Level 2 and the set PLT.

At this stage we are interested in the combined effect of the three factors on the replenishment lead-time. An important consideration is the crossed effects or interactions that occur when the simple main effects of one predictor are not the same at different levels of the other predictors. The GLM procedure allows the inclusion of an interaction term to test the hypothesis that the effect of one predictor does not depend on the levels of the other predictors in the interaction. The results of a full factorial model, including two and three-way interactions, are presented in Table 5.20, where predictors joined by asterisks denote interaction effects. The Type III SS provide corrected $F$ Ratios given the multiple terms in the model.

**Table 5.20: Interaction Effects.**

| Source of Variation | Type III SS Test Statistics | | | $F_{0.01}$ Value | Factor Significant |
|---|---|---|---|---|---|
| | *F* Ratio | $df_n$ | $df_d$ | | |
| Manufacturer | 3.17 | 1,236 | 29,132 | 1.10 | Yes |
| SMB L2 | 1.36 | 208 | 34,669 | 1.24 | Yes |
| Set PLT | 4.36 | 33 | 34,669 | 1.66 | Yes |
| Manuf'r ∗ SMB L2 | 0.90 | 262 | 4,851 | 1.22 | No |
| Manuf'r ∗ Set PLT | 1.21 | 109 | 4,851 | 1.35 | No |
| SMB L2 ∗ Set PLT | 1.05 | 1,771 | 34,669 | 1.08 | No |
| Manuf'r ∗ SMB L2 ∗ Set PLT | 1.32 | 118 | 4,851 | 1.33 | No |

The complete specification of effects using all variables simultaneously required excessive computational resources in this case. Two alternative means of gaining reductions in time and memory requirements were used to obtain full interaction effects:

(i) Limiting the raw data to only include a sample of line items.

(ii) The GLM absorption technique allows the effect of one or more predictors to be adjusted out before the construction and solution of the rest of the model.

One and two-way classifications have been obtained by absorbing the additional predictors, while the three-way interaction has been obtained by limiting the raw data to only include line items which have four or more lead-time observations.

The analysis shows there are significant differences among the manufacturers, the range managers, as well as among the set PLT values. However, there are no differences among the crossed effects of the predictors so the null hypothesis of no interaction is not rejected. As there are no interactions, the crossed effects term is dropped from the analysis and a new ANOVA model with only main effects is generated, as presented in the SAS output of Figure 5.4.

**Figure 5.4: SAS-Generated Three-Way ANOVA Table.**

```
                 General Linear Models Procedure

Dependent Variable: ACTPLT

Source               DF     Sum of Squares    F Value    Pr > F
Model              2011      813656.391129      12.26    0.0001
Error             59332     1958067.832932
Corrected Total   61343     2771724.224021

           R-Square               C.V.           ACTPLT Mean
           0.293556           71.87658            7.99248723

Source               DF        Type III SS    F Value    Pr > F
MANUF'R            1583      224235.040055       4.29    0.0001
SMB_L2              384       84329.073038       6.65    0.0001
SET_PLT              34       34275.980811      30.55    0.0001
```

With the calculated $F$ value for the manufacturer of 4.29 greater than the tabulated value of 1.09 at the 1 percent significance level, the manufacturer is considered a significant predictor. The calculated values of 6.65 and 30.55 for the range manager and set PLT are also greater than the tabulated values of 1.18 and 1.65 respectively. Therefore, these two predictors are likewise considered significant.

As the calculated $F$ ratio of 12.26 exceeds the tabulated $F$ value of 1.08 at the 1 percent level with 2,011 and 59,332 degrees of freedom, the main effects model itself is considered significant as a whole. The coefficient of determination indicates that 29.4 percent of the variation in the replenishment lead-time is explained by a combination of the manufacturer, the range manager and the set PLT.

## 5.3 Lead-Time Grouping Methodology

The next stage of the lead-time grouping procedure is to perform a cluster analysis and place the 72,712 lead-time observations into groups as suggested by the data, not defined a priori, such that observations in a given group tend to be similar to each other, and observations in different groups tend to be dissimilar. Again SAS software was utilised for forming disjoint clusters such that each lead-time observation is placed in one and only one cluster. The clustering was obtained through two methods, namely PROC CLUSTER, using PROC TREE to derive the tree diagrams, or dendrograms, with the required number of clusters, and PROC FASTCLUS.

PROC CLUSTER performs hierarchical clustering using a choice of agglomerative methods whereby each observation begins in a cluster by itself and the two closest clusters are merged to form a new cluster that replaces the two existing ones. Merging of the two closest clusters is repeated until only one cluster is left. The various methods differ in how the distance between two clusters is computed. PROC TREE uses the output from the CLUSTER procedure to obtain the resulting cluster membership at the desired level. This method is not practical for very large data sets due to the CPU time varying as the square or even cube of the number of observations.

As an alternative, PROC FASTCLUS is a non-hierarchical procedure that finds an initially specified number of disjoint clusters using a *k*-means method. A set of points referred to as cluster seeds are selected as a first guess of the means of the clusters. Each observation is assigned to the nearest seed to form temporary clusters. The seeds are then replaced by the means of the temporary clusters and the process is repeated until no further changes occur. This procedure tends to produce similarly sized clusters although the method is sensitive to outliers that may appear as clusters with only one member. The FASTCLUS procedure requires CPU time proportional to the number of observations and is particularly suitable for large data sets. With small data sets the results may be highly sensitive to the initial ordering of the observations.

The mean and standard deviation of the replenishment lead-times have been calculated for each category of the selected variables. For example, a mean and standard deviation are calculated from all lead-time observations with a set PLT of one month as the first category, all observations with a set PLT of two months as another, and so on. The clustering procedure then groups the categories on the basis of this mean and standard deviation. The standard deviation was chosen as opposed to the variance because the variance tended to dominate the clustering and result in elongated clusters.

As will be illustrated in subsequent sections, the mean and standard deviation do not provide well separated clusters, particularly in the case of the large number of categories for both the manufacturer and the range manager. The aim is therefore to find clusters comprising roughly the same number of observations. In this instance, PROC FASTCLUS has the advantage of being biased toward identifying compact clusters of equal size once outliers have been removed. In the interest of succinctness, it is desirable to limit the number of clusters for each predictor to six similarly sized groupings.

### 5.3.1 Clustering - Purchasing Lead-Time

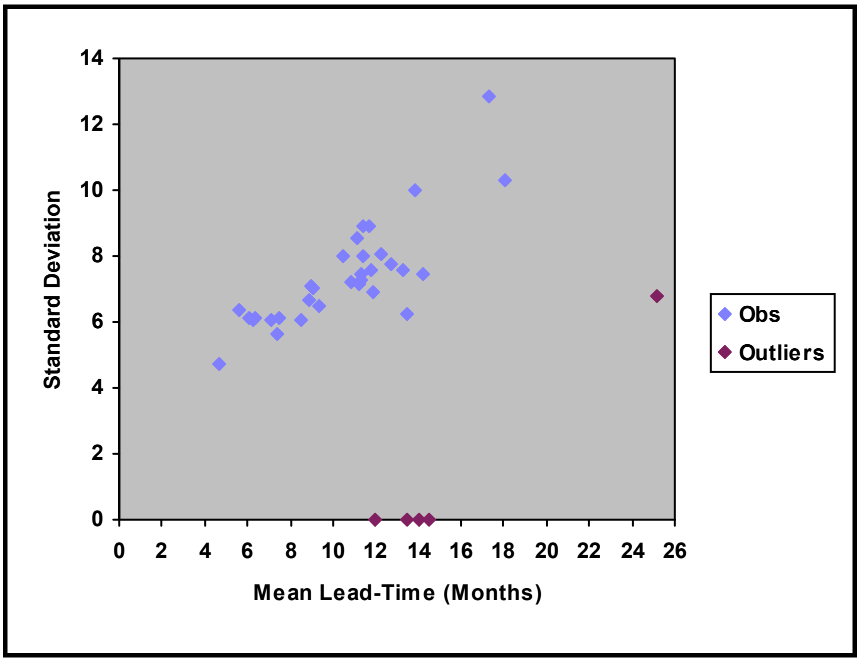
The first predictor for which a cluster analysis has been undertaken is the purchasing lead-time. Statistics for each set PLT value, comprising the number of observations, the mean and the standard deviation of the replenishment lead-times, are presented in Table 5.21. It is seen that there were no lead-time observations for set PLT values at 34, 38, 39 or above 40 months, while other high-end PLT values had only one observation.

**Table 5.21: Replenishment Lead-Time Statistics for Set PLT.**

| Set PLT | *n* | Mean | Std Dev | Set PLT | *n* | Mean | Std Dev |
|---|---|---|---|---|---|---|---|
| 1 | 298 | 6.304 | 6.037 | 21 | 196 | 14.186 | 7.474 |
| 2 | 908 | 5.577 | 6.389 | 22 | 66 | 11.647 | 8.914 |
| 3 | 11,077 | 4.633 | 4.748 | 23 | 35 | 11.316 | 7.275 |
| 4 | 2,608 | 6.382 | 6.122 | 24 | 342 | 13.293 | 7.569 |
| 5 | 806 | 7.134 | 6.056 | 25 | 22 | 13.877 | 10.018 |
| 6 | 17,781 | 6.100 | 6.115 | 26 | 31 | 13.513 | 6.263 |
| 7 | 1,081 | 7.447 | 6.092 | 27 | 34 | 18.010 | 10.273 |
| 8 | 1,875 | 7.388 | 5.642 | 28 | 14 | 17.315 | 12.853 |
| 9 | 14,779 | 8.525 | 6.047 | 29 | 5 | 25.150 | 6.763 |
| 10 | 1,445 | 8.863 | 6.696 | 30 | 16 | 11.369 | 8.028 |
| 11 | 673 | 9.323 | 6.486 | 31 | 3 | 11.111 | 8.566 |
| 12 | 11,106 | 9.048 | 7.060 | 32 | 4 | 8.938 | 7.072 |
| 13 | 641 | 11.308 | 7.463 | 33 | 1 | 14.000 | - |
| 14 | 646 | 11.261 | 7.179 | 34 | - | - | - |
| 15 | 3,138 | 10.512 | 8.011 | 35 | 1 | 14.000 | - |
| 16 | 277 | 11.910 | 6.915 | 36 | 18 | 11.380 | 8.930 |
| 17 | 188 | 10.861 | 7.188 | 37 | 1 | 15.000 | - |
| 18 | 2,297 | 11.819 | 7.603 | 38 | - | - | - |
| 19 | 90 | 12.232 | 8.033 | 39 | - | - | - |
| 20 | 208 | 12.713 | 7.743 | 40 | 1 | 12.000 | - |

Figure 5.5 presents the lead-time mean plotted against the standard deviation. Outliers are illustrated as points that have only one lead-time observation and therefore do not have a standard deviation, along with other points generally detached from the remainder. Such outliers have been removed prior to performing any cluster analysis.

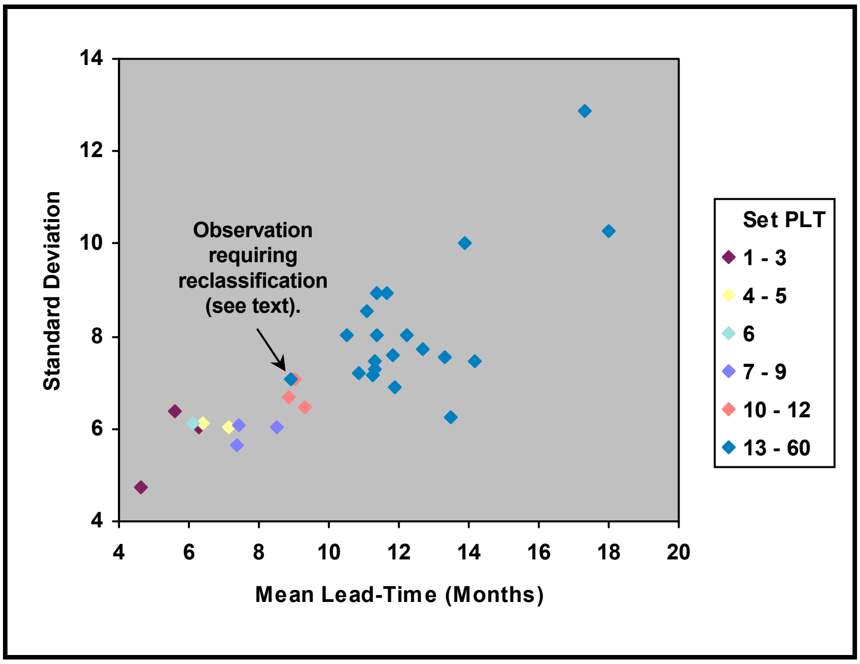
**Figure 5.5: Replenishment Lead-Time by Set PLT.**




The CLUSTER procedure with the two-stage density linkage option has been used for grouping by set PLT. This algorithm is similar to the single linkage algorithm ordinarily used with density linkage, where the distance between two clusters is the minimum distance between an observation in one cluster and an observation in another cluster. The two-stage option differs by ensuring all points are assigned to modal clusters before the modal clusters themselves are allowed to join. The advantage is that a minimum number of observations in a cluster can be obtained by specifying that when two clusters are joined, the result must have at least $n$ members to be designated as a modal cluster.

PROC TREE is used to generate the 6-cluster solution and the resulting cluster membership is illustrated in Figure 5.6. The groupings are identified by the different colours and it is seen, for example, that all line items with a PLT value of 1 to 3 months form a grouping. One interesting occurrence, as indicated in the plot, refers to a set PLT value of 34 months. Due to its locality it was grouped with the 10 to 12 category by the SAS clustering procedure. However, as this point lies beyond the cut-off of 20 months, and the parameters were generated from a small sample size, such a point would be better placed in a 13 to 60 category as discussed previously in Section 5.2.4.

**Figure 5.6: Lead-Time Clustering by Set PLT.**




By using the two-stage option of the clustering procedure, the methodology considers the number of lead-time observations within each category and ensures that a final grouping contains a specified minimum number. Specifying a minimum cluster size of 2,500 produces the aggregate lead-time statistics presented in Table 5.22.

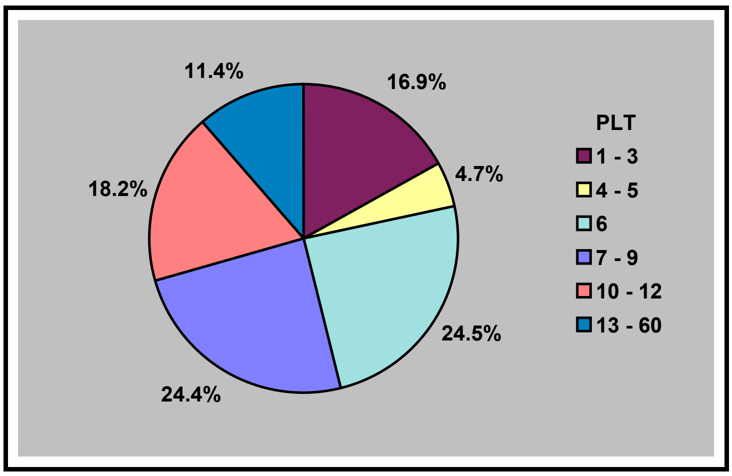
**Table 5.22: Lead-Time Clustering by PLT.**

| Set PLT Group | Number of Categories | Lead-Time Statistics | | |
|---|---|---|---|---|
| | | *n* | Mean | Std Dev |
| A: 1 – 3 | 3 | 12,283 | 4.745 | 4.937 |
| B: 4 – 5 | 2 | 3,414 | 6.500 | 6.050 |
| C: 6 | 1 | 17,781 | 6.100 | 6.115 |
| D: 7 – 9 | 3 | 17,735 | 8.339 | 6.022 |
| E: 10 – 12 | 3 | 13,224 | 9.041 | 6.993 |
| F: 13 – 60 | 48 | 8,275 | 11.414 | 7.793 |

The large number of observations with a set PLT value of 6 months forms its own grouping, even though the parameter values clearly lie within the 1 to 3 grouping as illustrated in Figure 5.6.

Figure 5.7 presents the proportion of the total observations allocated to each grouping.

**Figure 5.7: Lead-Time Frequencies by PLT Cluster.**




### 5.3.2 Clustering - Manufacturer

A second cluster analysis has been performed according to the manufacturer. The RAF procures line items from over 1,500 different manufacturers, although details have not been recorded against 15 percent of the procurements. Descriptive statistics for the replenishment lead-time, according to whether the manufacturer is known or not, are presented in Table 5.23.

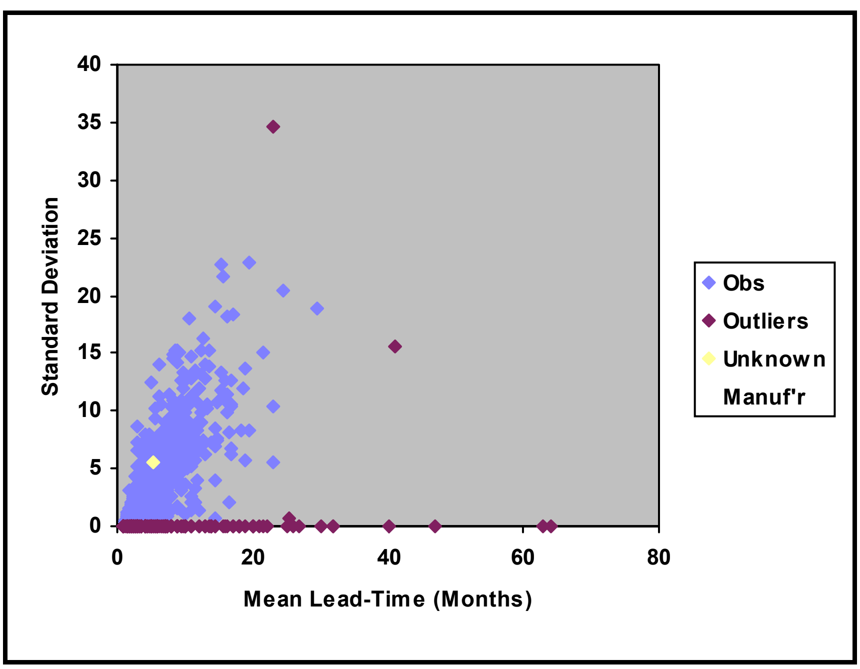
**Table 5.23: Replenishment Lead-Time Statistics for Manufacturer.**

| Manuf'r Details | Number of Categories | Lead-Time Statistics | | |
|---|---|---|---|---|
| | | *n* | Mean | Std Dev |
| Known | 1,521 | 61,344 | 7.992 | 6.722 |
| Unknown | ? | 11,368 | 5.327 | 5.497 |
| **Total** | **?** | **72,712** | **7.576** | **6.617** |

For the purpose of this analysis, those manufacturers with only one procurement observation are not considered for the clustering procedure. It is felt that the limited data for such one-off contracts would not provide representative lead-time parameters, and certainly there is no standard deviation to cluster upon. On the other hand, a large number of observations are without a known manufacturer and to exclude such a large pool of data may be imprudent. In this case, all observations without a known manufacturer will be grouped together and treated as one cluster.

The lead-time mean by manufacturer is plotted against the standard deviation in Figure 5.8. Manufacturers with only one procurement observation, as well as other points generally detached from the remainder, are illustrated as outliers. The group of lead-time observations without known manufacturers can be located in this plot.

**Figure 5.8: Replenishment Lead-Time by Manufacturer.**




The large number of manufacturers necessitates the use of the FASTCLUS procedure to perform the clustering. Once again, a total of six groups are required. However, as the

FASTCLUS procedure does not permit the specification of a minimum number of observations in a cluster, the generated groups tend to contain vastly differing numbers of observations. It was therefore necessary to specify a requirement for more than six clusters and manually combine groups to produce six more evenly sized clusters. One cluster was coded to comprise entirely of observations without a known manufacturer.

**Table 5.24: Lead-Time Clustering by Manufacturer.**

| Manuf'r Group | Number of Categories | Lead-Time Statistics | | |
|---|---|---|---|---|
| | | *n* | Mean | Std Dev |
| **A** | **544** | **8,196** | **3.534** | **2.901** |
| **B** | **209** | **12,131** | **4.985** | **4.899** |
| **C** | **1** | **11,368** | **5.327** | **5.497** |
| **D** | **144** | **29,700** | **8.910** | **5.920** |
| E-1 | 57 | 2,177 | 8.376 | 9.644 |
| E-2 | 29 | 1,107 | 14.544 | 11.480 |
| E-3 | 7 | 24 | 15.326 | 17.915 |
| E-4 | 2 | 17 | 26.892 | 19.229 |
| **E** | **95** | **3,325** | **10.574** | **10.894** |
| F-1 | 31 | 6,991 | 12.815 | 7.208 |
| F-2 | 7 | 432 | 19.890 | 9.110 |
| **F** | **38** | **7,423** | **13.227** | **7.516** |

Cluster membership, as a result of requesting nine groups in addition to the unknown manufacturer group, is detailed in Table 5.24 and illustrated in Figure 5.9. The number of clusters is reduced to six by manually combining four clusters to form Group E and another two to form Group F. Observations without a known manufacturer comprise Group C.

**Figure 5.9: Lead-Time Clustering by Manufacturer.**

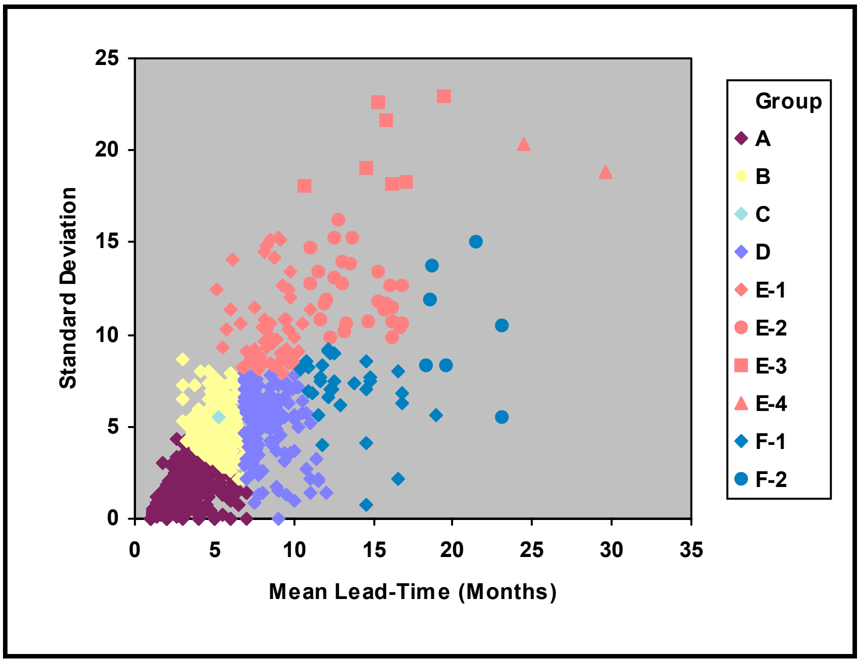
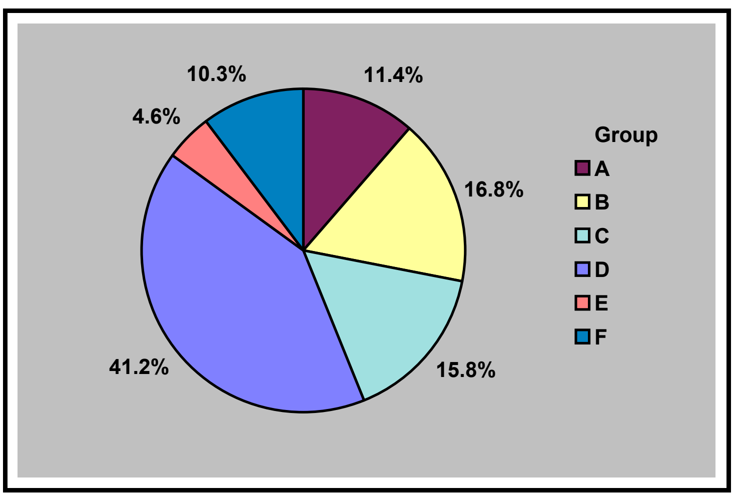



Figure 5.10 presents the proportion of lead-time observations within each grouping.

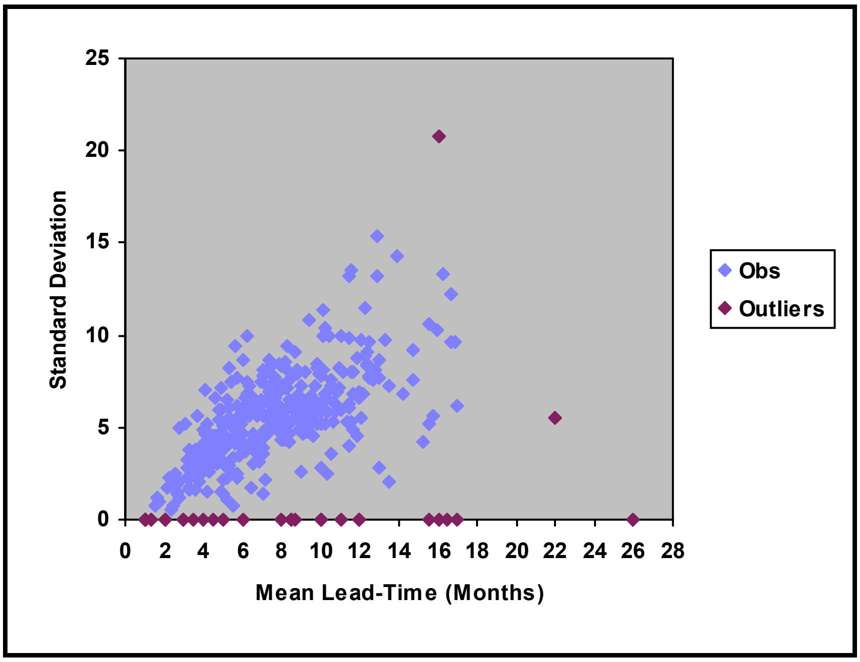
**Figure 5.10: Lead-Time Frequencies by Manufacturer Cluster.**




### 5.3.3 Clustering - Range Manager

A final cluster analysis has been performed using the 404 RM categories. Figure 5.11 presents the lead-time mean plotted against the standard deviation, with the outliers distinguished from the remaining observations.

**Figure 5.11: Replenishment Lead-Time by Range Manager.**

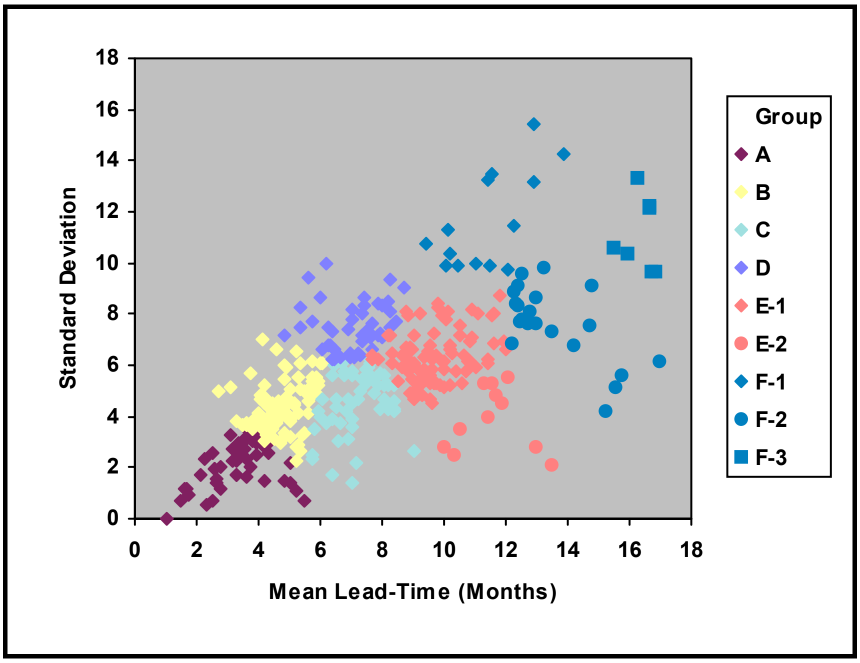



Observations have been grouped into nine clusters using the FASTCLUS procedure. The resultant cluster membership is presented in Table 5.25 and illustrated in Figure 5.12.

**Table 5.25: Lead-Time Clustering by Range Manager.**

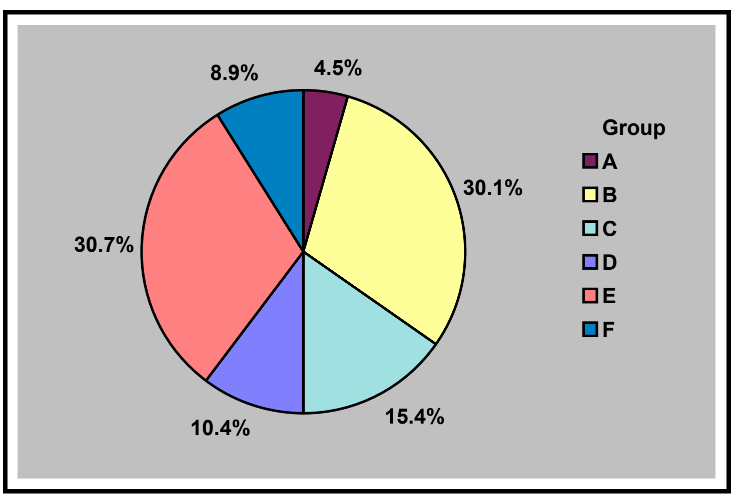
| Group | Number of Categories | Lead-Time Statistics | | |
|---|---|---|---|---|
| | | *n* | Mean | Std Dev |
| **A** | **47** | **3,304** | **3.451** | **2.853** |
| **B** | **86** | **21,886** | **4.782** | **4.825** |
| **C** | **53** | **11,172** | **7.224** | **4.975** |
| **D** | **45** | **7,545** | **6.640** | **7.375** |
| E-1 | 86 | 21,752 | 9.702 | 6.412 |
| E-2 | 11 | 525 | 11.718 | 4.961 |
| **E** | **97** | **22,277** | **9.750** | **6.388** |
| F-1 | 14 | 935 | 10.638 | 10.890 |
| F-2 | 21 | 4814 | 13.367 | 7.509 |
| F-3 | 7 | 741 | 16.414 | 11.479 |
| **F** | **42** | **6,490** | **13.321** | **8.720** |

**Figure 5.12: Lead-Time Clustering by Range Manager.**




In this instance, Group E is formed by combining two clusters and Group F combines three clusters. Figure 5.13 presents the proportion of observations within each grouping.

**Figure 5.13: Lead-Time Frequencies by RM Cluster.**




### 5.3.4 Lead-Time Grouping

At this stage, the groupings from each variable are combined to form a 6×6×6 matrix as illustrated by the lead-time grouping cube in Figure 5.14. Each of the 216 cells is assigned a lead-time mean and variance calculated from all lead-time observations falling within that cell.

**Figure 5.14: Lead-Time Grouping Cube.**

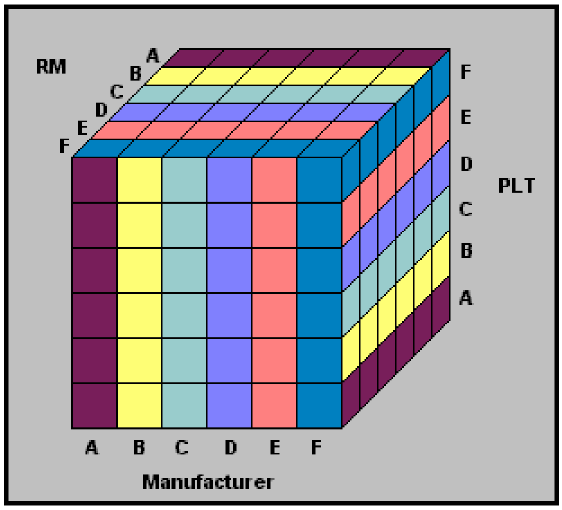



Table 5.26 illustrates the manner in which lead-time parameters are assigned to each grouping based on the observations within that group. As an example, all line items falling within the first group are given the lead-time mean and variance calculated from all 673 observations in this group.

**Table 5.26: Lead-Time Grouping Table.**

| Group Number | Grouping Variable | | | Lead-Time Observations | | |
|---|---|---|---|---|---|---|
| | Set PLT | Manuf'r | RM | *n* | Mean | Variance |
| 1 | A | A | A | 673 | 3.060 | 7.384 |
| 2 | A | A | B | 521 | 3.028 | 6.122 |
| 3 | A | A | C | 19 | 3.725 | 5.543 |
| 4 | A | A | D | 94 | 3.848 | 18.206 |
| 5 | A | A | E | 57 | 3.681 | 4.607 |
| 6 | A | A | F | 15 | 3.100 | 2.757 |
| 7 | A | B | A | 147 | 3.180 | 7.414 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 216 | F | F | F | 810 | 15.290 | 74.633 |

As a result of this analysis, all line items can be assigned lead-time parameters regardless of whether they have any actual historic lead-time observations or not. This is a useful feature for new line items entering service.

## 5.4 Concluding Remarks

Despite being a fundamental component of any inventory management system, the replenishment lead-time distribution is difficult to quantify. This is particularly the case for the RAF where the less frequent usage of erratic and slow-moving demand items means that few replenishment orders are placed. The long lead-times in the defence industry also means there is little lead-time data available.

A modified chi-square goodness-of-fit testing method was developed during the course of this research and several lead-time distributions have been investigated. This modified methodology provides automatic distribution fitting to large quantities of data across a range of probability distributions. Unfortunately the low number of lead-time observations per line item prevents the generation of definitive results on an individual basis and a range of theoretical distributions are identified as candidates. Results from the limited data at this stage of the analysis indicate each of the geometric, negative exponential, negative binomial and gamma distributions provide a reasonable fit. However, the analysis suggests that the normal distribution, which frequently finds favour in the literature, is somewhat removed from reality.

When a large number of line items do not have any lead-time observations it is appropriate to group line items which have a similar lead-time pattern and calculate summary statistics applicable to the entire grouping. In this research, suitable variables for grouping were investigated using regression and ANOVA analysis. The selected variables in this instance were the set purchasing lead-time, the manufacturer and the range manager. Groupings within each variable were determined through a cluster analysis such that observations in a group are similar to each other, and observations in different groups are dissimilar. A lead-time grouping cube containing some 216 cells was created and aggregated statistics from all observations within each cell were generated. An important feature resulting from the use of this methodology is the ability to assign lead-time parameters to all line items according to their location within the lead-time grouping cube.

Chapters 4 and 5 have provided a detailed examination of the constituent parts for a demand classification scheme. The next chapter attempts to classify the observed demand based on the transaction variability, the demand size variability and the lead-time variability.

# 6. DEMAND CLASSIFICATION

This chapter examines demand over a lead-time and classifies the entire RAF inventory by demand pattern. A more formal means of identifying erratic demand is utilised, in contrast to the somewhat naive approach of Section 4.5.2 which only considered the demand frequency. On this occasion, the lead-time demand is decomposed into the constituent causal parts of demand frequency, demand size, and lead-time.

## 6.1 RAF Demand Classification

With all line items assigned lead-time parameters from the previous chapter, each line in the RAF inventory can be classified by the observed lead-time demand in accordance with equation (2) as introduced in Chapter 2. The underlying variance partition equation for the variable lead-time case was defined as:

$$C_{LTD}^2 = \frac{C_n^2}{\overline{L}} + \frac{C_z^2}{\overline{n}\overline{L}} + C_L^2$$

where $\overline{n}$ is the mean number of transactions per unit time,

$\overline{L}$ is the mean replenishment lead-time, and

$C_z$ is the coefficient of variation for the demand size, etc.

An evaluation of the three constituent parts, previously translated as transaction variability, demand size variability and lead-time variability, led Williams [90] to propose a demand classification with four demand patterns, namely smooth, slow-moving, erratic and erratic with highly variable lead-time.

Through an initial analysis of RAF data, it was perceived that the Williams' classifications did not adequately describe the observed demand structure. In particular, it was not considered sufficient to distinguish a smooth demand pattern from the remainder simply on the basis of the transaction variability. Consequently, a revised classification scheme sub-divides line items with low transaction variability into smooth and irregular, according to the demand size variability. As an aid to simplification the erratic demand pattern, and the erratic with highly variable lead-time demand pattern, have been re-designated as mildly erratic and highly erratic respectively. The revised classifications are presented in Table 6.1.

**Table 6.1: Revised Classification of Demand.**

| Lead-Time Demand Component | | | Type of Demand Pattern |
|---|---|---|---|
| Transaction Variability | Demand Size Variability | Lead-Time Variability | |
| Low | Low | | Smooth |
| Low | High | | Irregular |
| High | Low | | Slow-moving |
| High | High | Low | Mildly Erratic |
| High | High | High | Highly Erratic |

In allocating a line item to a particular demand pattern, each of the components are considered sequentially in this analysis. Thus, line items are classified by transaction variability first and then the two resultant groups are further divided according to a common demand size variability parameter. Finally, the erratic demand classifications are divided according to a lead-time variability parameter. As a result, a high value for one of the components will not dominate the final classification. With each of the variability measures being continuous variables there are unlikely to be distinct break-points that would naturally separate the demand patterns. At the extremes it is easier to identify the demand patterns while in the middle there is a considerable grey area.

Under a parts and supplies inventory system, such as required for aircraft maintenance, a smooth demand pattern may be interpreted as resulting from a full replacement of items under a planned maintenance schedule; an irregular pattern as a result of replacing only faulty items under a similar schedule; a slow-moving pattern results from replacing faulty items as and when they fail; and an erratic demand pattern occurs through various combinations of the above.

A data conversion has been necessary to classify all line items in the RAF inventory. If a line item has only one demand observation, in the absence of remedial action the null value for the demand size standard deviation would lead to a null value for the demand size variability. On the other hand, a single demand observation gives a squared transaction per unit time CV of 72.0, thus clearly placing the line item in a *high* transaction variability category. Assigning a demand size standard deviation of zero then ensures all line items with one demand observation are classified as slow-moving.

In an attempt to identify what values constitute *high* and *low* in RAF terms, the summary statistics of Table 6.2 were obtained from the 223,746 line items experiencing at least one demand transaction over the six year period. Statistics were calculated using a monthly aggregation.

**Table 6.2: Summary Statistics for Monthly Demand.**

| Statistic | Demand Component | | |
|---|---|---|---|
| | Transaction Variability | Demand Size Variability | Lead-Time Variability |
| Mean | 4.29422 | 0.62187 | 0.63261 |
| Maximum | 72.00000 | 51.55030 | 1.92606 |
| Upper Quartile | 6.18786 | 0.55525 | 0.84865 |
| Median | 2.31918 | 0.10127 | 0.52537 |
| Lower Quartile | 0.74403 | 0.00000 | 0.36590 |
| Minimum | 0.00392 | 0.00000 | 0.03333 |

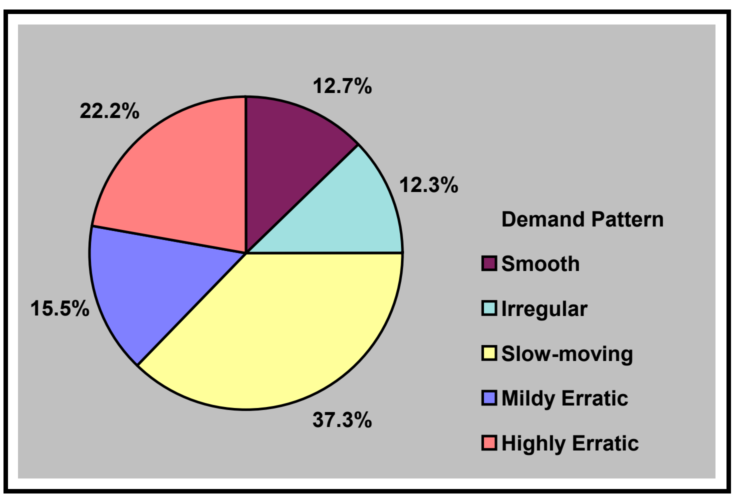
This table is useful for setting the boundaries between each of the demand categories. For example, defining 25 percent of line items as either smooth or irregular demand simply requires setting the boundary for transaction variability to the lower quartile. Boundaries for demand size variability and lead-time variability can also be individually set at management's discretion.

For the purpose of analysis it is useful to have a similar proportion of line items within each category in order to provide an adequate sample size. Firstly, this would mean setting the boundary for transaction variability to the fortieth percentile to ensure two-fifths of the line items have a low transaction variability, thereby being classed as either smooth or irregular. Then setting the demand size variability boundary to the median would ensure approximately half of the low transaction variability line items would be classed as smooth with the remainder classed as irregular. However, with the demand size variability boundary set at the overall median, approximately half of the high transaction variability line items would be classed as slow-moving with the mildly erratic and highly erratic line items comprising the remainder between them. The boundary for the demand size variability would therefore need to be set at the thirty-third percentile to ensure one-fifth of the total line items are classed as slow-moving, although in the process this would mean that the smooth and irregular line items are no longer evenly split if the same boundary was used. A precisely even categorisation would require establishing two demand size variability boundaries once line items have been split according to the transaction variability. Finally, an equal categorisation between the two erratic demand patterns could be obtained by setting the lead-time variability boundary to equal the median.

Rather than placing an exactly equal proportion of line items in each category, this research has adopted a simpler approach of setting the boundary for transaction variability to the lower quartile and singular boundaries for each of demand size variability and lead-time variability at their respective medians. Most organisations would view the majority of RAF demand as slow-moving or erratic and therefore it is reasonable to classify only 25 percent of the inventory as smooth or irregular, while still giving a sufficient sample size. About half of the remainder are classified as slow-moving with the same proportion classed as erratic, either mildly erratic or highly erratic.

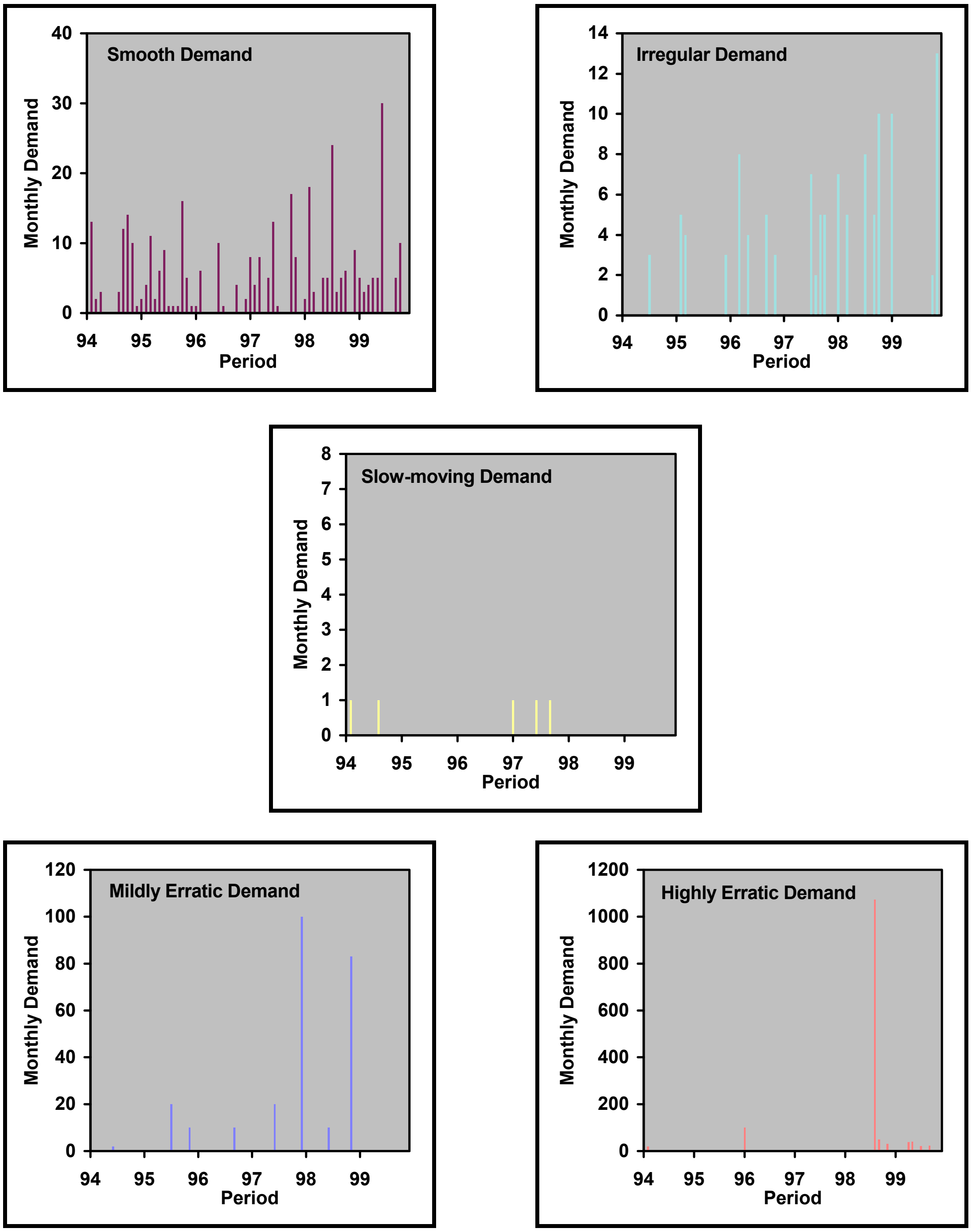
The final proportion of RAF line items falling within each demand pattern is illustrated in Figure 6.1. Taken as a whole, the demand classifications are achieved in an objective manner with the patterns being identified through an analysis of the data.

**Figure 6.1: Classification of RAF Demand.**




Examples of actual fitted demand patterns are illustrated in Figure 6.2 for monthly data, noting the differing scales on the vertical axis. The smooth demand pattern is characterised by frequent transactions of a not too dissimilar size and few months have zero demand. In contrast, an irregular demand pattern has a somewhat consistent transaction frequency but the demand size is seen to vary with many zero demands occurring, though not to the scale of the erratic demand patterns.

**Figure 6.2: Sample Demand Patterns.**




The slow-moving demand pattern is seen to have infrequent transactions with low demand sizes. The two erratic demand patterns will appear similar in this context, each exhibiting infrequent transactions with variable demand sizes, but they are differentiated by the lead-time variability as described in a later section.

## 6.2 Demand Pattern Fragmentation

This section investigates the common traits among line items within each demand pattern and seeks to determine whether differences exist between the patterns using a number of factors, including lead-time, cluster grouping, and demand frequency and demand size. The determination of common traits within a grouping would allow the categorisation of new line items in a straight-forward manner.

### 6.2.1 Lead-Time

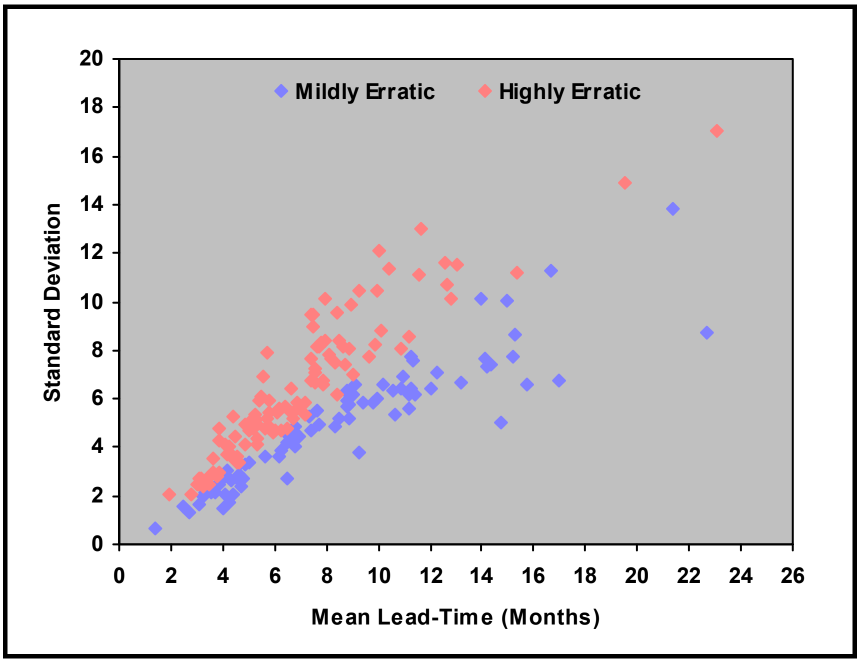
As far as the demand patterns are concerned, the lead-time is only a factor in differentiating the two erratic patterns. As shown in Table 6.3, the mildly erratic demand pattern has an average lead-time mean and standard deviation of 8.95 and 5.26 respectively, while the highly erratic demand pattern has a smaller mean of 5.70 months, although the standard deviation of 5.43 does not differ substantially.

**Table 6.3: Lead-Time by Demand Pattern.**

| Demand Pattern | Lead-Time Statistics (months) | | |
|---|---|---|---|
| | *n* | Mean | Std Dev |
| **Smooth** | 28,372 | 10.21 | 6.33 |
| **Irregular** | 27,565 | 8.46 | 5.74 |
| **Slow-moving** | 83,501 | 8.22 | 5.84 |
| **Mildly Erratic** | 34,603 | 8.95 | 5.26 |
| **Highly Erratic** | 49,705 | 5.70 | 5.43 |
| **Overall** | **223,746** | **8.05** | **5.71** |

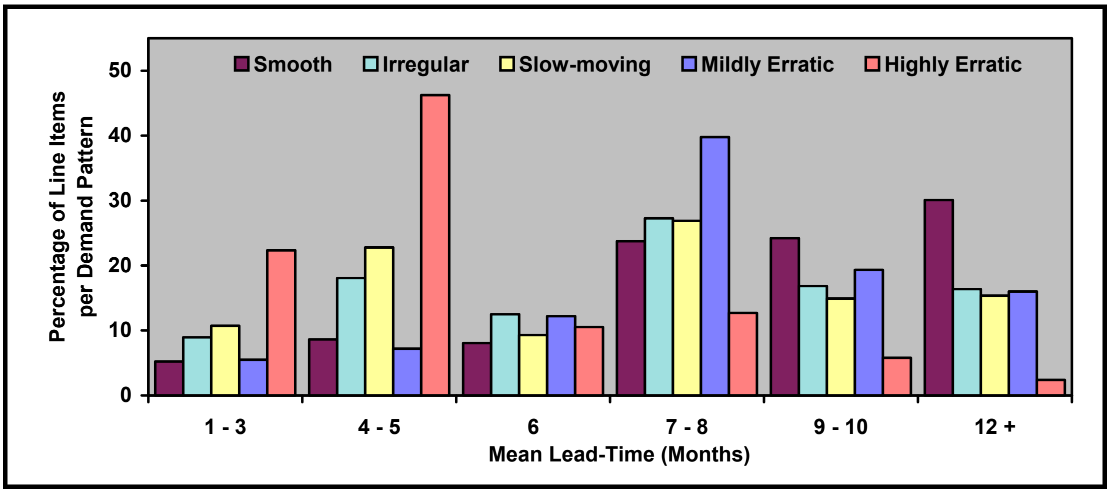
In practice, however, it is the coefficient of variation for the lead-time that provides the distinction between the two erratic demand groupings. Thus, if the standard deviation as a proportion of the mean, all squared, is in excess of the overall median of 0.52537 (from Table 6.2) then the line item will be classified as highly erratic rather than mildly erratic as illustrated in Figure 6.3.

**Figure 6.3: Erratic Demand Lead-Time Variation.**




The distribution of all line items by demand pattern against the lead-time is illustrated in Figure 6.4.

**Figure 6.4: Lead-Time by Demand Pattern.**



The plot indicates the frequency of a mildly erratic demand pattern peaks at a higher lead-time value than does the highly erratic demand pattern. This is because the shorter the mean lead-time the more likely the standard deviation will be high in comparison, as it is starting from a lower base level, hence a higher lead-time CV. The other three demand patterns, including smooth, irregular and slow-moving, show similar proportions of line items under each lead-time period, although the smooth demand pattern has a higher proportion with longer lead-times through sheer chance alone.

### 6.2.2 Cluster Grouping

The fragmentation of groupings by predictor variable, comprising set PLT, manufacturer and range manager at SMB Level 2, allows further comparisons between demand patterns. A starting point is provided by Table 6.4, which presents the observed proportion of RAF line items falling within each grouping, as previously shown in the pie charts of Figures 5.7, 5.10 and 5.13.

**Table 6.4: Grouping Percentages.**

| Grouping | Predictor | | |
|---|---|---|---|
| | Set PLT | Manuf'r | SMB L2 |
| A | 16.9% | 11.4% | 4.5% |
| B | 4.7% | 16.8% | 30.1% |
| C | 24.5% | 15.8% | 15.4% |
| D | 24.4% | 41.2% | 10.4% |
| E | 18.2% | 4.6% | 30.7% |
| F | 11.4% | 10.3% | 8.9% |

In this form, the tabulated values can be used as expected frequencies in a three-variable contingency table for testing the hypothesis that the data classification methods are independent. Under the null hypothesis, independence of classification exists between the groupings for each predictor and the final demand pattern.

Once the line items have been allocated a grouping, placed in the lead-time grouping cube, and finally identified with a demand pattern, a fragmentation by cluster grouping may provide insight into how the groupings lead to a particular demand classification. In effect, this examines the demand pattern breakdown of each of the rows in Table 5.26 on page 123. A total of 72,712 line items are included, which represents all mature line items currently in the RAF inventory for which lead-time observations are available.

Using a presentational technique similar to that used by Cooke [18] for dependence modelling and risk management with UNICORN software, a cobweb plot is used in this instance to graphically illustrate the relationships that lead to a demand pattern classification. A cobweb plot usually compares the joint distribution of the percentiles from rank correlations of a set of variables, such that when percentiles are shown on the vertical axis in a cobweb plot with $x$ and $y$ adjacent and having rank correlation $\tau(x, y)$:

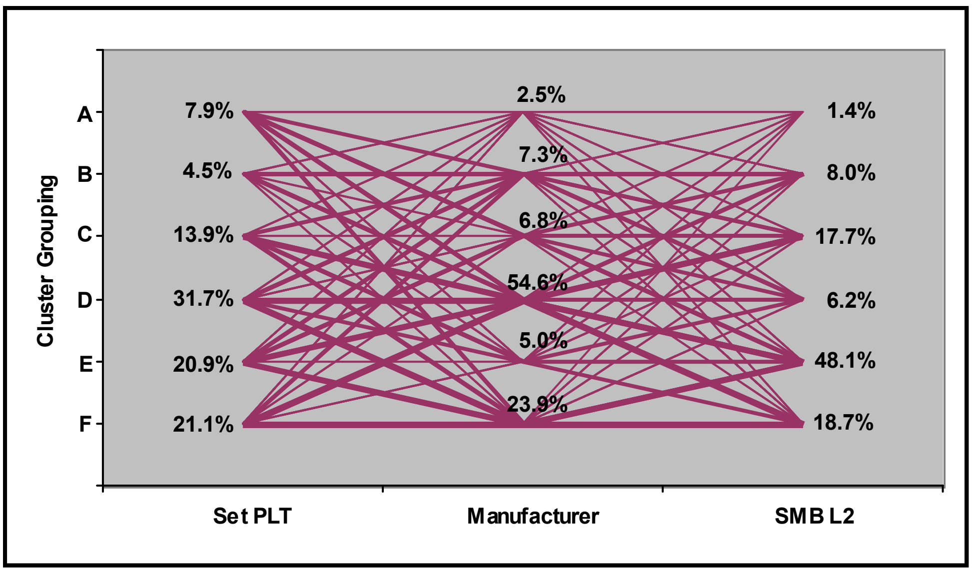
(i) If $\tau = 1$, then all lines between $x$ and $y$ are horizontal,

(ii) If $\tau = 0$, then the lines criss-cross in such a manner that the density of crossings between $x$ and $y$ is triangular, and

(iii) If $\tau = -1$, then all lines cross in one point.

A cobweb plot, such as the one shown for smooth demand in Figure 6.5, illustrates the observed pathways that lead to a particular demand pattern classification. The pathway density allocates the traversing observations to one of three frequency classifications, with the heaviest lines portraying in excess of 250 lead-time observations (accounting for approximately 5 to 10 percent of the flows), the intermediary lines portraying between 51 and 250 observations (25 to 45 percent of the flows), and the lightest lines portraying 50 or fewer observations (45 to 70 percent of the flows). The percentages on the nodes indicate the proportion of observations that fall into each cluster grouping. These percentages can be compared with those of Table 6.4 for significant differences.

**Figure 6.5: Cobweb Plot for Smooth Demand.**




A preliminary examination of the cobweb plot indicates that 31.7 percent of the observations share a common set PLT grouping in group D (7 to 9 months), while a large number of observations share a common manufacturer grouping with 54.6 percent of observations similarly in group D. The observations also tend to be unevenly spread amongst the range manager groupings at SMB Level 2 with the fifth grouping comprising 48.1 percent of the observations.

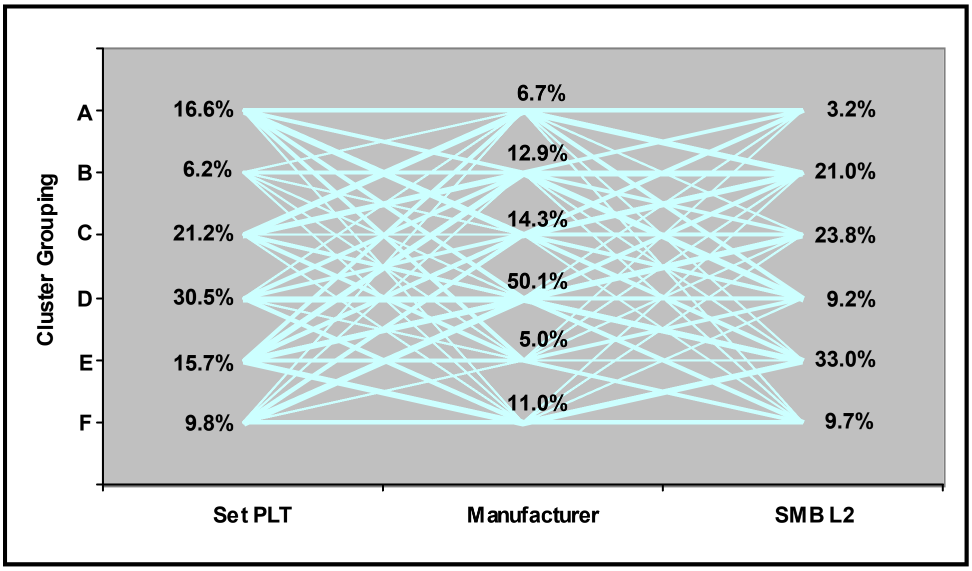
However, as the observations are in fact unevenly spread amongst the cluster groupings as a whole, it is necessary to examine how the observed frequencies for each demand pattern differ from the expected frequencies. This can be done using a chi-square contingency table test, which will test the null hypothesis that the grouping for each predictor and the demand pattern classification are independent. Under this test a $\chi^2$ value is calculated and compared with a tabulated value with $(r-1)(c-1)$ degrees of freedom, where $r$ is the number of rows in the contingency table and $c$ is the number of columns.

Calculating a chi-square statistic for each predictor for the smooth demand pattern gives values of 20.23, 39.82 and 41.16 for the set PLT, the manufacturer and SMB L2 respectively. With ten degrees of freedom the tabulated $\chi^2$ value is 18.31 at the 5 percent significance level. As the calculated value for the set PLT is greater than the tabulated value, we conclude that the smooth demand pattern is dependent upon the set PLT value. The below expected percentages in the lower PLT categories, combined with the higher than expected percentages in the upper categories, leads to the rejection of the null hypothesis. Therefore, a higher set PLT value, without further consideration, may predispose a smooth demand classification.

Similarly, as the calculated values for both the manufacturer and SMB L2 are greater than the tabulated value the demand pattern is not independent of either of these predictors. A higher than expected percentage of items fall in category F for the manufacturer and category B for SMB L2, although the interpretation with these categories is not immediately obvious.

Figure 6.6 presents a cobweb plot for the irregular demand classification. The calculated $\chi^2$ statistics are 3.00, 4.97 and 8.14 in this instance. As each of these are in fact less than the tabulated $\chi^2$ value of 18.31, the conclusion is that the attainment of an irregular demand pattern is not dependent upon the groupings of any of the three predictor variables.

**Figure 6.6: Cobweb Plot for Irregular Demand.**




A cobweb plot for slow-moving demand is shown in Figure 6.7. With calculated $\chi^2$ statistics of 1.80, 0.43 and 2.90 in this case, the slow-moving demand pattern is also not dependent upon the groupings of any of the predictor variables.

**Figure 6.7: Cobweb Plot for Slow-Moving Demand.**

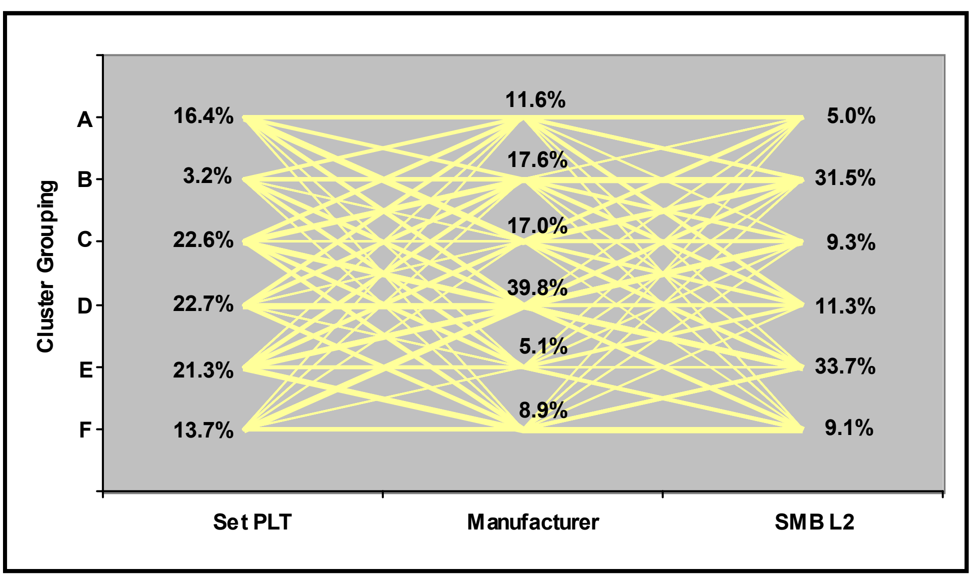
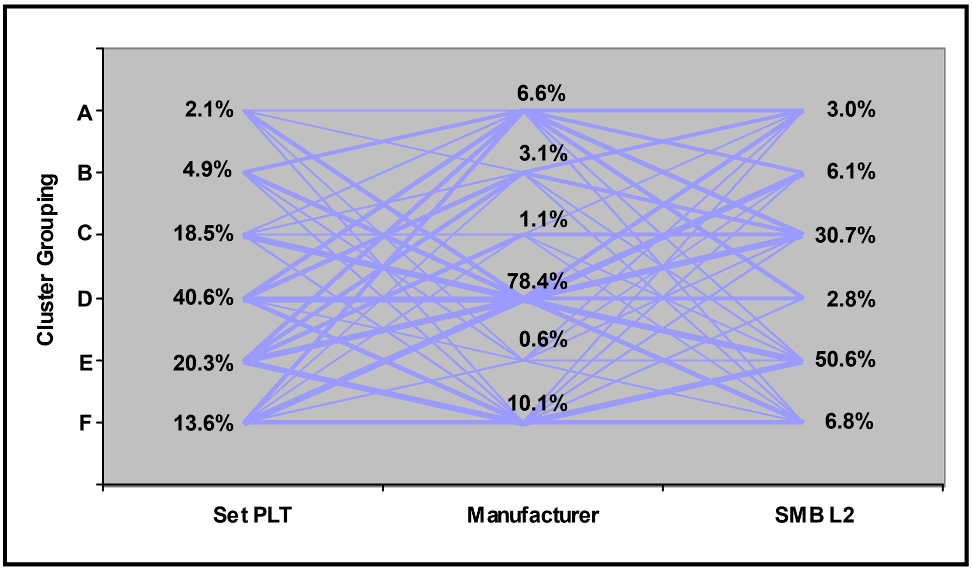


Figure 6.8 presents a cobweb plot for line items classified with a mildly erratic demand pattern. With chi-square values calculated as 25.88, 63.79 and 53.77, the mildly erratic demand classification depends on each of the set PLT, manufacturer and range manager groupings respectively.

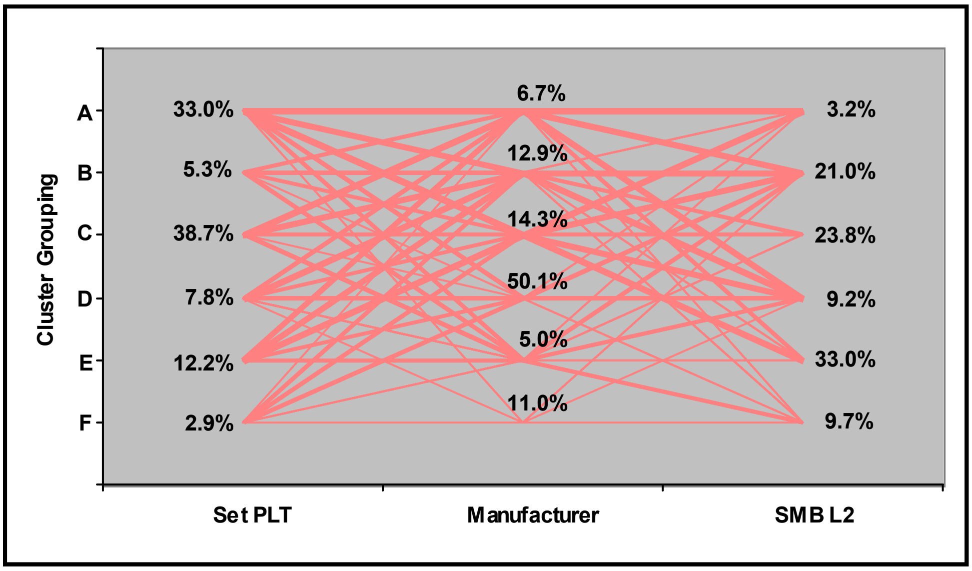
**Figure 6.8: Cobweb Plot for Mildly Erratic Demand.**




In this case the main contributor to the calculated value for the set PLT is the lower than expected percentage of low PLT observations. For the manufacturer and the range manager it is the higher than expected number of observations in category D and category C respectively, and likewise lower than expected observations in category C and category B that led to a rejection of the null hypothesis.

Finally, the cobweb plot for line items classified as having a highly erratic demand pattern is shown in Figure 6.9. The diagram indicates that the observations are not universally spread amongst the cluster groupings with few observations in the category F groupings in all instances. This coincides with a previous observation that line items classified as highly erratic tend to have lower lead-time values and this would restrict their placement in the naturally high lead-time category F groupings.

**Figure 6.9: Cobweb Plot for Highly Erratic Demand.**




Calculated chi-square values in this instance are 43.38, 94.11 and 86.86, indicating the demand pattern classification is not independent of any of the predictor classifications. Not surprisingly, the distribution of observations among the cluster groupings tends to be the opposite of what is seen with the previous erratic demand pattern, as it is the lead-time CV that provides the distinction between the two patterns and it is the lead-time parameters that determine the original cluster groupings. Hence, the highly erratic demand pattern tends to be over-represented in the low PLT groupings, while in the case of the manufacturer there are fewer than expected observations in category D and more than expected in category C. Similarly, in the case of SMB L2 there are fewer than expected observations in category C and more than expected in category B.

To summarise these observations, Table 6.5 presents the calculated chi-square values against the predictors for each demand pattern. With a tabulated $\chi^2$ value of 18.31 it is seen that for smooth, mildly erratic, and highly erratic demand patterns all calculated values exceed the tabulated value. Therefore, the attainment of these three patterns is dependent on the predictors. On the other hand, all calculated values for irregular and slow-moving demand patterns are less than the tabulated value, and therefore the attainment of these patterns is independent of the predictors.

**Table 6.5: Cluster / Demand Pattern Independency Test.**

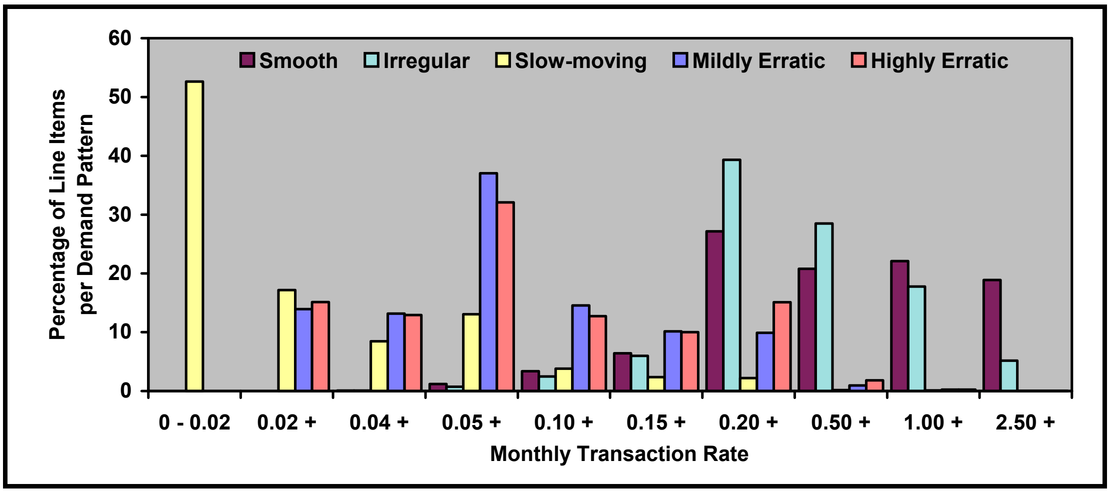
| Demand Pattern | Chi-Square Values | | |
|---|---|---|---|
| | Set PLT | Manuf'r | SMB L2 |
| **Smooth** | 20.23 | 39.82 | 41.16 |
| **Irregular** | 3.00 | 4.97 | 8.14 |
| **Slow-moving** | 1.80 | 0.43 | 2.90 |
| **Mildly Erratic** | 25.88 | 63.79 | 53.77 |
| **Highly Erratic** | 43.38 | 94.11 | 86.86 |

One conclusion drawn from the demand fragmentation is that although, the lead-time demand pattern is not wholly independent of the set PLT, the manufacturer and the range manager, there tends to be no clear-cut influence from the predictors that would assist in determining the demand pattern. On this basis, it appears necessary to consider the other factors of demand frequency and size.

### 6.2.3 Demand Frequency and Size

This section further investigates the demand pattern classifications, this time in respect of demand frequency and size. The proportion of line items falling within each classification over a range of monthly transaction rates is presented in Figure 6.10. All line items with a transaction rate of less then 0.02 per month, or alternatively one observation over a six year period, are naturally classed as slow-moving. Once the transaction rate exceeds 0.50 per month, the slow-moving classification is no longer observed. A smooth or irregular demand pattern is not observed until the transaction rate is in excess of 0.05 per month and rates beyond 1.00 per month become their sole preserve, with the smooth demand pattern dominating at the latter stages.

**Figure 6.10: Transaction Frequency by Demand Pattern.**




The average transaction rate for a slow-moving demand classification is 0.04 while the average for a smooth demand classification is 2.28 transactions per month. The irregular demand classification experiences 0.90 transactions per month on average. Both the mildly erratic demand and the highly erratic demand peak at 0.05 transactions per month, and the average transaction rates are also similarly matched at 0.11 and 0.13 respectively. This similarity between the mildly erratic and highly erratic demand patterns corresponds with the fact that the classification procedure does not distinguish between the two erratic demand patterns based on the transaction frequency. Therefore, the two erratic demand patterns could be combined as one in this instance.

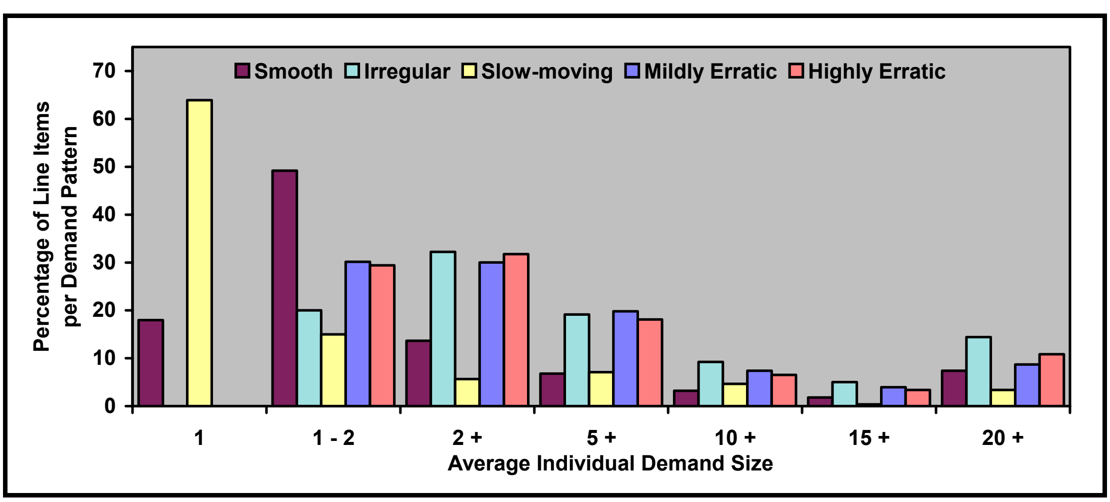
Table 6.6 presents the proportion of demand periods with positive demand for quarterly, monthly and weekly demand aggregations. Results were obtained from a sample of 18,750 line items, where the selected items comprise a stratified random sample with equal representation between the five identified demand patterns. Smooth demand is observed to have the highest percentage of positive demand while slow-moving demand has the lowest. The proportion of periods with zero demand increases for all demand patterns as the data moves from quarterly, to monthly and on to weekly aggregation.

**Table 6.6: Percentage of Periods with Positive Demand.**

| Demand Pattern | Quarterly Data | | Monthly Data | | Weekly Data | |
|---|---|---|---|---|---|---|
| | Positive Demand | Zero Demand | Positive Demand | Zero Demand | Positive Demand | Zero Demand |
| Smooth | 78.4% | 21.6% | 53.8% | 46.2% | 23.7% | 76.3% |
| Irregular | 70.2% | 29.8% | 40.8% | 59.2% | 14.1% | 85.9% |
| Slow-moving | 28.9% | 71.1% | 11.6% | 88.4% | 3.1% | 96.9% |
| Mildly Erratic | 30.4% | 69.6% | 12.3% | 87.7% | 3.3% | 96.7% |
| Highly Erratic | 34.1% | 65.9% | 14.2% | 85.8% | 3.8% | 96.2% |
| **Overall** | **48.4%** | **51.6%** | **26.5%** | **73.5%** | **9.6%** | **90.4%** |

Figure 6.11 presents the proportion of line items within each demand pattern classification over a range of demand sizes. For an average demand size of one unit it is only the smooth and slow-moving demand patterns that occur, with the majority of slow-moving line items fitting in this category.

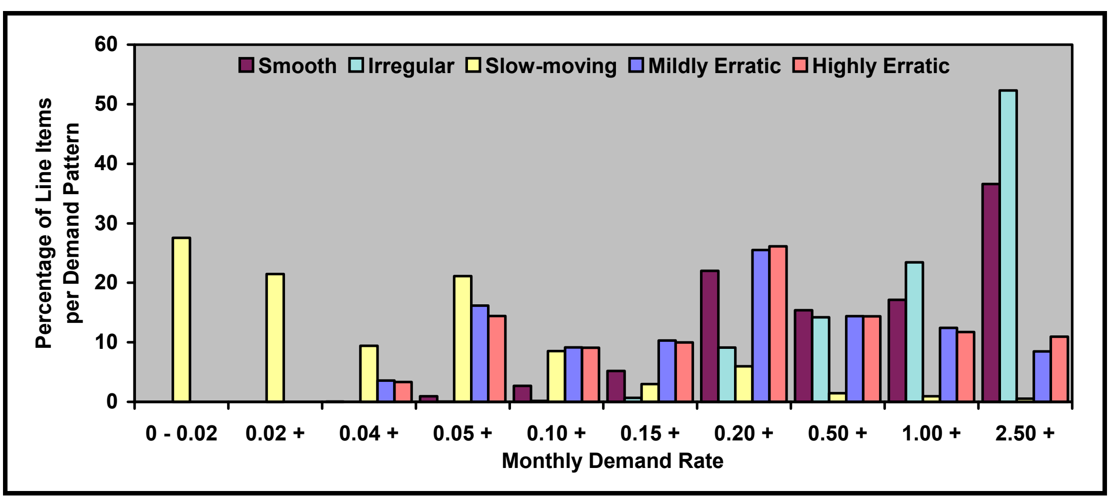
**Figure 6.11: Demand Size by Demand Pattern.**



The percentage of slow-moving items quickly decreases as the average demand size increases, although the average demand size across all line items with this pattern is 4.44 and the pattern still arises when the average demand size exceeds 20 units. The smooth demand pattern is most frequent when the average demand size is between one and two units although this distribution is also skewed to the right as the average demand size is 7.00 units. The majority of irregular and erratic demand line items have an average demand size between one and five units, although the overall average demand size is somewhat higher at 14.34, 9.32 and 12.31 for irregular, mildly erratic and highly erratic demand patterns respectively.

The demand pattern cannot satisfactorily be explained by examining the demand size and frequency in isolation, but rather through their joint consideration. Figure 6.12 presents the proportion of line items falling within each demand classification according to the monthly demand rate.

**Figure 6.12: Demand Rate by Demand Pattern.**




Demand rates of less than 0.04 units per month lead to a slow-moving demand classification, although this classification is still observed when the demand rate is in excess of 2.50, the average being 0.15 units per month. An erratic demand classification peaks at 0.20 units per month, with the mildly erratic demand classification having a mean of 1.20 and the highly erratic demand pattern having a mean of 1.85 units per month. The average demand rate for a smooth demand classification is 38.45 units per month, and the average is 16.51 for an irregular demand pattern.

The demand frequency and size is summarised for each demand pattern in Table 6.7. It is seen that the smooth demand pattern experiences the highest transaction rate on average followed by the irregular demand pattern, while, as expected, the slow-moving demand pattern experiences the lowest. In terms of demand size, it is the irregular demand pattern that has the highest average and again the slow-moving demand has the lowest. The monthly demand rate presents the same ordering of the demand patterns as the monthly transaction rate.

**Table 6.7: Demand Frequency and Size by Demand Pattern.**

| Demand Pattern | Mean Monthly Transaction Rate | Mean Individual Demand Size | Mean Monthly Demand Rate |
|---|---|---|---|
| **Smooth** | 2.28 | 7.00 | 38.45 |
| **Irregular** | 0.90 | 14.34 | 16.51 |
| **Slow-moving** | 0.04 | 4.44 | 0.15 |
| **Mildly Erratic** | 0.11 | 9.32 | 1.20 |
| **Highly Erratic** | 0.13 | 12.31 | 1.85 |
| **Overall** | **0.46** | **8.49** | **7.56** |

Though not suitable for allowing any categorisation of new line items, the unit price by demand pattern is worthy of examination; the subject of the next section.

### 6.2.4 Unit Price

Summary statistics for the unit price are presented in Table 6.8 for each demand pattern. It is observed that line items with a smooth demand pattern have the highest average unit value. Such items are fast-movers by definition, so it is surprising they are the high-value items, although the slow-moving line items also have a high average value as well as the highest maximum value, as would be expected.

**Table 6.8: Summary Statistics for Unit Price.**

| Statistic | Demand Pattern | | | | |
|---|---|---|---|---|---|
| | Smooth | Irregular | Slow-Moving | Mildly Erratic | Highly Erratic |
| ***n*** | **28,372** | **27,565** | **83,501** | **34,603** | **49,705** |
| Mean (£) | 460.83 | 115.84 | 343.75 | 137.02 | 107.52 |
| Standard Dev | 1,792.44 | 571.16 | 3,015.80 | 756.04 | 804.88 |
| Maximum | 102,837.54 | 25,581.95 | 654,938.27 | 53,211.37 | 87,299.31 |
| Upper Quartile | 279.11 | 49.34 | 183.08 | 69.07 | 42.93 |
| Median | 62.68 | 9.14 | 41.29 | 14.90 | 9.14 |
| Lower Quartile | 10.29 | 1.72 | 8.09 | 2.85 | 1.53 |
| Minimum | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |

Both the smooth demand pattern and the slow-moving demand pattern have a high standard deviation, indicating wide variation in the individual unit prices. Values for the quartiles and median for these two patterns are also substantially higher than those for the other demand patterns, which suggests that higher values occur throughout. Thus, the high average values are not governed simply by a small number of extreme values.

### 6.2.5 Comparison With Initial Demand Classification

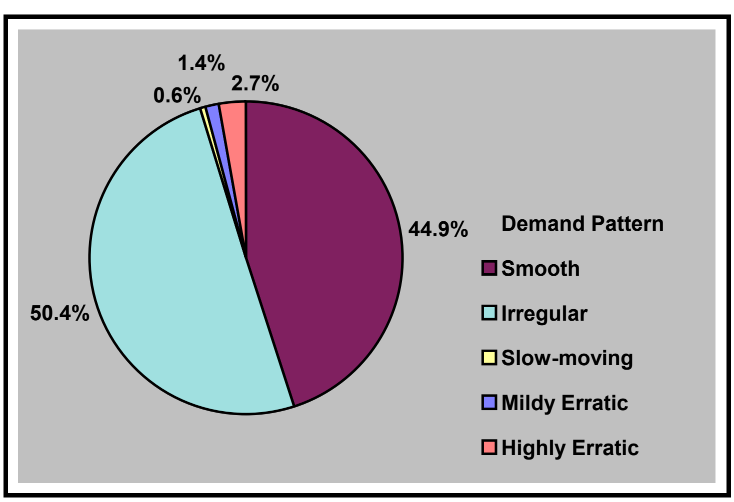
Section 4.5.2 provided an analysis of 12,644 RAF line items initially considered to have an erratic demand pattern. On that occasion, arbitrary boundaries of between 50 and 99 demand transactions over a 72 month period, or equivalently between 0.69 and 1.38

transactions per month on average, were used to identify line items as erratic. At that time it was recognised that the transaction rates were likely to be too high to capture the erratic demand, but it was necessary to ensure that there were enough observations for the analysis.

It is now possible to ascertain whether the demand classification methodology utilised in this chapter has defined the same line items as erratic. A cursory glance at Table 6.7 would suggest otherwise, as the mean monthly transaction rate for the mildly erratic demand pattern is 0.11 and for the highly erratic demand pattern it is 0.13, both very much less than the selected boundaries. In addition, Figure 6.10 indicated previously that very few line items classified as erratic have a transaction rate in excess of 0.5 per month.

Therefore, it is expected that only a small proportion of the 12,644 line items, arbitrarily selected as erratic in Section 4.5.2, would in fact have an erratic demand pattern. Confirmation is provided by Figure 6.13, which indicates only 4.1 percent (1.4 + 2.7) of the line items are in fact classified as erratic.

**Figure 6.13: Initial Erratic Demand Classification.**



The observations made in this chapter are derived solely from RAF demand data. It is likely that data from other non-military industries would differ significantly. On the whole, it would be expected that lead-times would be considerably shorter elsewhere, and the overall demand is likely to be higher. What has been classified as smooth demand in this instance could well be considered as erratic in other industries. However, the methods employed in both clustering lead-time observations and classifying the line items by demand pattern are certainly applicable elsewhere.

### 6.2.6 Autocorrelation and Crosscorrelation

This section returns to the issue of autocorrelation and crosscorrelation in the demand data, and considers the wider sample of 18,750 line items used previously. Line items have been classified according to the significance of their autocorrelations and crosscorrelations of the logarithmic transformation:

(i) If the correlations are not significant on the whole a *not signif* classification is given, otherwise

(ii) If the sum of the individually significant correlations is less than zero a *negative* classification is given, or

(iii) If the sum of the individually significant correlations is greater than zero a *positive* classification is given, alternatively

(iv) If the correlations are significant on the whole but there are no individually significant correlations a *no sign* classification is given.

Table 6.9 presents the proportions classed as not significantly correlated, negatively correlated and positively correlated, as well as the percentage for which no sign was determined. Statistics are shown for the demand size, the interval between transactions and the combined size and interval correlations. In each case the correlation statistics are divided into the five equally represented demand patterns.

**Table 6.9: Correlation by Demand Pattern.**

| Statistic | Demand Pattern | Percentage of Line Items | | | |
|---|---|---|---|---|---|
| | | Negative | Not Signif | Positive | No Sign |
| Demand Size Autocorrelation | Smooth | 4.99 | 86.19 | 7.17 | 1.65 |
| | Irregular | 6.19 | 83.41 | 8.32 | 2.08 |
| | Slow-moving | 6.83 | 84.13 | 6.27 | 2.77 |
| | Mildly Erratic | 4.27 | 87.36 | 7.25 | 1.12 |
| | Highly Erratic | 4.72 | 85.79 | 7.76 | 1.73 |
| | **Overall** | **5.40** | **85.38** | **7.35** | **1.87** |
| Interval Between Demands Autocorrelation | Smooth | 8.83 | 81.57 | 7.28 | 2.32 |
| | Irregular | 9.17 | 79.12 | 9.36 | 2.35 |
| | Slow-moving | 9.47 | 81.04 | 5.92 | 3.57 |
| | Mildly Erratic | 7.57 | 81.92 | 7.97 | 2.53 |
| | Highly Erratic | 8.19 | 81.55 | 7.81 | 2.45 |
| | **Overall** | **8.65** | **81.04** | **7.67** | **2.65** |
| Demand Size and Interval Crosscorrelation | Smooth | 11.63 | 77.65 | 10.40 | 0.32 |
| | Irregular | 13.15 | 74.61 | 11.89 | 0.35 |
| | Slow-moving | 13.23 | 73.55 | 12.67 | 0.56 |
| | Mildly Erratic | 11.73 | 79.25 | 8.77 | 0.24 |
| | Highly Erratic | 12.05 | 75.79 | 11.84 | 0.32 |
| | **Overall** | **12.36** | **76.17** | **11.11** | **0.36** |

In the case of demand size, some 85.4 percent of line items are not significantly autocorrelated as a whole at the 5 percent significance level, while 5.4 percent are negatively autocorrelated and 7.4 percent are positively autocorrelated. No sign was determined for 1.9 percent of the line items. When considering the interval between demands some 81.0 percent are not significantly autocorrelated, while 8.7 percent are negatively autocorrelated, 7.7 percent are positively autocorrelated, and no sign was determined for 2.7 percent. The percentage of line items not significantly crosscorrelated falls to 76.2 percent, with 12.4 percent negatively crosscorrelated and 11.1 percent positively crosscorrelated, with no sign determined for 0.4 percent.

These results are reasonably consistent with the autocorrelation and crosscorrelation results of Section 4.5.6 where the correlations among 12,251 line items were analysed. In the previous analysis some 74.5 percent of line items did not have statistically significant demand size autocorrelations compared with 85.4 percent on this occasion. Similarly, some 80.8 percent did not have autocorrelation in the interval between demands and 82.2 percent did not have significant crosscorrelation compared with 81.0 and 76.2 percent respectively on this occasion.

## 6.3 Fitting Distributions to RAF Data

In this section, goodness-of-fit tests are performed on the demand size distribution, the demand interval distribution and the lead-time distribution using large quantities of data. Individual line items are utilised in fitting distributions to the demand size and demand interval data, although line items have been grouped to provide sufficient data for fitting lead-time distributions.

The purpose of this section is not an exact test of a particular hypothesis, but rather to determine the reasonableness of models put forward in the literature. It should be borne in mind that the nature of the chi-square statistic itself means that relatively small differences between observed and expected frequencies will lead to large chi-square values if the sample is large.

### 6.3.1 Demand Size Distribution

A goodness-of-fit test was conducted on the demand sizes of the 18,750 line items utilised in previous chapters. However, as these line items comprise a range of demand patterns, including slow-moving, some of the sample sizes are inadequate for analysis purposes. A total of 6,795 line items with over 20 demand transactions during the six year period were selected from the data set and a maximum of 200 demand size observations were randomly selected for each line item.

The results presented in Table 6.10 show that the geometric distribution provides the most frequent fit, with 63 percent of line items fitting at the 5 percent significance level. The logarithmic distribution closely follows with 61 percent.

**Table 6.10: Goodness-of-Fit Test Results - Demand Size.**

| Probability Distribution | Goodness-of-Fit Statistics ($n$ = 6,795) | | | | | |
|---|---|---|---|---|---|---|
| | Alpha 0.10 | | Alpha 0.05 | | Alpha 0.01 | |
| | Count | Percent | Count | Percent | Count | Percent |
| Geometric | 3,858 | 56.8% | 4,249 | 62.5% | 4,908 | 72.2% |
| Logarithmic | 3,795 | 55.8% | 4,126 | 60.7% | 4,730 | 69.6% |
| Log Normal | 2,432 | 35.8% | 2,861 | 42.1% | 3,584 | 52.7% |
| Negative Exponential | 2,040 | 30.0% | 2,487 | 36.6% | 3,195 | 47.0% |
| Gamma | 2,068 | 30.4% | 2,408 | 35.4% | 2,968 | 43.7% |
| Laplace | 1,142 | 16.8% | 1,371 | 20.2% | 1,917 | 28.2% |
| Negative Binomial | 925 | 13.6% | 1,176 | 17.3% | 1,697 | 25.0% |
| Normal | 789 | 11.6% | 1,006 | 14.8% | 1,425 | 21.0% |
| Poisson | 663 | 9.8% | 837 | 12.3% | 1,327 | 19.5% |

### 6.3.2 Interval Between Transactions Distribution

Using the same 6,795 line items considered in the previous section, Table 6.11 presents the results of a goodness-of-fit test on the interval between transactions. Once again, a maximum of 200 observations were randomly selected for each line item. The results of the analysis indicate that the log normal distribution provides a very good fit with 93 percent of line items at the 5 percent significance level, followed closely by the geometric distribution with a 91 percent fit.

**Table 6.11: Goodness-of-Fit Test Results - Interval Between Transactions.**

| Probability Distribution | Goodness-of-Fit Statistics (*n* = 6,795) | | | | | |
|---|---|---|---|---|---|---|
| | Alpha 0.10 | | Alpha 0.05 | | Alpha 0.01 | |
| | Count | Percent | Count | Percent | Count | Percent |
| Log Normal | 6,131 | 90.2% | 6,330 | 93.2% | 6,570 | 96.7% |
| Geometric | 6,001 | 88.3% | 6,189 | 91.1% | 6,421 | 94.5% |
| Negative Exponential | 5,769 | 84.9% | 5,983 | 88.1% | 6,244 | 91.9% |
| Gamma | 5,109 | 75.2% | 5,326 | 78.4% | 5,648 | 83.1% |
| Negative Binomial | 4,820 | 70.9% | 5,071 | 74.6% | 5,436 | 80.0% |
| Logarithmic | 2,996 | 44.1% | 3,625 | 53.3% | 4,743 | 69.8% |
| Normal | 1,136 | 16.7% | 1,588 | 23.4% | 2,421 | 35.6% |
| Laplace | 915 | 13.5% | 1,407 | 20.7% | 2,609 | 38.4% |
| Poisson | 1 | 0.0% | 3 | 0.0% | 41 | 0.6% |

The results of this goodness-of-fit test reveal that the Poisson distribution provides a very poor fit for the interval between transactions. At first glance this result undermines the choice of the compound Poisson distribution that has found widespread usage in the erratic demand environment. However, some clarification is required at this point; Table 6.11 has been generated using the interval between transactions rather than the number of arrivals in a fixed interval, which would be required of a Poisson arrival process.

There is a close connection between the negative exponential distribution and the Poisson distribution. If the interval between transactions has a negative exponential distribution with parameter $\lambda$, then the number of arrivals in one unit of time has a

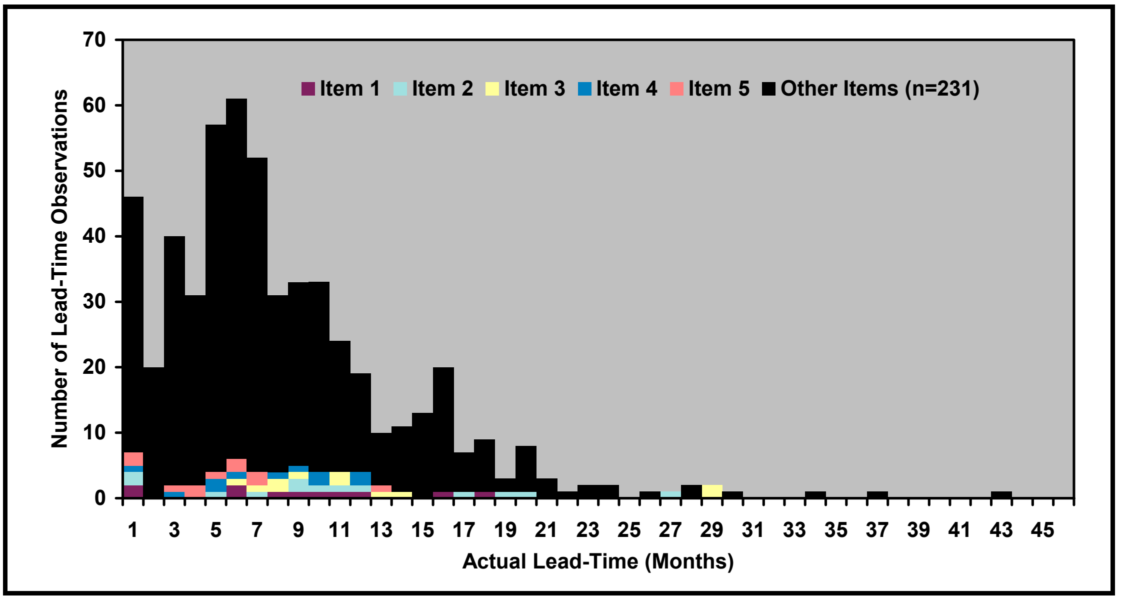
Poisson distribution also with parameter $\lambda$. Therefore, some 88 percent of observations would be expected to fit a Poisson process at the 5 percent level.

### 6.3.3 Lead-Time Distribution

In examining the lead-time distribution, this section revisits the lead-time grouping cube created in the previous chapter. On that occasion, RAF lead-time observations were combined to form 216 groupings on the basis of the set PLT, the manufacturer and the range manager. The grouping of lead-time observations, and an associated increase in sample size, allows a more conclusive goodness-of-fit test on the lead-time observations over the one conducted in Section 5.1. Previously 161 individual line items with twelve or more lead-time observations were analysed, although the low number of observations per line item meant a range of theoretical distributions were candidates.

Actual lead-times from a sample grouping combination (PLT group B, manufacturer group D and range manager group E from Table 5.26) are illustrated as a histogram in Figure 6.14, where the lead-times for five line items with more than ten lead-time observations each are shown individually while the remaining 231 line items are combined. On average, the combined line items experience 2.1 lead-time observations each. It is observed that the line items shown individually encompass the range of lead-time values and therefore there is no evidence to suggest the observations should not be grouped in this manner. In total, there are 546 lead-time observations in the sample.

**Figure 6.14: Actual Lead-Times for a Sample Grouping (Group B-D-E).**



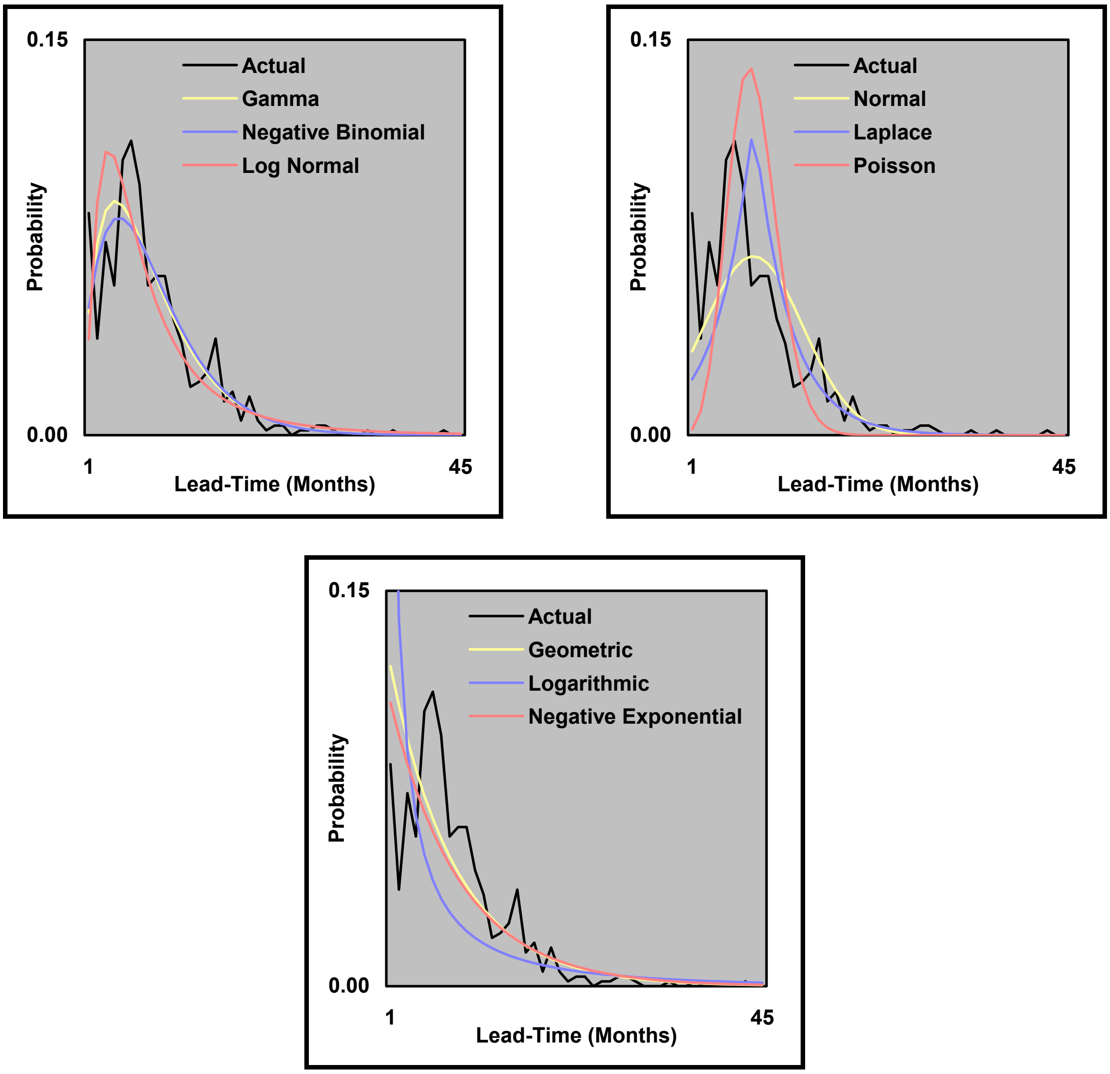

The shape of the plot in Figure 6.14 appears somewhat removed from a normal distribution. In fact, the summary results presented in Table 6.12 confirm non-normality with the relatively high values for skewness and kurtosis, as well as a Kolmogorov-Smirnov *D* statistic of 0.146 which is in excess of the critical value of 0.038 at the 5 percent significance level.

**Table 6.12: Analysis of Sample Lead-Time Data.**

| *n* | Mean | Median | Std Dev | Variance | Skewness | Kurtosis | *D* Statistic |
|---|---|---|---|---|---|---|---|
| 546 | 8.24 | 7.00 | 5.86 | 34.38 | 1.61 | 4.34 | 0.146 |

The plots presented in Figure 6.15 all compare the actual lead-time sample histogram of Figure 6.14 against a range of possible probability distributions. Conducting a chi-square goodness-of-fit test using the GoodFit methodology indicates that the gamma distribution fits at the 5 percent significance level, the negative binomial distribution fits at the 1 percent level, while the remaining distributions do not fit at all.

**Figure 6.15: Fitting Probability Distributions to Lead-Time Data.**




It is observed that the log normal distribution provides a similar shape to the actual data although the positioning of the peaks differ. This distribution may provide a good fit against other sample groupings, particularly if an adjustment to the mean is introduced.

A goodness-of-fit test was conducted on a sample of 82 groupings from the 216 corresponding to the lead-time grouping cube. The groupings were selected as those with 200 or more lead-time observations such that there was sufficient data for analysis. Exactly 200 observations were randomly selected from each grouping.

Results presented in Table 6.13 indicate that none of the distributions offer a complete description of the replenishment lead-time. The log normal is seen to be suitable in 52 percent of cases at the 5 percent significance level (without any adjustment to the mean) and provides the best fit more frequently than any other distribution. The geometric distribution provides the next best fit with 37 percent. On the other hand, the normal distribution fits in only 11 percent of cases, providing strong evidence that lead-times are not normal in reality.

**Table 6.13: Goodness-of-Fit Test Results - Grouped Lead-Time Observations.**

| Probability Distribution | Goodness-of-Fit Statistics (*n* = 82) | | | | | |
|---|---|---|---|---|---|---|
| | Alpha 0.10 | | Alpha 0.05 | | Alpha 0.01 | |
| | Count | Percent | Count | Percent | Count | Percent |
| Log Normal | 30 | 36.6% | 43 | 52.4% | 57 | 69.5% |
| Geometric | 25 | 30.5% | 30 | 36.6% | 44 | 53.7% |
| Gamma | 18 | 22.0% | 24 | 29.3% | 34 | 41.5% |
| Negative Binomial | 18 | 22.0% | 21 | 25.6% | 30 | 36.6% |
| Logarithmic | 14 | 17.1% | 15 | 18.3% | 17 | 20.7% |
| Normal | 8 | 9.8% | 9 | 11.0% | 12 | 14.6% |
| Negative Exponential | 5 | 6.1% | 7 | 8.5% | 14 | 17.1% |
| Laplace | 3 | 3.7% | 5 | 6.1% | 8 | 9.8% |
| Poisson | 0 | 0.0% | 1 | 1.2% | 2 | 2.4% |

### 6.3.4 Probability Models

The results from fitting probability distributions to RAF data provides empirical evidence to support the models in the literature to some degree. The popular stuttering Poisson distribution, where demand arrivals follow a Poisson process and demand sizes are given by a geometric distribution, is applicable in many cases. There is also some support for a logarithmic-Poisson compound distribution as utilised by Nahmias and Demmy [57]. However, what the results do not support are normally distributed lead-times. A more suitable candidate in this case is the log normal distribution, which applies when the log values of the observations are normally distributed.

## 6.4 Concluding Remarks

A demand classification methodology has been applied to the entire RAF inventory. Line items have been classed as smooth, irregular, slow-moving, mildly erratic or highly erratic. The particular numerical values for the boundaries between the demand patterns are likely to be specific to the RAF inventory although the method itself is applicable elsewhere. What is classed as a smooth demand pattern in this instance may well be considered erratic in another industry.

Despite investigating whether there are common traits among line items within one demand pattern, which differentiate them from line items within another demand pattern, no single factors emerge to assist in categorising a new item. Therefore, evaluating the three constituent parts of the lead-time demand remains a necessary part of the categorisation procedure. New items can be assigned lead-time parameter values using the methodology provided, although demand parameters will have to be based on similar items or obtained by other means.

Although the normal distribution is frequently used for modelling the lead-time distribution, analysis in this chapter has shown it provides a poor representation of reality. The log normal distribution provides a better fit and may be better suited for use within probability models. On the other hand, a Poisson arrival process with demand sizes following a geometric distribution, which results in the popular stuttering Poisson distribution, does provide a reasonable representation of reality. Alternatively, the logarithmic is suitable for modelling the demand sizes and the log normal distribution is suitable for modelling the interval between demands.

The following two chapters compare the accuracy of a range of forecasting methods in toto and by demand pattern. The methods include Croston's method, which was put forward as particularly suitable for forecasting erratic demand, as well as three variations on this method which sought to further improve the performance. More traditional forecasting methods, including exponential smoothing and moving average, are also included in the analysis.

# 7. FORECASTING ERRATIC DEMAND

Traditional forecasting methods are often based on assumptions that are deemed inappropriate for items with erratic demand. Croston [19] identifies the inadequacy of exponential smoothing (ES) for estimating the underlying erratic demand pattern. With erratic items, the observed demand during many periods is zero interspersed by occasional periods with irregular non-zero demand. ES places most weight on the more recent data, giving estimates that are highest just after a demand and lowest just before a demand. Since the replenishment level will be broken by a demand occurrence, the replenishment quantity is likely to be determined by the biased estimates that immediately follow a demand as a consequence. This tends to lead to unnecessarily high stocks. Johnston [38] suggests the inadequacies of ES on erratic data become apparent when the mean interval between transactions is greater than two time periods.

Under a situation of infrequent transactions, it is often preferable to forecast two separate components of the demand process:

(i) The interval between consecutive transactions.

(ii) The magnitude of individual transactions.

The aim being to estimate the mean demand per period as $\mu/\rho$, where $\mu$ is the mean demand size and $\rho$ is the mean interval between transactions.

## 7.1 Croston's Forecasting Method

The method developed by Croston [19], and corrected by Rao [58], separately applies exponential smoothing to the interval between demands and the size of the demands. When the demand is stable, the aim of this method is to estimate the mean demand per period. Updating only occurs at moments of positive demand; if a period has no demand, the method simply increments the count of time periods since the last demand. Let:

$y_t$ = demand for an item at time $t$

$p_t$ = Croston's estimate of mean interval between transactions

$z_t$ = Croston's estimate of mean demand size

$\hat{y}_t$ = Croston's estimate of mean demand per period

$q$ = time interval since last demand

$\alpha$ = smoothing parameter between 0 and 1

If $y_t = 0$,

$$p_t = p_{t-1}$$

$$z_t = z_{t-1}$$

$$q = q + 1$$

Else,

$$p_t = p_{t-1} + \alpha(q - p_{t-1})$$

$$z_t = z_{t-1} + \alpha(y_t - z_{t-1})$$

$$q = 1$$

Combining the estimates of size and interval provides an estimate of the mean demand per period of $\hat{y}_t = z_t / p_t$. When demand occurs every period, Croston's method is identical to conventional ES. In his original paper, Croston used the same smoothing value for updating the mean interval between transactions and the mean demand size. Subsequent researchers have also tended to follow this convention, although there is no necessity to do so.

Example calculations for Croston's method are presented in Table 7.1 using a quarterly demand series from a sample line item.

**Table 7.1: Example Croston's Method Forecast Calculations.**

| Quarter | Actual Demand ($y_t$) | Interval Between Transactions ($q$) | Croston's Mean Estimates | | |
|---|---|---|---|---|---|
| | | | Demand Size ($z_t$) | Interval ($p_t$) | Demand Per Period ($\hat{y}_t$) |
| 1 | 37 | | | | |
| 2 | 5 | 1 | | | |
| 3 | 0 | | | | |
| 4 | 14 | 2 | 18.667 | 1.500 | 12.444 |
| 5 | 5 | 1 | 17.300 | 1.450 | 11.931 |
| 6 | 0 | | 17.300 | 1.450 | 11.931 |
| 7 | 10 | 2 | 16.570 | 1.505 | 11.010 |
| 8 | 10 | 1 | 15.913 | 1.455 | 10.941 |
| 9 | 0 | | 15.913 | 1.455 | 10.941 |
| 10 | 0 | | 15.913 | 1.455 | 10.941 |
| 11 | 6 | 3 | 14.922 | 1.609 | 9.274 |
| 12 | 20 | 1 | 15.430 | 1.548 | 9.966 |
| 13 | 32 | 1 | 17.087 | 1.493 | 11.442 |
| 14 | 5 | 1 | 15.878 | 1.444 | 10.996 |
| 15 | 25 | 1 | 16.790 | 1.400 | 11.996 |
| 16 | 38 | 1 | 18.911 | 1.360 | 13.909 |
| 17 | 15 | 1 | 18.520 | 1.324 | 13.991 |
| 18 | 6 | 1 | 17.268 | 1.291 | 13.373 |
| 19 | 70 | 1 | 22.541 | 1.262 | 17.859 |
| 20 | 0 | | 22.541 | 1.262 | 17.859 |
| 21 | 0 | | 22.541 | 1.262 | 17.859 |
| 22 | 0 | | 22.541 | 1.262 | 17.859 |
| 23 | 10 | 4 | 21.287 | 1.536 | 13.859 |
| 24 | 0 | | 21.287 | 1.536 | 13.859 |

A number of periods are required for initialising the forecast parameters and one year, or four quarters, is used for this purpose. The initial mean demand size is calculated from the positive actual demands in the first year, and an initial mean interval is also calculated from the individual intervals between transactions. Beyond period 4, the mean demand size and the mean interval are updated using a smoothing parameter of 0.1 in each case. It can be seen that updating only occurs in periods of positive demand, otherwise the previous values just roll forward. The mean demand per period is then calculated as the mean demand size divided by the mean interval.

## 7.2 Validation and Limitations of Croston's Method

Given that the infrequent updating of this method can introduce a significant lag in response to actual changes in the underlying parameters, Croston stresses the importance of control signals to identify forecast deviations. He suggests a number of indicators to use before updating:

(i) If $(1-1/p_t)^q < k_1$, for all values of $t$, with $k_1$ say 0.01, then the time interval since the last transaction is significantly greater than expected.

(ii) If $q/p_t < k_2$, for all non-zero demands, with $k_2$ say 0.2, then the transaction occurred earlier than expected.

(iii) With forecasting errors $e_t = y_t - \hat{y}_{t-1}$ and mean absolute deviation (MAD) $m_t = (1-\alpha)m_{t-1} + \alpha\,|\,e_t\,|$; if $|\,e_t\,| > k_3 m_t$, for all non-zero demands, with $k_3$ say 3.0 to 5.0, then the size of the demand is out of control.

(iv) With smoothed errors $s_t = (1-\alpha)s_{t-1} + \alpha e_t$ and tracking signal $\gamma_t = s_t/m_t$; if $|\,\gamma_t\,| > k_4$, for all non-zero demands, with $k_4$ say 0.5 to 0.7, then the model is inadequate or there is bias in the data.

In such situations, it is probable either that the demand pattern has suddenly changed or the forecasts are not responding fast enough to recent values of demand and corrective action may be to increase the value of $\alpha$.

The suggested control indicators, presented as Test (i) to Test (iv), are implemented for the sample line item in Table 7.2 alongside the quarterly demand series and Croston's estimate of mean demand per period. Indicators are calculated prior to the forecast updates.

**Table 7.2: Example Control Indicators for Croston's Method.**

| Qtr | Actual Demand ($y_t$) | Croston's Forecast ($\hat{y}_t$) | Control Indicators | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | Test (i) | Test (ii) | Error ($e_t$) | MAD ($m_t$) | Test (iii) | SMER ($s_t$) | Test (iv) |
| 1 | 37 | | | | | | | | |
| 2 | 5 | | | | | | | | |
| 3 | 0 | | | | | | | | |
| 4 | 14 | 12.444 | | | | 14.000 | | 7.000 | |
| 5 | 5 | 11.931 | 0.310 | 0.690 | -7.444 | 13.344 | 0.558 | 5.556 | 0.416 |
| 6 | 0 | 11.931 | 0.096 | 1.379 | -11.931 | 13.203 | 0.904 | 3.807 | 0.288 |
| 7 | 10 | 11.010 | 0.336 | 0.664 | -1.931 | 12.076 | 0.160 | 3.233 | 0.268 |
| 8 | 10 | 10.941 | 0.312 | 0.688 | -1.010 | 10.969 | 0.092 | 2.809 | 0.256 |
| 9 | 0 | 10.941 | 0.098 | 1.375 | -10.941 | 10.966 | 0.998 | 1.434 | 0.131 |
| 10 | 0 | 10.941 | 0.031 | 2.063 | -10.941 | 10.964 | 0.998 | 0.196 | 0.018 |
| 11 | 6 | 9.274 | 0.379 | 0.621 | -4.941 | 10.362 | 0.477 | -0.317 | -0.031 |
| 12 | 20 | 9.966 | 0.354 | 0.646 | 10.726 | 10.398 | 1.032 | 0.787 | 0.076 |
| 13 | 32 | 11.442 | 0.330 | 0.670 | 22.034 | 11.562 | 1.906 | 2.912 | 0.252 |
| 14 | 5 | 10.996 | 0.307 | 0.693 | -6.442 | 11.050 | 0.583 | 1.976 | 0.179 |
| 15 | 25 | 11.996 | 0.286 | 0.714 | 14.004 | 11.345 | 1.234 | 3.179 | 0.280 |
| 16 | 38 | 13.909 | 0.265 | 0.735 | 26.004 | 12.811 | 2.030 | 5.462 | 0.426 |
| 17 | 15 | 13.991 | 0.245 | 0.755 | 1.091 | 11.639 | 0.094 | 5.025 | 0.432 |
| 18 | 6 | 13.373 | 0.226 | 0.774 | -7.991 | 11.274 | 0.709 | 3.723 | 0.330 |
| 19 | 70 | 17.859 | 0.208 | 0.792 | 56.627 | 15.809 | 3.582 * | 9.013 | 0.570 * |
| 20 | 0 | 17.859 | 0.043 | 1.585 | -17.859 | 16.014 | 1.115 | 6.326 | 0.395 |
| 21 | 0 | 17.859 | 0.009 * | 2.377 | -17.859 | 16.199 | 1.102 | 3.908 | 0.241 |
| 22 | 0 | 17.859 | 0.002 * | 3.169 | -17.859 | 16.365 | 1.091 | 1.731 | 0.106 |
| 23 | 10 | 13.859 | 0.349 | 0.651 | -7.859 | 15.514 | 0.507 | 0.772 | 0.050 |
| 24 | 0 | 13.859 | 0.122 | 1.302 | -13.859 | 15.349 | 0.903 | -0.691 | -0.045 |

Test (i) indicates whether the interval since the last demand is greater than expected, and it is not until period 21 that the measure has fallen below 0.01, suggesting a demand was expected and one did not occur. In fact, a demand did not occur until period 23.

Alternatively, Test (ii) indicates whether a transaction occurs earlier than expected and, on this occasion, none of the values are less than 0.2, thus no early demands are indicated.

Test (iii), which indicates whether the size of the demand is out of control, requires the calculation of the error value, as the actual minus the forecast from the previous period, as well as the mean absolute deviation (MAD). The initial MAD in this case has been calculated simply as the average demand from the first year (quarters 1 to 4). With an error value more than three times greater than MAD, Test (iii) indicates the demand for 70 units in period 19 is larger than expected. Such an observation offers an explanation for the results from Test (i), where a long transaction interval was indicated following this same demand. It would appear that extra units were ordered in period 19 such that no further units were required in the subsequent periods.

Test (iv) indicates whether the model is inadequate or there is bias in the data, and requires a smoothed errors (SMER) calculation along with MAD. The two combine to produce tracking signal $\gamma_t$. SMER has been initialised as half the value of MAD. With a value for $|\gamma_t|$ in excess of 0.5 in period 19, there is an overall suggestion that the model is inadequate. However, in the analysis, both Test (iii) and Test (iv) have been interpreted using the tightest specifications prescribed by Croston. Under his more relaxed specifications, neither test indicates any lack of control in the forecasting procedure.

An examination of the results from the tests in unison with the actual demand does not indicate any serious inadequacy in the forecasting model. An interval greater than what was expected was identified in period 21, arising after a second consecutive zero demand. However, two consecutive periods of zero demand had been observed previously (periods 9 and 10). The fact that there are three consecutive zero demands in periods 20 to 22 is not cause for concern, particularly as a large demand occurs in period 19. The request for 70 units could have been used to satisfy demand over a number of future periods and a demand of this size is not exceptional under these circumstances. The identification of forecast deviations need not concern the forecaster in this instance and no adjustments are required. The results from this sample line item illustrate the usefulness of the methodology for drawing the forecaster's attention to demand pattern changes and particular attention can be given to this item if the tests are triggered again.

## 7.3 Forecasting Model

The development of a forecasting model for comparing the performance of the various methods is introduced in this section. Firstly, the individual performance of the methods needs to be gauged and several measures of accuracy are investigated. A second consideration is the forecast implementation in terms of when the comparisons are made; whether it is at every point in time or only after a demand has occurred. Finally, smoothing parameters are required for the smoothing methods and a method of selecting optimal parameter values is investigated.

This research focuses exclusively on smoothing methods, representing relatively simple means of forecasting. This is primarily due to the fact that most time series contain large numbers of zero demands. Under this situation it is not possible to identify any trend or seasonality in the data and the standard tools for time series analysis, such as first differences and autocorrelation analysis, do not identify any actionable patterns.

### 7.3.1 Measurement of Accuracy

In order to compare the performance of each of the forecasting methods various measures of accuracy are utilised. One measure commonly used in inventory control is the mean absolute deviation (MAD), calculated simply as the average of the absolute forecast errors:

$$MAD = \frac{\sum_{t=1}^{n} |e_t|}{n}$$

A desirable feature of MAD is that it is less affected by outliers than other measures, which Wright *et al*. [92] noted as being of particular importance in practical forecasting situations where outliers are a frequent occurrence.

Kling and Bessler [44] suggest that if large errors do in fact have a greater than proportional cost compared to small errors then a measure that places a heavier penalty on large errors is more appropriate. The mean square error (MSE) and the root mean square error (RMSE) place more weight on large errors:

$$MSE = \frac{\sum_{t=1}^{n} e_t^2}{n}$$

$$RMSE = \sqrt{MSE}$$

Mean square error measures have often been criticised as unreliable and sensitive to outliers. In addressing the reliability of these measures, Armstrong and Collopy [4] examined the extent to which the RMSE produces the same accuracy rankings when applied to different samples taken from a set of data series, including quarterly and annual observations with differing periods looking ahead. They found that rankings based on the RMSE were highly unreliable except where the comparisons involved many series.

None of these measures allow comparison across time series as they are all absolute measures related to the specific series. To objectively compare forecasts from different series with widely differing sizes Makridakis and Hibon [52] suggest a unit free metric such as the mean absolute percentage error (MAPE) which relates the size of the error to the actual observation on a proportional basis:

$$MAPE = \frac{\sum_{t=1}^{n} \frac{|e_t|}{y_t} \times 100}{n}$$

MAPE also finds favour with Lawrence *et al*. [46] for several reasons, "*First, being less affected than squared measures by extreme errors, it becomes a good relative measure for comparisons among techniques. Secondly, the metric is independent of scale, enabling a comparison to be made between different time series. Additionally, it is a common measure used to assess relative accuracy*". This measure also has its disadvantages. Armstrong and Collopy [4] indicate that MAPE is only relevant for ratio-scale data whereby the data has an absolute zero, as is the case for most economic data, and, as the method puts a heavier penalty on forecasts that exceed the actual value than those that are less, it is biased in favour of low forecasts. A further disadvantage of MAPE is identified by Gardner [31] for time series similar to those encountered in this study; it is often left undefined due to zero observations in the series and is therefore sensitive to errors in such cases.

All the measures mentioned thus far, except perhaps MAD, offer poor protection against outliers and a single observation may dominate the analysis because it has a much larger or smaller error than the other observations in the series. Armstrong and Collopy suggest the effect of outliers can be reduced by trimming so as to discard high and low errors and an extreme way to trim is to use medians to remove all values higher and lower than the middle value. They recommend the median absolute percentage error (MdAPE) as a means for comparing methods when many series are available:

$$MdAPE = \text{ } Observation \text{ } \frac{n+1}{2} \text{ } if \text{ } n \text{ } is \text{ } odd, \text{ } or \text{ } the \text{ } mean \text{ } of \text{ } observations \text{ } \frac{n}{2} \text{ } and \text{ } \frac{n}{2}+1 \text{ } if \text{ } n \text{ } is \text{ } even, \text{ } where \text{ } observations \text{ } are \text{ } ordered \text{ } by \text{ } APE.$$

The MdAPE reduces the bias in favour of low forecasts and therefore offers an additional advantage over MAPE.

In the early part of this study the four measures of MAD, RMSE, MAPE and MdAPE are utilised for all forecast comparisons and, as there is some justification for each measure, the consistency of results between the measures is examined.

All of the suggested measures have their advantages and disadvantages in the context of this research. MAPE is a widely utilised metric and being independent of scale it enables comparisons between different time series. The measure is less affected by extreme errors than squared measures although a single observation with a much larger or smaller error may still dominate the results. The most common MAPE formulation cannot be defined in periods when the actual observation is zero, which tends to happen frequently in a spare parts environment. However, when comparing over multiple periods, such as over a lead-time duration, this problem diminishes.

A disadvantage of MAPE is that it penalises an equivalent absolute error associated with a low demand more than a correspondingly high demand. This will have implications in an inventory control environment in particular. As MAPE is biased in favour of reducing the percentage errors for the low demands, a 10 percent error say, with a high demand leads to higher excess stock or more units out of stock than a 10 percent error with a low demand. With a bias towards the improved forecasting of the low demands rather than the high demands this could result in higher absolute levels of inventory. MAPE, however, complements a percentage of demand satisfied criterion with respect to customer service. As selecting a certain percentage of demand to be satisfied is equivalent to fixing a cost per unit out of stock, we may conclude that MAPE is probably satisfactory for the type of customer service measures most commonly encountered.

Stock-holding is normally measured in terms of the costs incurred, related directly to the absolute level of stock carried. Chapter 9 introduces an alternative means for comparing forecasts which will seek to alleviate the problems associated with the traditional measures in an inventory context. This measure takes into account the levels of safety stock necessary to give an equivalent customer service.

There is no singularly correct measure of accuracy that can be used for comparing forecasting methods. MAPE is frequently reported in the literature and from a purely forecasting perspective it is considered to strike a favourable balance between the advantages and the disadvantages. Thus MAPE has been selected for establishing optimal smoothing parameters and making comparisons in the latter part of the study. However, whilst this chapter utilises MAPE for the most part it also simultaneously presents results for MAD, RMSE and MdAPE for comparative purposes.

### 7.3.2 Forecast Implementation

As well as numerous measures for making comparisons between forecast accuracy, there are also two methods of implementation for consideration:

(i) Measuring the errors observed at *every* point in time, or

(ii) Only measuring the errors immediately *after a demand* has occurred.

The first implementation is suitable for stock replenishment systems employing a periodic review approach or product group review with a forecasting method that updates every period as a reorder could occur at any stage in the inventory cycle, either before or after a demand, based on the forecasted value at that time. On the other hand, the second implementation may be more applicable to a continuous review approach where a reorder will only be placed after a demand occurs. This implementation only considers the forecast accuracy in periods where an order *might be placed* and excludes those periods where an order *will not be placed*. Such an implementation would also be suitable under a periodic review approach if the forecasting method is only updated after a demand occurrence, as occurs with Croston's method.

The RAF utilises a periodic review system where the period between reviews is equal to one month, in which case it would seem appropriate to measure the forecast errors at every point in time. However, it may be argued that so few months record any demand (only 8 percent of line items experience a demand rate in excess of one per month) that the RAF operates something akin to a continuous review system. As the two implementations for measuring the forecast errors have their merits in reality, they will both be considered in this research.

A model has been written with SAS software, called FORESTOC, which compares various forecasting methods using RAF demand data covering the six year period from January 1994 to December 1999. The model caters for a number of options for comparative purposes:

(i) Aggregation of individual demand transactions can be performed over three time spans - quarterly, monthly and weekly aggregation.

(ii) Forecast errors are generated for every point in time, as well as only after a demand has occurred.

(iii) As well as comparing the forecast value with the traditional one-period ahead actual value, the FORESTOC methodology recognises the purpose for which forecasts are made and compares the forecast value with the actual demand over a forward-looking lead-time period.

Using the same sample line item from Section 7.1 for demonstrating Croston's method, Table 7.3 illustrates the manner in which the errors are measured. The measuring is done using the mean absolute percentage error (MAPE) and the median absolute percentage error (MdAPE) in this example. By necessity, the absolute percentage error for the one-period ahead comparison can only be calculated after a demand has occurred, as the actual demand value must be greater than zero.

The selected line item has a lead-time period of 5 quarters, comprising set ALT and PLT plus one period for review. Comparisons between the actual value and the forecast value are not made until quarter 5 as the first year is used for initialisation. The last lead-time period is also not available for any comparisons as it is required in calculating the forward-looking lead-time demand. Actual demand for each quarter is shown in column (2), while the one-period ahead demand shown in column (3), is simply the actual demand from the next period. Cumulative demand over a forward lead-time period is shown in column (4), thus the first value for five quarters ahead is equal to (0 + 10 + 10 + 0 + 0). The average lead-time demand of column (5) is the average demand for the five quarters. Column (6) provides Croston's estimate of mean demand per period, taken from Table 7.1.

**Table 7.3: Example Measures for Croston's Method (5 Quarter Lead-Time).**

| Quarter | Demand | | | | Demand Forecast | Absolute Percentage Error | | |
|---|---|---|---|---|---|---|---|---|
| | Actual Demand | One-Period Ahead | Forward Lead-Time | | | One-Period Ahead | Forward Lead-Time | |
| | | | Cumu-lative | Average Demand | | | All Periods | Demand Only |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| 1 | 37 | 5 | | | | | | |
| 2 | 5 | 0 | | | | | | |
| 3 | 0 | 14 | | | | | | |
| 4 | 14 | 5 | | | 12.444 | | | |
| 5 | 5 | 0 | 20 | 4.0 | 11.931 | - | 198.280 | 198.280 |
| 6 | 0 | 10 | 26 | 5.2 | 11.931 | 19.310 | 129.440 | - |
| 7 | 10 | 10 | 36 | 7.2 | 11.010 | 10.100 | 52.916 | 52.916 |
| 8 | 10 | 0 | 58 | 11.6 | 10.941 | - | 5.685 | 5.685 |
| 9 | 0 | 0 | 63 | 12.6 | 10.941 | - | 13.170 | - |
| 10 | 0 | 6 | 88 | 17.6 | 10.941 | 82.342 | 37.838 | - |
| 11 | 6 | 20 | 120 | 24.0 | 9.274 | 53.632 | 61.360 | 61.360 |
| 12 | 20 | 32 | 115 | 23.0 | 9.966 | 68.855 | 56.668 | 56.668 |
| 13 | 32 | 5 | 89 | 17.8 | 11.442 | 128.840 | 35.720 | 35.720 |
| 14 | 5 | 25 | 154 | 30.8 | 10.996 | 56.017 | 64.299 | 64.299 |
| 15 | 25 | 38 | 129 | 25.8 | 11.996 | 68.431 | 53.502 | 53.502 |
| 16 | 38 | 15 | 91 | 18.2 | 13.909 | 7.274 | 23.577 | 23.577 |
| 17 | 15 | 6 | 76 | 15.2 | 13.991 | 133.190 | 7.952 | 7.952 |
| 18 | 6 | 70 | 80 | 16.0 | 13.373 | 80.896 | 16.422 | 16.422 |
| 19 | 70 | 0 | 10 | 2.0 | 17.859 | - | 792.950 | 792.950 |
| 20 | 0 | 0 | | | 17.859 | | | |
| 21 | 0 | 0 | | | 17.859 | | | |
| 22 | 0 | 10 | | | 17.859 | | | |
| 23 | 10 | 0 | | | 13.859 | | | |
| 24 | 0 | | | | 13.859 | | | |
| **Mean Absolute Percentage Error (MAPE)** | | | | | | **64.444** | **103.319** | **114.111** |
| **Median Absolute Percentage Error (MdAPE)** | | | | | | **68.431** | **52.916** | **53.209** |

Comparisons are made over quarters 5 to 19 between the forecast value and the one-period ahead demand and the average lead-time demand, assessed in all periods and upon demand occurrence only. Column (7) shows the absolute percentage error (APE) for the one-period ahead forecast, calculated from column (3) as the actual and column (6) as the forecast. As previously mentioned, APE can only be calculated when the actual observation is greater than zero and, therefore, some periods have a missing value. Columns (8) and (9) show the APE for the forward lead-time forecasts; firstly for all periods and secondly on occasions of positive demand only as determined by column (2). In both cases, the forecast of column (6) is compared with the average lead-time demand of column (5). Finally, the respective mean absolute percentage errors and median absolute percentage errors are calculated from the non-missing values for the last three columns.

The MAPE values in Table 7.3 are observed to differ substantially depending on whether comparisons are made with the one-period ahead demand or the lead-time demand, either in all periods or in periods of demand only. It is observed that the relatively poor performance for MAPE when comparing the forecast with lead-time demand is mostly due to the large percentage error occurring in quarter 19. As the demand in quarter 20 was zero the one-period ahead comparison receives a null value on this occasion. This raises questions as to which implementation should be used for conveying the results and, indeed, which periods should be included in the calculations. If the calculations only went as far as quarter 18 the results would be quite different.

In addition, the rankings of the forecast implementations differ between MAPE and MdAPE, which now raises the question of which performance measure should be used as well as which forecast implementation should be used. Different researchers may well report different results given a situation like this depending on which methodology they choose.

As a starting point, the implementation to be considered should depend on the purpose of the comparisons. If it is purely a forecasting accuracy issue then the traditional one-period ahead comparison may be satisfactory. Alternatively, if the forecasting is part of an inventory management system, and would be used for making replenishment decisions, it would be sensible to consider one of the lead-time comparisons. In addition, as replenishments will only be made after a demand occurs, it would be sensible to only measure the forecast performance after a demand occurrence. As for which measure to report it is not possible to say, as both measures are valid and they both have their merits, so perhaps they should both be considered.

Such observations illustrate weaknesses of the traditional measures of accuracy, such as MAPE and MdAPE, when considering an inventory management system. A problem arises as there is no singularly correct method for applying the measures and, as the results differ depending on the method of application, there is no confirmation of the results through any consensus.

This chapter continues to examine the forecast performance according to the identified measures of accuracy and the results are examined for general trends. A later chapter suggests an alternative approach to measuring performance. First, however, it is necessary to determine smoothing parameter values that provide the best results for the smoothing methods.

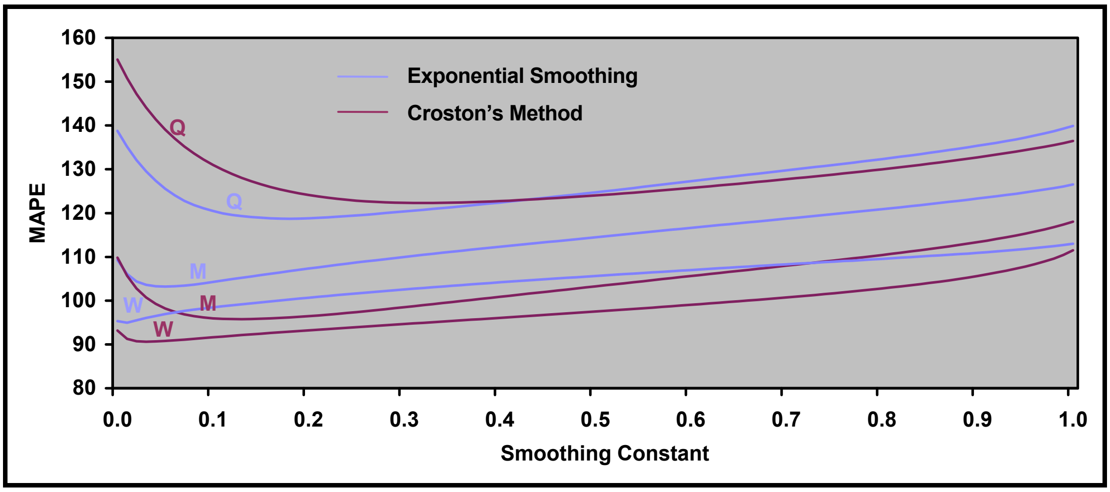
### 7.3.3 Selecting Smoothing Parameters

Through an analysis of the widely available M-Competition time series data, Fildes *et al*. [27] suggest the forecasting performance of smoothing methods is dependent on how the smoothing parameters are estimated. Rather than using fixed arbitrary values suggested in the forecasting literature, improvements in performance can be obtained by using information drawn from the time series themselves. Fixed optimal smoothing parameters from a cross-sectional study of the time series were found to perform well across other series, and this is the selection methodology used in this analysis. Using a hold-out sample of 500 line items from the RAF consumable inventory with equal representation between the five identified demand patterns from Chapter 6, average MAPEs across all line items for a range of smoothing constant values are computed. The smoothing constant in which the average MAPE is at a minimum is taken as the optimal value.

The effect of the smoothing constant on the forecasting performance of exponential smoothing (ES) and Croston's method is examined in the following figures. Results are shown for weekly, monthly and quarterly aggregations for both forecasting methods. In the case of Croston's method, the smoothing parameters for the demand size and the interval between transactions are set at the same value. The average forecasting performance has been monitored for 500 sample line items using increments of 0.01 in the smoothing constant.

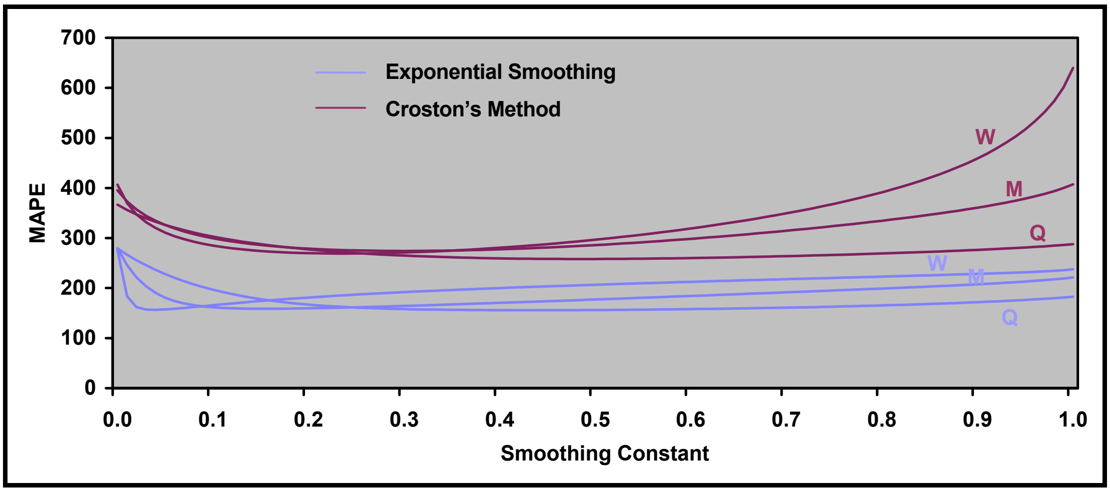
Figure 7.1 presents the results from comparing the forecast value with the one-period ahead demand, where three general patterns are observed. Firstly, it is observed that for both methods the forecasting performance deteriorates as the demand moves from weekly (W) to monthly (M) and on to quarterly (Q) aggregation. Secondly, after initial rapid improvements in forecasting performance as the smoothing constant increases, once a minimum MAPE is obtained the deterioration in performance is less severe. Finally, Croston's method provides better results than ES at most parameter values, although ES is initially significantly better when considering quarterly data and does in fact provide the lower MAPE in this case.

**Figure 7.1: Effect of Smoothing Constant - One-Period Ahead (All Periods).**



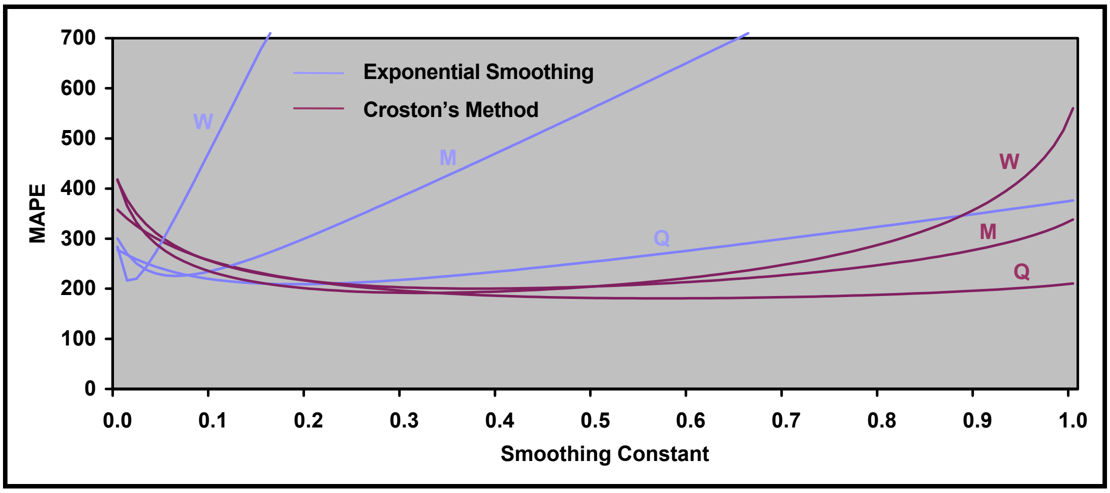

The same observations do not hold true when comparing the forecast value with the lead-time demand for all periods as shown by Figure 7.2. A major difference in this instance is that ES now provides consistently better results than Croston's method. In addition, the quarterly data provides lower MAPE values in most instances. The weekly data provides a better performance than monthly data for very low smoothing constant values, although the weekly performance quickly deteriorates.

**Figure 7.2: Effect of Smoothing Constant - Lead-Time Demand (All Periods).**



A different set of observations occur when comparing the forecast value with the lead-time demand in periods with positive demand only as shown by Figure 7.3.

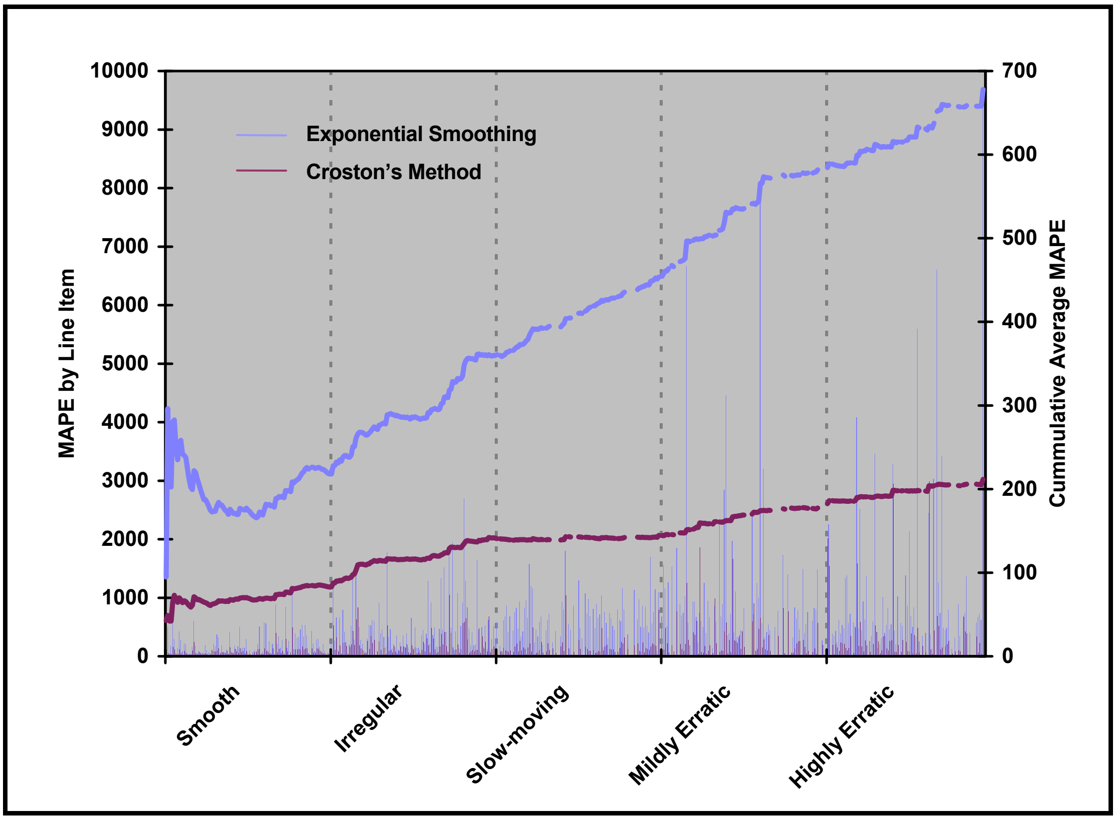
**Figure 7.3: Effect of Smoothing Constant - Lead-Time Demand (Demand Only).**




In this instance, quarterly data tends to provide better results than weekly or monthly data, and Croston's method tends to be better than ES. Furthermore, ES is very sensitive to changes in the value of the smoothing constant and deterioration in performance occurs rapidly with weekly and monthly data in particular.

The reason behind the severe deterioration in the forecasting performance for ES is initially explored in Figure 7.4. The calculated MAPEs for each of the 500 line items in the sample are presented (in no particular order within a demand pattern) for both ES and Croston's method, where the first 100 line items were classified as having a smooth demand pattern, the second an irregular demand pattern, etc.

**Figure 7.4: Comparative MAPE for Lead-Time Demand (Demand Only).**



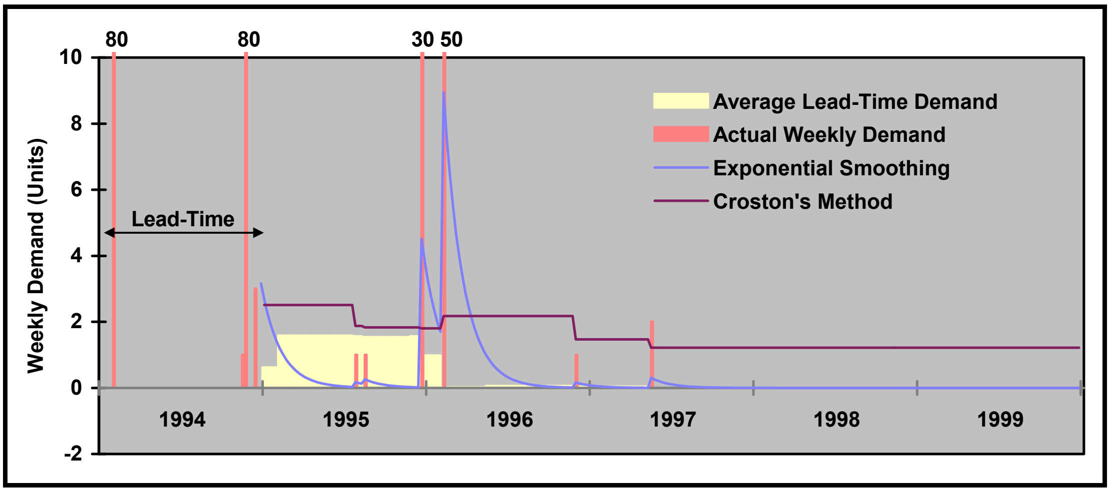

A smoothing constant of 0.15 was used with weekly data for this analysis (a value where the forecast was particularly poor) and comparisons were made with lead-time demand in periods with positive demand. It is observed that the individual MAPEs for the smooth demand pattern are considerably less than those for the erratic demand patterns in particular.

Also shown in Figure 7.4 are the cumulative average MAPEs obtained by successively including individual MAPE results. The cumulative average MAPEs initially fluctuate, although they settle with a continuous increase as more line items are considered. A continual increase occurs as the individual MAPEs tend to increase as the demand pattern changes from smooth to highly erratic. Gaps in both lines occur when MAPEs for individual line items could not be calculated. The average cumulative MAPEs conclude at values of 678 and 211 for ES and Croston's method respectively, which correspond with the values shown for weekly data in Figure 7.3 at the 0.15 point.

It was observed that the comparatively poor MAPE performance of ES occurs for the most part when forecasting erratic demand. In examining this issue further, the forecasting of one individual line item is now examined in greater detail. The selected line item is that which provided the highest individual MAPE in Figure 7.4, namely observation 500 with a MAPE of 9,454 for ES. In comparison, the corresponding MAPE is 3,009 for Croston's method.

Figure 7.5 illustrates demand circumstances that lead to high values for MAPE. Over the six year period, weekly demand occurs as 80, 80, 30 and 50 units, interspersed by a few demands of one, two or three units and a large number of zeros. Forecasts are made using ES and Croston's method (with a smoothing constant of 0.15 in all cases) and compared with the average forward-looking lead-time demand. In this instance, the lead-time is exactly one year, as shown in the figure.

**Figure 7.5: Example Forecasts for Erratic Demand.**



As specified by the FORESTOC forecasting model, the first lead-time period is used for initialising the parameters. Successive demands of 80, 1, 80 and 3 units occur in the first year, giving an average of 3.15 units per week to initialise the ES forecast at the start of 1995. Likewise, the average demand size and average demand interval combine to initialise Croston's method at 2.51 units per week. With successive demands of 1, 1 and 30 units during 1995, the initial forward-looking lead-time demand is 0.62 units per week.

Beyond initialisation, the ES forecast continues to decrease until a demand of a single unit occurs, while Croston's forecast maintains the same level until the same demand occurrence. The average lead-time demand first increases when the demand for 30 units enters the moving one-year horizon and then marginally decreases when the first single unit demand exits the horizon. As a demand has occurred, a MAPE is calculated comparing each of the forecast values against the reduced average lead-time demand. In this manner, comparisons are made with the average lead-time demand commencing one-period ahead.

The calculated MAPEs for the selected line item are presented in Table 7.4. Absolute percentage errors compare the average lead-time demand and the forecasts only when a demand occurs. A substantial APE results for ES, in particular, in week 110 when a demand for 50 units occurs. This demand significantly increases the ES forecast value, as previously illustrated in Figure 7.5, while the average lead-time demand falls significantly as the 50 unit demand is no longer included in the calculation and only minor demands subsequently occur. On the other hand, Croston's forecast does not increase to such an extent as the ES forecast, and therefore the associated APE is not as large.

**Table 7.4: Comparative MAPE for Lead-Time Demand (Demand Only).**

| Week of Demand | Actual Demand | Average Lead-Time Demand | Exponential Smoothing | | Croston's Method | |
|---|---|---|---|---|---|---|
| | | | Forecast | APE | Forecast | APE |
| 82 | 1 | 1.558 | 0.174 | 88.83 | 1.873 | 20.26 |
| 85 | 1 | 1.538 | 0.257 | 83.30 | 1.831 | 19.01 |
| 103 | 30 | 0.981 | 4.514 | 360.23 | 1.804 | 83.95 |
| 110 | 50 | 0.019 | 8.947 | 46,424.00 | 2.174 | 11,206.00 |
| 152 | 1 | 0.038 | 0.160 | 315.25 | 1.468 | 3,715.90 |
| 176 | 2 | 0.000 | 0.303 | - | 1.217 | - |
| **Mean Absolute Percentage Error** | | | | **9,454.32** | | **3,009.02** |

The observations of this section have confirmed in reality the inadequacy of ES when forecasts are made from an erratic demand series as first identified by Croston [19]. As ES places most weight on the more recent data, the forecast estimates are highest just after a demand, leading to substantial errors in the periods in which replenishment orders are placed. Thus, there is a tendency for the holding of unnecessarily high stocks when this method is used for inventory management.

The next section concludes the search for optimal smoothing parameters. It is expected that the selected values would largely be determined by the results from forecasting the erratic demand patterns as they tend to produce the greatest errors.

### 7.3.4 Optimal Smoothing Parameters

The optimal values of the smoothing constants for the sample data are presented in Table 7.5. At this stage of the analysis, the values for MAPE are considered provisional and will be recalculated in a later section using a significantly larger sample size. In terms of the smoothing constants, it is observed that the optimal values decrease as the demand moves from quarterly to monthly and down to weekly aggregation. In addition, the optimal values for Croston's method are seen to be higher than those for ES.

**Table 7.5: Optimal Smoothing Constant Values for MAPE.**

| Type of Forecast | Demand Aggregation | Exponential Smoothing | | Croston's Method | |
|---|---|---|---|---|---|
| | | Smoothing Constant | Provisional MAPE | Smoothing Constant | Provisional MAPE |
| One-Period Ahead Demand - All Periods | Quarterly | 0.18 | 118.71 | 0.32 | 122.33 |
| | Monthly | 0.05 | 103.20 | 0.13 | 95.77 |
| | Weekly | 0.01 | 94.96 | 0.03 | 90.62 |
| Lead-Time Demand - All Periods | Quarterly | 0.43 | 155.71 | 0.48 | 258.33 |
| | Monthly | 0.16 | 158.65 | 0.30 | 274.34 |
| | Weekly | 0.04 | 156.40 | 0.24 | 269.17 |
| Lead-Time Demand - Demand Only | Quarterly | 0.19 | 208.94 | 0.57 | 180.73 |
| | Monthly | 0.06 | 225.48 | 0.38 | 200.17 |
| | Weekly | 0.01 | 216.34 | 0.32 | 191.59 |

Some of the smoothing parameters are observed to be very low while others are very high (ranging between 0.01 and 0.57). Smoothing methods form a new estimate of demand by taking a weighted average of the current demand and the previous smoothed estimate. With the smoothed estimate being a linear combination of all historic demands, the selected smoothing parameters can have a considerable impact on the forecast values. When the smoothing constant $\alpha$ is large, the most recent observations are given more weight and therefore have a stronger influence on the smoothed estimate. Conversely, when $\alpha$ is small the new observations have little influence. Thus, a large $\alpha$ provides little smoothing whereas a small $\alpha$ gives considerable smoothing. When the smoothing constant for ES is equal to one, this is equivalent to using the last observation as the forecast value, referred to as naïve forecasting.

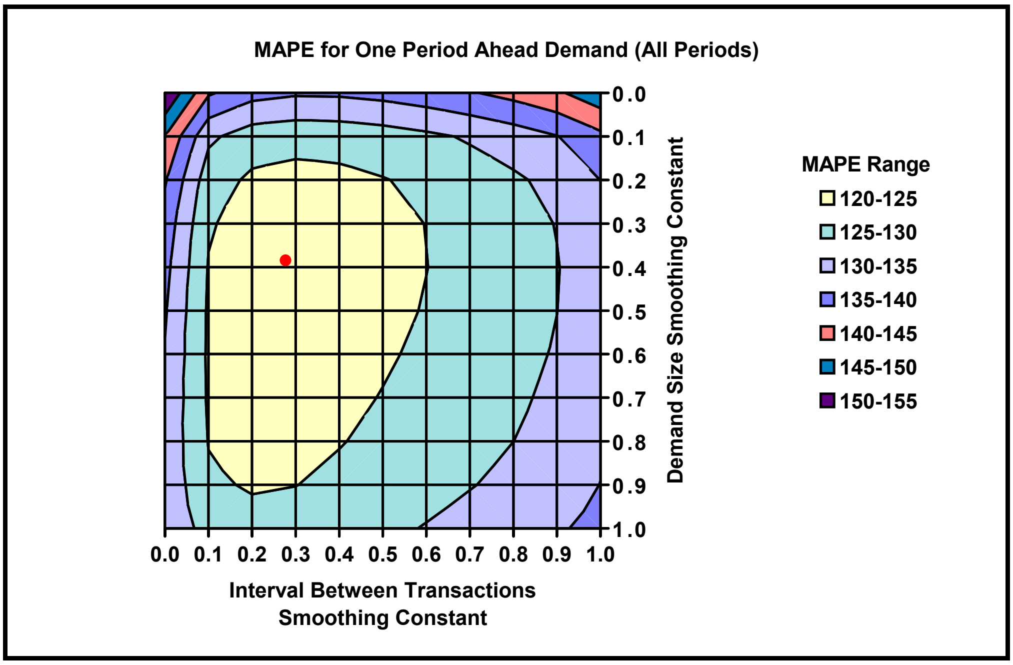
Also, the initialisation methodology plays a more prominent role for a smaller $\alpha$ than it does for a larger $\alpha$. If the smoothing parameter is close to one, the influence of the initialisation process rapidly becomes less significant. However, if $\alpha$ is close to zero the initialisation process can play a significant role for many time periods ahead.

In this analysis, forecasts with weekly data have lower optimal smoothing constants and therefore they tend to give the more recent observations less influence than the quarterly forecasts do. As a result the weekly forecasts are smoothed to a greater degree by the smoothing process. This is likely to occur because the quarterly data is pre-smoothed by aggregation and requires less smoothing when forecasting. The weekly forecasts are also more significantly affected by the initialisation values. This may be due to the fact that the weekly data has a higher proportion of zero demands throughout and if the initialisation procedure has provided a starting value close to zero through the choice of $\alpha$ then this value would likely be maintained. It will be of interest to observe how the forecasts with low smoothing parameters compare with outright averaging methods.

The optimal smoothing constants for ES presented in Table 7.5 are suitable for implementation with the full sample of line items. However, with Croston's method, it is likely the forecasting performance can be improved by allowing the two constituent smoothing constants to vary independently. All the academic research observed to date utilise the same smoothing value for both components of Croston's method, although, as they represent independent processes, the values should be allowed to differ.

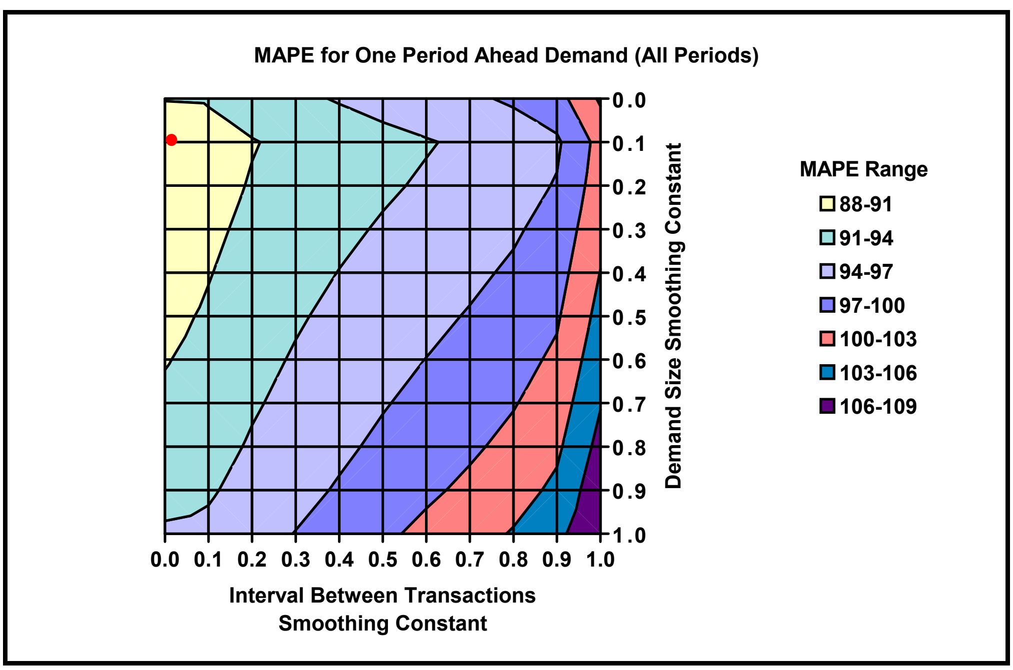
The two-dimensional surface map in Figure 7.6 illustrates the variability in the performance of Croston's method when the two smoothing constants are allowed to vary. MAPE is again used as the performance measure and in this example the results come from comparing the forecast value with the one-period ahead demand using quarterly demand data. The red dot indicates the minimum value for MAPE, where the demand size smoothing constant and interval between transactions smoothing constant are set to 0.39 and 0.28 respectively. From this minimum point, MAPE increases in a circular manner as illustrated by the surrounding contour lines.

**Figure 7.6: Effect of Smoothing Constants - Croston's Method (Quarterly Data).**




An altogether different pattern is shown by Figure 7.7 where the one-period ahead demand is again considered but this time using weekly demand data.

**Figure 7.7: Effect of Smoothing Constants - Croston's Method (Weekly Data).**



In this instance the optimal smoothing constant for the interval between transactions falls close to zero and the surrounding contour lines are seen to be less circular with a sharp *gully* along the 0.1 demand size smoothing constant value.

The optimal smoothing constant values for the three forecast implementations under the various demand aggregations are presented in Table 7.6. Some of the smoothing values are again very low while others are very high (ranging between 0.01 and 0.92). When the smoothing constants are low there is greater smoothing and the smoothed values will be close to the average value provided by the initialisation procedure. Alternatively, when the smoothing constants are high there is little smoothing and the smoothed values will be close to the demand from the previous period.

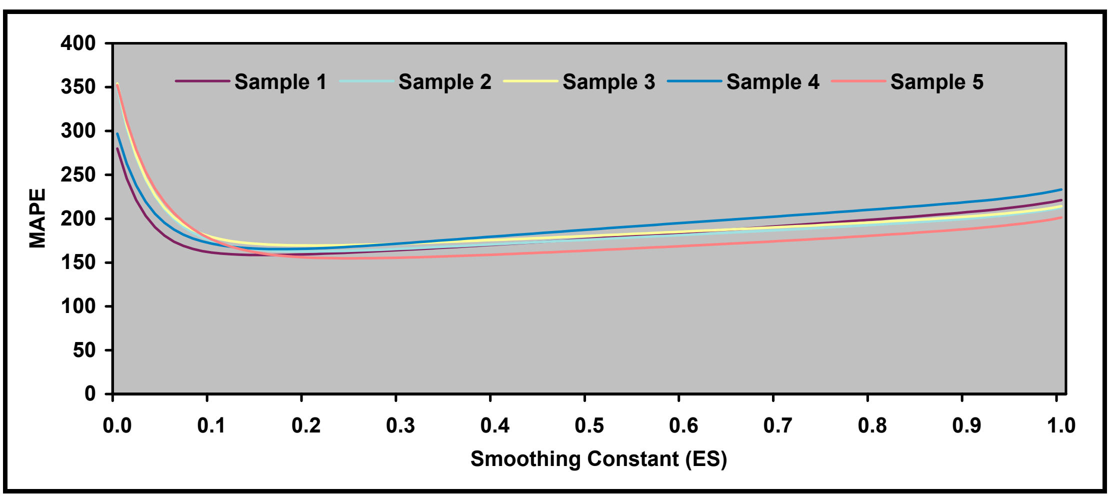
**Table 7.6: Optimal Smoothing Constant Values for MAPE - Croston's Method.**

| Type of Forecast | Demand Aggregation | Smoothing Constants | | Provisional MAPE |
|---|---|---|---|---|
| | | Demand Size | Demand Interval | |
| One-Period Ahead Demand - All Periods | Quarterly | 0.39 | 0.28 | 122.11 |
| | Monthly | 0.18 | 0.08 | 95.45 |
| | Weekly | 0.10 | 0.01 | 89.86 |
| Lead-Time Demand - All Periods | Quarterly | 0.92 | 0.40 | 252.56 |
| | Monthly | 0.10 | 0.34 | 272.31 |
| | Weekly | 0.02 | 0.25 | 263.53 |
| Lead-Time Demand - Demand Only | Quarterly | 0.92 | 0.46 | 174.56 |
| | Monthly | 0.50 | 0.36 | 199.67 |
| | Weekly | 0.58 | 0.30 | 190.16 |

Once again it is observed that the optimal values decrease as the demand moves from quarterly to monthly and down to weekly aggregation. In general the optimal demand size smoothing constants are greater than the optimal interval between transactions smoothing constants.

In order to gauge whether the calculated optimal smoothing constants have any prescriptive power, alternative optimal values for exponential smoothing have been calculated using additional samples of 500 line items. MAPE results from comparing the monthly forecast value with the lead-time demand in all periods are illustrated in Figure 7.8 for five different random samples of 500 line items each. Results from the original 500 line items are identified as Sample 1. It is observed that the five samples all share a similar optimal value (ranging between 0.16 and 0.22) and they all have similar MAPE values throughout. This suggests the predictive powers of the selected smoothing constants are very good.

**Figure 7.8: Effect of Smoothing Constant on Alternative Samples of 500 Line Items.**




Confirmation of the high-quality predictive powers of the original smoothing constants is provided by Table 7.7. In this instance the optimal smoothing constants for ES and the associated MAPE values are presented for the five samples by forecast measurement type. The original 500 line items are again identified as Sample 1. The optimal smoothing constants under each forecast measurement type are closely matched as are the corresponding MAPE values.

**Table 7.7: Forecast Results from Alternative Samples of 500 Line Items.**

| Exponential Smoothing Statistics | Monthly Demand Series | Type of Forecast Measurement | | |
|---|---|---|---|---|
| | | One-Period Ahead Demand - All Periods | Lead-Time Demand - All Periods | Lead-Time Demand - Demand Only |
| Optimal Smoothing Constants | Sample 1 | 0.05 | 0.16 | 0.06 |
| | Sample 2 | 0.08 | 0.20 | 0.08 |
| | Sample 3 | 0.09 | 0.21 | 0.06 |
| | Sample 4 | 0.04 | 0.17 | 0.05 |
| | Sample 5 | 0.09 | 0.22 | 0.08 |
| Calculated MAPEs | Sample 1 | 103.20 | 158.65 | 225.48 |
| | Sample 2 | 95.19 | 162.68 | 256.78 |
| | Sample 3 | 100.51 | 169.53 | 239.18 |
| | Sample 4 | 97.14 | 165.37 | 260.50 |
| | Sample 5 | 96.19 | 154.92 | 218.29 |

The original optimal smoothing values are deemed suitable for implementation with the full sample of line items from which definitive performance measures will be determined. In the next section, the smoothing constants for ES presented in Table 7.5 and the smoothing constants for Croston's method presented in Table 7.6, which were both optimally obtained from the original hold-out sample of 500 line items, will be applied across a substantially larger sample.

## 7.4 Forecast Results

Forecast comparisons have been made using the FORESTOC forecasting model described in Section 7.3.2 on the sample of 18,750 line items considered previously. The selected items comprise a random sample with equal representation from the five identified demand patterns. Each line item has a different lead-time, taken as the set purchasing lead-time (PLT) value. Initially four forecasting methods have been included in the study: exponential smoothing, Croston's method, a one year moving average, and a simple previous year average.

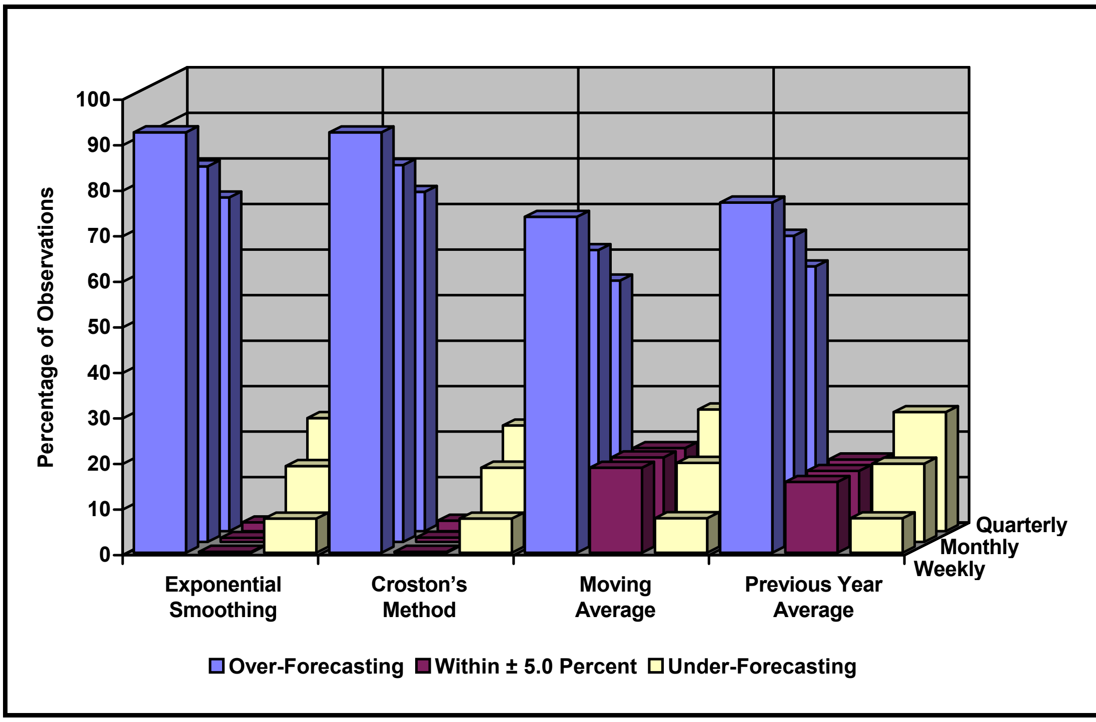
Comparative evaluations between the methods are illustrated in the following sections, starting with the frequency of over and under forecasting followed by measures of forecasting accuracy.

### 7.4.1 Frequency of Over and Under Forecasting

Over and under forecasting have differing consequences to the RAF. The affect of over-forecasting demand is the holding of too much stock with the associated purchasing, handling and holding costs, together with increased risks of wastage, pilferage and obsolescence. On the other hand, under-forecasting demand leads to the holding of too little stock and items are not available when required, incurring stock-out or shortage costs. As with most parts and supplies inventory systems, the shortage costs to the RAF take the form of *backorder* costs. The lack of available parts can cause aircraft to be grounded, thus reducing operational capability, while other costs are incurred through expediting and cannibalisation. Stock shortages are usually more costly than idle stock.

Figure 7.9 presents the percentage of occasions on which each of the forecasting methods over-estimates, under-estimates and approximately matches (forecast within ± 5.0 percent of actual) the one-period ahead demand. Measurements are taken across all periods regardless of whether a demand was recorded in a period or not. All of the forecasting methods tend to over-estimate rather than under-estimate the one-period ahead demand, with less frequent over-estimation as the demand period lengthens. ES and Croston's method over-estimate demand to a similar degree. The moving average and the previous year average methods over-estimate less frequently. They also approximately match the actual demand substantially more often, with the proportion being constant between quarterly, monthly and weekly aggregations.

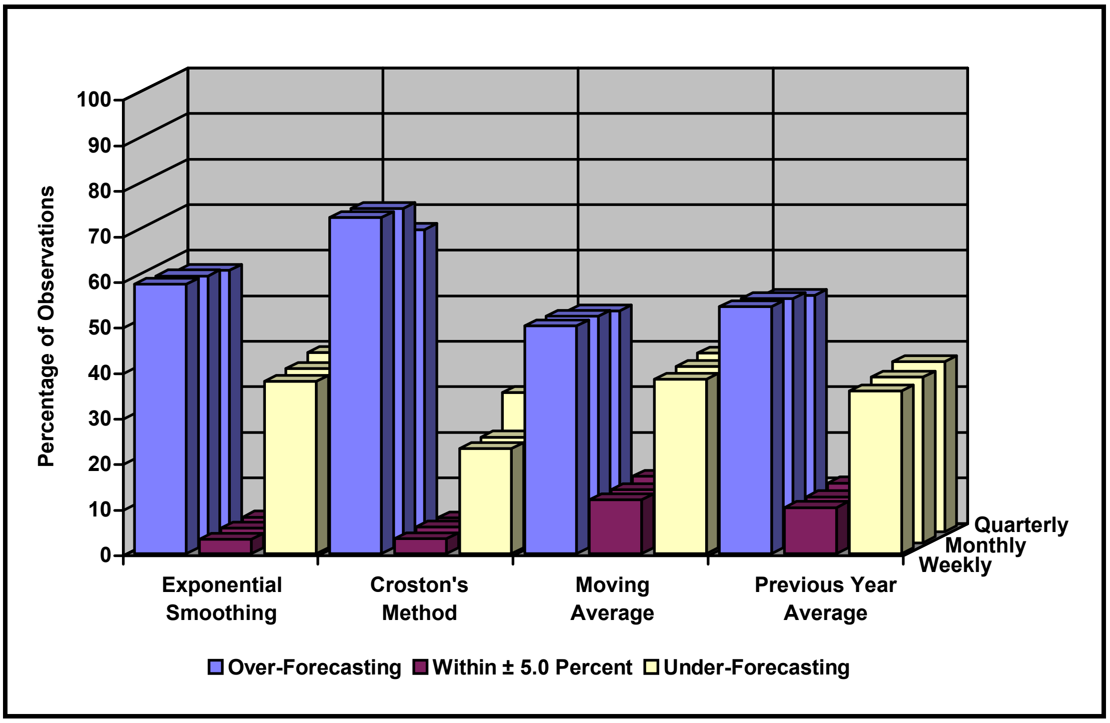
**Figure 7.9: One-Period Ahead Demand Over and Under Forecasting - All Periods.**




Such observations are understandable given the fact that sixty percent of line items are classed as erratic or slow-moving, and therefore contain many periods with zero demand. In such cases, if the predicted demand is not similarly zero, then the only alternative is over-estimation. By their very nature, ES and Croston's method tend to decay towards zero over a number of periods of zero demand and will therefore not match a series of zeros as often as the two averaging methods that drop to zero more readily. In Croston's case, the forecasts will not actually reach zero since they are only updated by a demand observation, thus neither the average demand interval nor the average demand size will equal zero. As the demand period lengthens from weekly aggregation through to monthly and on to quarterly aggregation, the relative frequency of periods with zero demand decreases and therefore there is less over-estimation.

Comparing the forecasting methods, using the demand over a forward lead-time, results in a different pattern, as illustrated by Figure 7.10. Croston's method over-estimates the demand the most frequently, though, like each of the other methods, at a lower rate than the one-period ahead demand comparison. In this instance, the actual demand is aggregated from a number of periods and the likelihood of zero actual demand is reduced, therefore there tends to be less over-estimation.

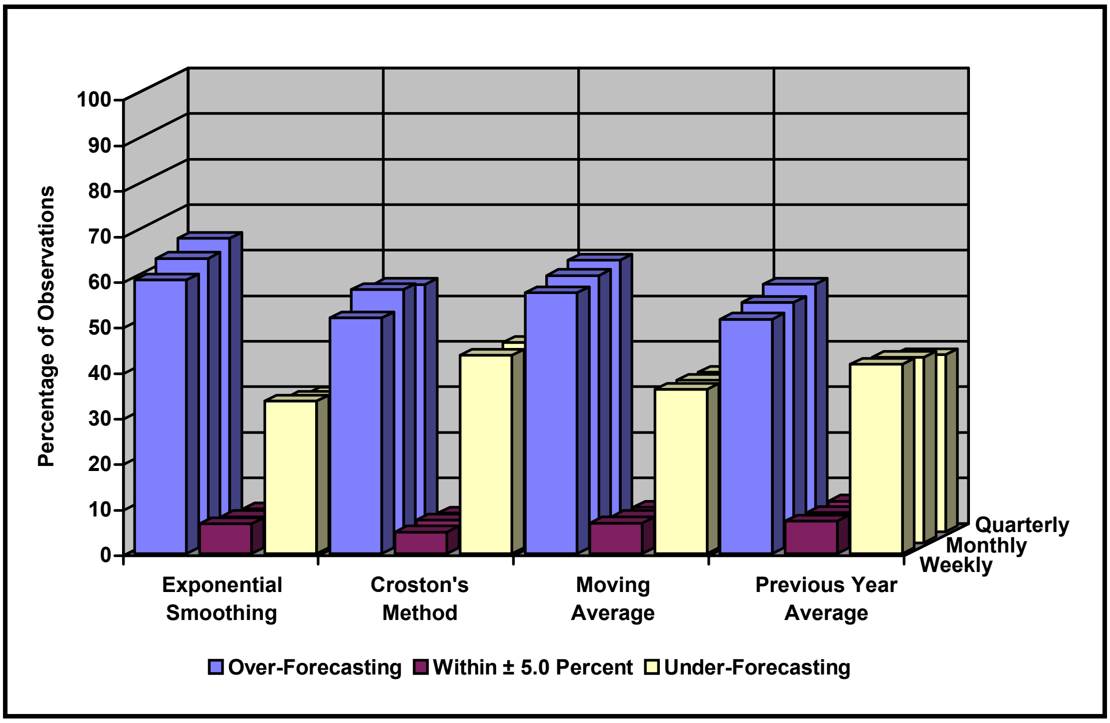
**Figure 7.10: Lead-Time Demand Over and Under Forecasting - All Periods.**




Once again the moving average and previous year average methods over-estimate less frequently. On the whole ES tends to over-estimate less than Croston's method as ES reduces to zero quicker than Croston's method, which only changes in periods of positive demand. Although there are less zeros in the average lead-time demand, any series of zeros serves to reduce the actual demand, as well as the forecast, and over-estimation still occurs.

Figure 7.11 presents the results of comparing the forecasts with the demand over a forward lead-time, but only after a demand has occurred. The results show a reversal of a previous observation that, as the demand period lengthened, there was proportionally less over-estimation. In this instance, it is observed that as the demand period lengthens, from weekly aggregation through monthly and on to quarterly aggregation, the relative frequency of over-estimation generally increases.

**Figure 7.11: Lead-Time Demand Over and Under Forecasting - Demand Only.**




Using the frequency of over-estimation as a rather simplistic forecasting performance measure, Croston's method would appear to fall short of the other three methods. Alternatively, consideration should be given to the scale to which each method either over-forecasts or under-forecasts using the previously mentioned measures of accuracy.

### 7.4.2 Forecasting Performance

The first set of performance comparisons examines the percentage of occasions on which each of the four forecasting methods attains a particular ranking. A rank of 1 is given to the best performing method down to a rank of 4 to the worst, with any tied results sharing the lower ranking of the group. As ties frequently occur, the sum of a row of ranks need not necessarily equal 100 percent.

Table 7.8 presents MAPE and MdAPE results for quarterly data: firstly for comparisons with the one-period ahead demand, secondly for comparisons with lead-time demand for all periods, and finally for comparisons with lead-time demand for periods with positive demand. Although, by definition, MAPE and MdAPE can only be calculated for periods with positive demand when considering only one period, the forecast type is sub-titled "*All Periods*" to remain consistent with tables in the appendices that also present MAD and RMSE, which can both be calculated when the actual demand is zero. In the case of the forward lead-time demand, the actual demand is the sum of many periods and the chance of having zero actual demand is very much lessened. Under this situation the actual demand in a particular period can be zero and it will still be possible to calculate a MAPE value.

**Table 7.8: Forecasting Method Rankings - Quarterly Data.**

| Type of Forecast | Error Measure | Rank Percent | Exponential Smoothing | Croston's Method | Moving Average | Prev Year Average |
|---|---|---|---|---|---|---|
| One-Period Ahead Demand - All Periods | MAPE | 1 | 26.1 | 42.9 | 15.3 | 16.9 |
| | | 2 | 40.7 | 21.1 | 19.6 | 20.9 |
| | | 3 | 21.4 | 16.7 | 36.0 | 34.1 |
| | | 4 | 11.8 | 19.4 | 29.2 | 28.1 |
| | Friedman's Test Statistic: 5,231.11 | | | | | |
| | MdAPE | 1 | 24.2 | 47.3 | 14.5 | 17.1 |
| | | 2 | 39.5 | 23.8 | 18.6 | 20.7 |
| | | 3 | 22.8 | 15.2 | 38.7 | 35.6 |
| | | 4 | 13.5 | 13.7 | 28.2 | 26.6 |
| | Friedman's Test Statistic: 7,538.83 | | | | | |

| Type of Forecast | Error Measure | Rank Percent | Exponential Smoothing | Croston's Method | Moving Average | Prev Year Average |
|---|---|---|---|---|---|---|
| Lead-Time Demand - All Periods | MAPE | 1 | 33.8 | 21.6 | 19.1 | 26.5 |
| | | 2 | 29.3 | 10.3 | 44.2 | 19.2 |
| | | 3 | 25.3 | 13.6 | 28.6 | 29.7 |
| | | 4 | 11.5 | 54.4 | 8.1 | 24.6 |
| | | Friedman's Test Statistic: 4,863.52 | | | | |
| | MdAPE | 1 | 35.2 | 29.8 | 16.0 | 21.9 |
| | | 2 | 31.5 | 14.0 | 38.2 | 28.0 |
| | | 3 | 22.0 | 14.3 | 33.3 | 26.3 |
| | | 4 | 11.4 | 41.9 | 12.5 | 23.8 |
| | | Friedman's Test Statistic: 2,599.54 | | | | |
| Lead-Time Demand - Demand Only | MAPE | 1 | 26.4 | 29.1 | 18.4 | 28.7 |
| | | 2 | 29.2 | 19.0 | 32.4 | 19.3 |
| | | 3 | 22.7 | 18.0 | 32.5 | 26.1 |
| | | 4 | 21.7 | 33.8 | 16.7 | 25.9 |
| | | Friedman's Test Statistic: 151.37 | | | | |
| | MdAPE | 1 | 28.1 | 29.0 | 21.2 | 26.1 |
| | | 2 | 24.8 | 22.0 | 33.5 | 21.2 |
| | | 3 | 22.3 | 22.0 | 29.2 | 24.8 |
| | | 4 | 24.9 | 27.0 | 16.1 | 28.0 |
| | | Friedman's Test Statistic: 184.16 | | | | |

In the first instance, Croston's method with 42.9 percent of line items in rank 1 attains the lowest MAPE value the most frequently. Next is ES with 26.1 percent with the highest ranking and 40.7 percent with the second ranking. The moving average method and the previous year average method share a similar spread amongst the rankings. The rankings for MdAPE tend to display a similar pattern as those for MAPE.

A suitable statistical test for determining whether $k$ related samples of an ordinal nature have been drawn from the same population is the non-parametric Friedman test, as described in Appendix D. Under this test the null hypothesis is rejected if the test statistic is greater than the tabulated chi-square value with $k-1$ degrees of freedom. The calculated statistics from the SAS FREQ procedure are presented. In each case the calculated statistics are substantially greater than the tabulated value of 7.81 at the 5 percent significance level and, therefore, there are differences in the forecasting performance between the methods.

With consideration given to the forecasting performance over a lead-time when measured at all periods, the ranking results change significantly. In this instance, ES attains the highest ranking more frequently when looking at both MAPE and MdAPE, while Croston's method now attains the lowest ranking substantially more often.

When the forecast value is compared with the lead-time demand when a demand has occurred, the spread of the rankings has diminished markedly and no method dominates to the extent observed previously. This observation is reflected by the calculated Friedman statistics, 151.37 and 184.16 for MAPE and MdAPE respectively, which are substantially reduced. However, these values are still greater than the tabulated value and differences in the rankings do exist. Croston's method attains the highest ranking marginally more frequently than the previous year average method in terms of MAPE.

Attention is now given to the comparisons when using monthly data and the performance results are presented in Table 7.9. In this case, Croston's method increases its dominance of the higher rankings when looking at the one-period ahead demand for both MAPE and MdAPE, while ES maintains a second place positioning. Once again, the Friedman statistics are all significant and they continue to suggest greater differences in the forecasting methods when forecasting one-period ahead, as opposed to lead-time demand forecasting, particularly when measured at periods of demand only.

**Table 7.9: Forecasting Method Rankings - Monthly Data.**

| Type of Forecast | Error Measure | Rank Percent | Exponential Smoothing | Croston's Method | Moving Average | Prev Year Average |
|---|---|---|---|---|---|---|
| One-Period Ahead Demand - All Periods | **MAPE** | **1** | 17.4 | 58.0 | 13.2 | 11.9 |
| | | **2** | 45.9 | 16.5 | 15.7 | 23.0 |
| | | **3** | 25.7 | 12.2 | 32.9 | 37.5 |
| | | **4** | 11.0 | 13.3 | 38.2 | 27.6 |
| | Friedman's Test Statistic: 10,448.66 | | | | | |
| | **MdAPE** | **1** | 14.5 | 62.8 | 11.8 | 12.5 |
| | | **2** | 47.9 | 17.5 | 14.8 | 21.5 |
| | | **3** | 25.6 | 11.4 | 38.2 | 37.5 |
| | | **4** | 11.9 | 8.4 | 35.2 | 28.5 |
| | Friedman's Test Statistic: 14,157.96 | | | | | |
| Lead-Time Demand - All Periods | **MAPE** | **1** | 36.6 | 22.0 | 19.1 | 22.7 |
| | | **2** | 27.6 | 15.2 | 40.4 | 19.0 |
| | | **3** | 19.3 | 15.9 | 28.7 | 34.2 |
| | | **4** | 16.5 | 46.9 | 11.8 | 24.1 |
| | Friedman's Test Statistic: 3,337.34 | | | | | |
| | **MdAPE** | **1** | 34.9 | 30.4 | 17.0 | 20.4 |
| | | **2** | 32.0 | 14.8 | 37.1 | 28.4 |
| | | **3** | 21.0 | 13.3 | 31.5 | 28.1 |
| | | **4** | 12.1 | 41.5 | 14.4 | 23.2 |
| | Friedman's Test Statistic: 2,490.82 | | | | | |
| Lead-Time Demand - Demand Only | **MAPE** | **1** | 22.3 | 26.4 | 19.0 | 33.6 |
| | | **2** | 30.1 | 26.3 | 25.0 | 18.2 |
| | | **3** | 25.0 | 21.8 | 31.6 | 21.6 |
| | | **4** | 22.6 | 25.5 | 24.4 | 26.6 |
| | Friedman's Test Statistic: 233.87 | | | | | |
| | **MdAPE** | **1** | 25.8 | 25.4 | 23.3 | 27.9 |
| | | **2** | 25.5 | 29.8 | 26.1 | 18.4 |
| | | **3** | 23.9 | 25.7 | 29.6 | 20.9 |
| | | **4** | 24.8 | 19.1 | 21.0 | 32.8 |
| | Friedman's Test Statistic: 281.95 | | | | | |

ES is viewed as the best performer when considering the lead-time demand across all periods but its positioning is curtailed when the forecast is compared at times of demand only. In the first instance, Croston's method results in comparatively poor results with a large proportion of low rankings. In the case of demand only comparisons, the results again show less differentiation among the forecasting methods. The previous year average method attains the highest rankings for both MAPE and MdAPE.

When looking at weekly data, Croston's method displays an overwhelming domination with regards to the one-period ahead demand, as shown by Table 7.10. Again, Friedman's statistics indicate significant differences in the performance rankings. When considering the lead-time forecasts measured across all periods, ES continues as the best performing method. Croston's method has a substantial proportion of the lowest rankings. In the case of demand only measurements, the spread of the rankings is more evenly balanced between the methods, although the previous year average method obtains the highest rankings.

**Table 7.10: Forecasting Method Rankings - Weekly Data.**

| Type of Forecast | Error Measure | Rank Percent | Exponential Smoothing | Croston's Method | Moving Average | Prev Year Average |
|---|---|---|---|---|---|---|
| One-Period Ahead Demand - All Periods | MAPE | 1 | 9.3 | 69.6 | 13.0 | 8.3 |
| | | 2 | 52.3 | 9.8 | 15.1 | 23.5 |
| | | 3 | 27.7 | 7.4 | 30.6 | 42.3 |
| | | 4 | 10.6 | 13.2 | 41.4 | 26.0 |
| | | Friedman's Test Statistic: 13,866.78 | | | | |
| | MdAPE | 1 | 9.3 | 72.6 | 10.1 | 8.9 |
| | | 2 | 55.5 | 11.1 | 14.4 | 20.9 |
| | | 3 | 26.0 | 8.1 | 37.4 | 40.9 |
| | | 4 | 9.2 | 8.2 | 38.0 | 29.3 |
| | | Friedman's Test Statistic: 18,694.12 | | | | |

| Type of Forecast | Error Measure | Rank Percent | Exponential Smoothing | Croston's Method | Moving Average | Prev Year Average |
|---|---|---|---|---|---|---|
| Lead-Time Demand - All Periods | MAPE | 1 | 34.8 | 24.4 | 19.0 | 21.9 |
| | | 2 | 29.8 | 11.0 | 40.5 | 20.5 |
| | | 3 | 20.1 | 12.6 | 29.9 | 36.1 |
| | | 4 | 15.4 | 52.0 | 10.6 | 21.5 |
| | | | Friedman's Test Statistic: 3,718.95 | | | |
| | MdAPE | 1 | 34.9 | 29.8 | 17.2 | 20.9 |
| | | 2 | 32.9 | 12.5 | 38.1 | 28.8 |
| | | 3 | 21.3 | 12.0 | 31.3 | 29.3 |
| | | 4 | 10.9 | 45.7 | 13.5 | 21.0 |
| | | | Friedman's Test Statistic: 2,920.55 | | | |
| Lead-Time Demand - Demand Only | MAPE | 1 | 25.5 | 22.4 | 18.8 | 34.0 |
| | | 2 | 31.2 | 23.4 | 23.5 | 21.8 |
| | | 3 | 20.7 | 21.8 | 33.8 | 23.6 |
| | | 4 | 22.7 | 32.4 | 23.9 | 20.6 |
| | | | Friedman's Test Statistic: 810.74 | | | |
| | MdAPE | 1 | 27.2 | 23.2 | 23.5 | 27.7 |
| | | 2 | 26.2 | 27.3 | 26.3 | 19.9 |
| | | 3 | 21.3 | 25.6 | 30.1 | 22.9 |
| | | 4 | 25.3 | 23.9 | 20.0 | 29.5 |
| | | | Friedman's Test Statistic: 71.11 | | | |

Overall results from the rankings indicate that Croston's method is the best performing forecasting method when considering the one-period ahead demand. Under this situation, the dominance of Croston's method increases as the demand aggregation moves from quarterly through monthly and down to weekly aggregation. Thus, Croston's method provides a relative improvement as the demand series tend towards a higher proportion of periods with zero demand.

In the case of lead-time demand with comparisons made in all periods, ES emerges with the best performance when considering either quarterly, monthly or weekly data. Meanwhile, Croston's method provides the worst performance by obtaining the lowest ranking the most frequently. The simpler methods, particularly the moving average method, provide a better overall performance than Croston's method in this case.

When the lead-time demand is compared at periods of demand only, the methods are more similar in the range of rankings obtained. Croston's method provides the best performance when considering quarterly data, closely followed by the previous year average method. In the case of monthly and weekly data, the previous year average method emerges as the best.

Additional measures of forecasting accuracy, including MAD and RMSE, are presented in Appendix E. Also shown are the MADs for forecast errors which exceed 5.0 percent above the actual value (referred to as MAD+) and the MAD for forecast errors more than 5.0 percent below the actual value (referred to as MAD-). In all cases the MAD+ value is greater than the MAD- value, indicating that each method tends to over-forecast rather than under-forecast the actual demand. This observation coincides with the results of Section 7.4.1 where it was noted that over-forecasting tends to occur more frequently.

When considering the one-period ahead comparisons, the moving average method provides the lowest MAD and lowest MAD+ for all demand aggregations, as well as the lowest RMSE for monthly and weekly data. Despite Croston's method providing the lowest MAPE and MdAPE values the most frequently for quarterly data as shown in Table 7.8, ES provides the lowest average values for both these measures, as well as for RMSE. This result is possible as Croston's method was also observed to provide the highest MAPE and MdAPE values the most frequently, demonstrating extremes in performance. Croston's method does however provide the lowest values for MAPE and

MdAPE for monthly and weekly data and the lowest value for MAD- throughout. The previous year average method fails to achieve the best result in all cases.

ES completely dominates the error measures when comparisons are made against lead-time demand for all periods. It is only in the case of MAD- for monthly and weekly data that another forecasting method provides the best result, and these are provided by Croston's method on both occasions.

The results are far more varied when the lead-time demand is compared only in periods with positive demand. The moving average method once again provides the best MAD results for all demand aggregations, and ES provides the best MAD- and RMSE results throughout. The best MAPE and MdAPE results are shared by Croston's method and the previous year average method. Croston's method provides the best MAPE and MdAPE for quarterly data and the best MdAPE for weekly data, while the previous year average method provides the best results on the other occasions.

The overall results across the range of error measures show a similar pattern as the ranking results. Croston's method performs well for the one-period ahead demand, particularly for monthly and weekly data, while ES performs well with quarterly data. ES provides the best performance in the majority of cases for lead-time demand comparisons made in all periods. When the lead-time demand is compared in periods of demand only the results are somewhat varied with no particular method dominating.

In Section 7.3.4 it was noted that some of the optimal smoothing constants had very low values and others had very high values. For both exponential smoothing and Croston's method it was with weekly data that the optimal values tended to be lowest, although Croston's method did not have any low values when considering lead-time demand in periods of demand only. When the smoothing values are low there will be greater smoothing of demand and the initialisation value will have the greatest impact on the forecasts.

It would be expected that when the smoothing methods have low optimal values that the previous year average method and the moving average method would be at their most competitive as all the methods would essentially be averaging methods in such cases. The overall results do not support this hypothesis, however, as a comparison of Table 7.8 with quarterly results against Table 7.10 with weekly results suggests the moving average method and the previous year average method have the same likelihood of giving the best results when demand is weekly as they do when it is quarterly. In general, ES and Croston's method provide better results than averaging methods even when the smoothing constants are low. That is not to say, however, that the averaging methods are not capable of producing the best results when conditions suit them.

A fundamental observation, given the results presented in Appendix E, is that all forecasts are rather abysmal. Extremely high values for the measurements of accuracy throughout the table indicate that none of the forecasting methods are good in the traditional sense. However, when faced with demand of this nature, the usage of ES or Croston's method is still preferable to the alternative of using a simple average of the historic demand. The earlier suggestion by Brown [14], that if demand is identified as erratic then the forecast be set simply to the historic average is unlikely to be appropriate in this case. ES is observed to provide the best results more frequently than both the moving average method and the previous year average method.

It should be borne in mind that none of the comparisons between methods has yet considered any demand pattern categorisation. As Croston's method was put forward as a method for providing improved forecasts under the situation of erratic demand, it is worthwhile to consider the performance across the range of demand patterns. The comparative forecasting performance by demand pattern is examined in the next section.

### 7.4.3 Forecasting Performance by Demand Pattern

In this section comparisons are made between the four forecasting methods across the range of identified demand patterns. Once again a total of 18,750 line items are considered, but in this section the performance measures are calculated for each of the five identified demand patterns. The optimal smoothing parameters, as used previously, were produced from a hold-out sample of 500 line items for the group as a whole, although each demand pattern was equally represented.

Using the mean absolute percentage error (MAPE) as the selected performance measure, comparative results for quarterly, monthly and weekly series are shown in the following three tables respectively. In each case the comparisons are made for the one-period ahead demand over all periods, the lead-time demand over all periods and the lead-time demand measured at periods of demand only. The lowest MAPE for each demand pattern is shown in bold-type.

Table 7.11 presents average MAPE results for quarterly data. Croston's method provides the best results for the smooth and slow-moving demand patterns when comparing the forecast value with the one-period ahead demand, while the moving average method provides the best results for the irregular and mildly erratic demand patterns, and ES provides the best result for the highly erratic demand pattern.

**Table 7.11: Mean Absolute Percentage Error by Demand Pattern - Quarterly Data.**

| Type of Forecast | Demand Pattern | Expon'l Smooth. | Croston's Method | Moving Average | Prev Year Average | Friedman Statistic |
|---|---|---|---|---|---|---|
| One-Period Ahead Demand - All Periods | **Smooth** | 90.20 | **87.68** | 89.33 | 94.81 | 547.52 |
| | **Irregular** | 200.42 | 207.63 | **194.43** | 202.56 | 229.13 |
| | **Slow-moving** | 70.98 | **54.22** | 81.14 | 78.89 | 4,524.21 |
| | **Mildly Erratic** | 113.21 | 146.60 | **111.83** | 118.66 | 1,249.60 |
| | **Highly Erratic** | **111.34** | 138.33 | 117.85 | 119.47 | 1,186.77 |
| Lead-Time Demand - All Periods | **Smooth** | **86.75** | 106.51 | 88.97 | 101.50 | 1,315.63 |
| | **Irregular** | **197.99** | 286.56 | 204.29 | 228.98 | 791.70 |
| | **Slow-moving** | **109.92** | 201.26 | 118.26 | 135.28 | 1,092.86 |
| | **Mildly Erratic** | **232.93** | 478.74 | 239.28 | 278.71 | 817.87 |
| | **Highly Erratic** | **239.24** | 449.38 | 255.02 | 279.28 | 951.75 |
| Lead-Time Demand - Demand Only | **Smooth** | 101.07 | 96.33 | **95.35** | 99.87 | 868.86 |
| | **Irregular** | 239.58 | 221.29 | 225.08 | **220.37** | 369.21 |
| | **Slow-moving** | 151.88 | **125.56** | 143.68 | 142.58 | 193.83 |
| | **Mildly Erratic** | 412.60 | 330.87 | 365.61 | **322.44** | 69.15 |
| | **Highly Erratic** | 399.00 | **330.85** | 368.38 | 331.46 | 81.64 |

In the case of lead-time demand comparisons at all periods, ES provides the best results for all five demand patterns and Croston's method performs particularly poorly with the two erratic patterns. However, in the case of lead-time demand comparisons at times of demand only, ES does not provide the best result for any demand pattern, while Croston's method provides the best results for the slow-moving demand pattern and the highly erratic demand pattern. The moving average method once again provides the best result for the smooth demand pattern, and the previous year average method provides the best results for the irregular and mildly erratic demand patterns.

When it comes to monthly data the results are somewhat different, as shown by Table 7.12. In this instance Croston's method provides the best results for the smooth demand pattern, as well as the slow-moving demand pattern for the one-period ahead demand comparison, while ES provides the best results for the erratic demand patterns.

**Table 7.12: Mean Absolute Percentage Error by Demand Pattern - Monthly Data.**

| Type of Forecast | Demand Pattern | Expon'l Smooth. | Croston's Method | Moving Average | Prev Year Average | Friedman Statistic |
|---|---|---|---|---|---|---|
| One-Period Ahead Demand - All Periods | **Smooth** | 99.88 | **94.93** | 98.77 | 101.73 | 894.01 |
| | **Irregular** | 143.18 | 142.97 | **139.82** | 145.86 | 562.02 |
| | **Slow-moving** | 84.38 | **69.79** | 88.82 | 87.15 | 5,634.30 |
| | **Mildly Erratic** | **90.90** | 96.80 | 94.32 | 95.34 | 2,714.08 |
| | **Highly Erratic** | **86.74** | 87.56 | 92.89 | 92.26 | 2,956.88 |
| Lead-Time Demand - All Periods | **Smooth** | 94.45 | 117.79 | **94.26** | 104.55 | 304.28 |
| | **Irregular** | **206.49** | 295.14 | 211.53 | 243.30 | 162.32 |
| | **Slow-moving** | **110.89** | 237.11 | 118.93 | 140.90 | 1,612.16 |
| | **Mildly Erratic** | **239.64** | 566.66 | 248.16 | 293.88 | 810.73 |
| | **Highly Erratic** | **222.21** | 496.15 | 239.42 | 279.73 | 1,303.38 |
| Lead-Time Demand - Demand Only | **Smooth** | 105.28 | 105.87 | 101.42 | **97.50** | 543.09 |
| | **Irregular** | 246.83 | 230.76 | 231.66 | **219.03** | 208.31 |
| | **Slow-moving** | 158.16 | **133.41** | 148.16 | 144.46 | 391.41 |
| | **Mildly Erratic** | 422.29 | 349.63 | 374.31 | **313.58** | 276.26 |
| | **Highly Erratic** | 380.30 | 311.69 | 335.31 | **300.45** | 264.51 |

In the case of lead-time demand comparisons at all periods, ES no longer has complete dominance for all demand patterns, as the moving average method provides a marginally better result for the smooth demand pattern, although Croston's method still performs particularly poorly with the two erratic patterns. The previous year average method, which has not previously shown much promise, now provides the best results for four of the demand patterns when the lead-time demand is compared at periods of demand only, and it is only Croston's method that provides a better result for slow-moving demand.

Results from the weekly data, as presented in Table 7.13, show Croston's method to be the best for all demand patterns for the one-period ahead demand, while ES is the best for all demand patterns when lead-time demand is compared at all periods, and Croston's method is again poor for the two erratic patterns.

Table 7.13: Mean Absolute Percentage Error by Demand Pattern - Weekly Data.

| Type of Forecast | Demand Pattern | Expon'l Smooth. | Croston's Method | Moving Average | Prev Year Average | Friedman Statistic |
|---|---|---|---|---|---|---|
| One-Period Ahead Demand - All Periods | **Smooth** | 97.60 | **94.38** | 97.14 | 98.16 | 802.38 |
| | **Irregular** | 94.87 | **94.03** | 96.04 | 96.83 | 1,008.62 |
| | **Slow-moving** | 95.22 | **89.62** | 96.52 | 96.06 | 5,611.02 |
| | **Mildly Erratic** | 92.26 | **88.47** | 94.67 | 94.33 | 4,074.61 |
| | **Highly Erratic** | 91.04 | **85.57** | 94.01 | 93.29 | 4,407.47 |
| Lead-Time Demand - All Periods | **Smooth** | **93.14** | 119.51 | 93.98 | 103.08 | 732.01 |
| | **Irregular** | **209.06** | 299.75 | 213.17 | 247.65 | 103.77 |
| | **Slow-moving** | **106.38** | 239.26 | 113.94 | 132.75 | 1,585.26 |
| | **Mildly Erratic** | **232.31** | 577.27 | 242.44 | 290.43 | 698.92 |
| | **Highly Erratic** | **215.55** | 507.36 | 232.70 | 281.70 | 1,166.12 |
| Lead-Time Demand - Demand Only | **Smooth** | 100.77 | 108.01 | 97.34 | **92.69** | 1,188.12 |
| | **Irregular** | 258.88 | 252.33 | 244.60 | **228.22** | 577.29 |
| | **Slow-moving** | 146.52 | **133.00** | 141.81 | 134.27 | 199.53 |
| | **Mildly Erratic** | 409.12 | 354.73 | 359.15 | **320.80** | 212.21 |
| | **Highly Erratic** | 370.22 | **295.73** | 314.80 | 319.44 | 178.24 |

In the case of lead-time demand when comparing at periods of demand only, the previous year average method provides the best results for the smooth, irregular and mildly erratic demand patterns, while Croston's method is best for the slow-moving and highly erratic demand patterns.

For each demand aggregation it is observed that the results for the one-period ahead are often substantially lower than those from the lead-time comparisons. This is particularly the case for the slow-moving and erratic demand patterns. The reason for this is that MAPE cannot be calculated when the actual demand is zero, which occurs more frequently when considering the one-period ahead demand as opposed to the lead-time demand. Therefore, some large errors are not included in calculations under the first implementation. In addition, the slow-moving and erratic demand patterns contain more zeros, leading to greater differences between the results for these patterns.

Prior to commenting on the overall performance of the forecasting methods by demand pattern, it is worthwhile examining another performance measure along with MAPE. Different measures are likely to provide different results, although it is hoped that general patterns will emerge. Equivalent statistics to those shown for MAPE in this section are presented in Appendix F for the median absolute percentage error (MdAPE), and the joint consideration of both sets of statistics provide the observations that follow.

Croston's method is observed to increasingly provide the best results for the one-period ahead demand as the data moves from quarterly to monthly and on to weekly aggregation. Initially it is only the smooth and slow-moving demand patterns for which Croston's method provides the best results, but once weekly data is considered the method is the best across the full range of demand patterns. ES provides the best results for the erratic demand patterns when considering quarterly data.

ES provides the best results across the range of demand patterns for each aggregation when comparing the forecast value with the lead-time demand in all periods. Very infrequently will another method better the results from ES under this situation, and it is only the moving average method that manages to do so. When it comes to lead-time demand compared at times of demand only, ES wholly loses the dominance it had for the lead-time demand in all periods. It is Croston's method that for the most part now provides the best results, particularly when considering MdAPE, although the previous year average method also provides good results.

Results from the analysis undertaken in this section indicate Croston's method is not as effective at forecasting erratic demand as suggested by particular sections of the academic literature. The method performs relatively better when considering weekly data, which contains proportionally more zero observations than monthly or quarterly data, although for the most part the better performance is limited to forecast comparisons with the one-period ahead demand. When it comes to forecasting lead-time demand, Croston's method comes a poor second to ES when comparisons are made at all periods, and the method shares the honours with the previous year average method when lead-time demand comparisons are made at periods of demand only.

## 7.5 Concluding Remarks

ES is commonly used for demand forecasting in an inventory control environment. However, this method has been shown to be unsuitable for items with erratic demand as it tends to lead to unnecessarily high stocks. An alternative method was developed by Croston in 1972 to separately smooth the interval between demands and the size of the demands in order to reduce the inherent bias. Since this time much of the academic literature has commented upon the desirability of this method. Forecasting methods based on a Box-Jenkins approach are not suitable as trends and seasonality cannot be discerned when demand is erratic or slow-moving.

The large scale comparative analysis undertaken in this chapter utilised 18,750 line items with equal demand pattern representation. Four forecasting methods were compared using various measures of accuracy, including MAD, RMSE, MAPE and MdAPE, with errors measured at every point in time as well as only after a demand has occurred. In recognising the purpose for which demand forecasts are made in reality, the forecast value is compared with the actual demand over a forward-looking lead-time period, in addition to comparisons with the more traditional one-period ahead demand. Optimal smoothing constant values were obtained from a hold-out sample of 500 line items with equal demand pattern representation.

All the forecasting methods tend to over-estimate, rather than under-estimate, the actual demand. With many zero observations in the data, if the forecasts are not likewise zero, then over-estimation occurs. Both ES and Croston's method over-estimate demand to a greater degree than the moving average method and the previous year average method. This is due to ES and Croston's method decaying towards zero over a number of periods of zero demand and therefore not matching a series of zeros as often as the two averaging methods that drop to zero more readily.

When comparing the overall forecasting performance using the standard measures of accuracy, the results vary and no single method emerges as the best. Croston's method performs well when comparing the forecast with the one-period ahead demand, particularly with monthly and weekly data. Alternatively, ES completely dominates when comparing against lead-time demand in all periods irrespective of the demand aggregation. With lead-time demand comparisons in periods of demand only, Croston's method performs well with quarterly data, while the previous year average method provides the best results with monthly and weekly data.

Although Croston's method was put forward as suitable for forecasting when facing erratic demand, the results using RAF data do not wholly reflect this. The method is often bettered by simpler methods when demand is erratic, as well as when demand follows any of the other identified patterns, including smooth, irregular and slow-moving. Syntetos and Boylan [78] recently observed similar findings in the course of their research. They made the conclusion that the method proposed by Croston reduced the bias associated with ES although it did not eliminate it completely.

The next chapter examines a recognised weakness of Croston's method and introduces various alternative methods that all seek to further reduce the inherent bias.

# 8. ALTERNATIVE FORECASTING METHODS

Croston's method has been claimed to have practical tangible benefits, although results on real data often show very modest benefits (Willemain *et al*. [88]). Moreover, inferior performance has recently been reported in the literature when the method is compared with less sophisticated methods, such as exponential smoothing (ES) or simple moving averages (Sani and Kingsman [60]). In fact, previous observations using industrial data prompted Syntetos and Boylan [76] to conduct a simulation experiment comparing Croston's method and ES. Their results indicate the estimates of demand per period are not unbiased in either method. This bias stems from the observation that the forecast is highest just after a demand and lowest just before a demand.

In an effort to identify the cause of the modest performance of Croston's method, Syntetos and Boylan [78] found a mistake was made in Croston's mathematical derivation of the demand estimate. Croston's method accurately estimates the expected interval between transactions, as well as the expected demand size, although, with them erroneously combined, the method fails to produce accurate estimates of the demand per period. The implementation of Croston's method reduces the bias of ES but it does not eliminate it completely. Subsequently, the authors developed modifications to Croston's method that theoretically eliminate the forecast bias.

## 8.1 Eliminating Forecasting Bias

Croston demonstrated that when demand estimates are updated every period using ES the expected demand per period is $\mu/\rho$, where $\mu$ is the mean demand size and $\rho$ is the mean interval between transactions, with variance:

$$\text{var} = \left(\frac{\alpha}{2-\alpha}\right)\left(\frac{\rho-1}{\rho^2}\mu^2 + \frac{\sigma^2}{\rho^2}\right)$$

When demand estimates are only updated at moments of positive demand, as specified by Croston's forecasting method, such calculations introduce bias. This was adequately recognised by Croston in the case of the variance with an approximation given as:

$$\text{var} \approx \left(\frac{\alpha}{2-\alpha}\right)\left(\frac{(\rho-1)^2}{\rho^4}\mu^2 + \frac{\sigma^2}{\rho^2}\right)$$

However, if $z_t$ is an estimate of mean demand size, updated only after demand occurs with ES, and $p_t$ is an estimate of the mean interval between transactions updated in the same manner then, according to Croston, the expected demand in the period immediately following one with demand is given by:

$$E(\hat{y}_t) = E\left(\frac{z_t}{p_t}\right) = \frac{E(z_t)}{E(p_t)}$$

If it is assumed that demand sizes and intervals are independent, then

$$E\left(\frac{z_t}{p_t}\right) = E(z_t)E\left(\frac{1}{p_t}\right)$$

However

$$E\left(\frac{1}{p_t}\right) \neq \frac{1}{E(p_t)}$$

Syntetos and Boylan [76] showed that (Part A of Appendix G) an unbiased expected demand per period for a smoothing constant of unity is:

$$E(\hat{y}) = \mu\left[-\frac{1}{\rho-1}\log\left(\frac{1}{\rho}\right)\right]$$

If, for example, the average size of demand when it occurs is $\mu = 6$, and the average interval between transactions is $\rho = 3$, then the average estimated demand per period using Croston's method is $\mu/\rho = 6/3 = 2$, whereas it should be $6 \times 0.549 = 3.295$ (a 64.75 percent bias implicitly incorporated in Croston's estimate).

The maximum bias is given by:

$$\mu\left[-\frac{1}{\rho-1}\log\left(\frac{1}{\rho}\right)\right]-\frac{\mu}{\rho}$$

for $\alpha$=1, while for $\alpha$ values less than 1 the magnitude of the bias is much smaller depending on the value of the smoothing constant.

Using an artificial data simulation the authors sought to quantify the bias implicitly incorporated into Croston's method. Apart from the fact that the bias increases as the value of $\alpha$ increases and is independent of the interval between transactions, no specific relationship could be determined. For all smoothing parameter values used (ranging between 0.1 and 1.0), Croston's method is recommended only for low values of $\alpha$, as values above 0.3 show considerable bias in all simulation runs.

Syntetos and Boylan [77] show the bias can be approximated by:

$$\frac{\alpha}{2-\alpha}\mu\frac{(\rho-1)}{\rho^2}$$

although the accuracy of the approximation deteriorates as $\alpha$ increases. However, for $\alpha \leq 0.3$, it is accurate to within 10 percent.

Fildes and Beard [26] indicate that methods that are consistently biased have the potential for improvement by simply subtracting the historical bias from the forecast. Given the limitations of Croston's method, Syntetos and Boylan [77,78] have developed three alternative methods that attempt to account for the historical bias in this manner.

The three methods are:

(i) Revised Croston's method.

(ii) Bias reduction method.

(iii) Approximation method.

The revised Croston's method updates the demand size in the usual manner while the updating of the interval is modified. Alternatively, the latter two methods update the parameters and generate a forecast according to the original method while introducing a factor to depress the result. Subsequent analysis will compare the performance of these modifications against Croston's original method, but first, the methods are introduced.

### 8.1.1 Revised Croston's Method

Under the first alternative, from Part B of Appendix G, an expectation for obtaining unbiased demand is given as:

$$E\{\hat{y}\} = E(z)E\left(\frac{1}{\rho c^{\rho-1}}\right) \approx \frac{\mu}{\rho}$$

where $c$ is an arbitrarily large constant.

Theoretically, $c$ has to be infinitely large, although the result is a good approximation, as determined by Syntetos and Boylan [78], if $c$ is sufficiently large, say $c > 100$. This method is exactly unbiased for $c$ set at infinity. Hence, for any finite value of $c$ the method is expected to have a very small bias.

The revised Croston's method updates the demand size and $\left(1/\rho c^{\rho-1}\right)$ instead of the actual interval between transactions after demand occurs with ES. The results are combined as:

$$\hat{y}_t = z_t \frac{1}{p_t c^{p_t - 1}}$$

in order to produce estimates of the future demand per period. As in Croston's method if no demand occurs the estimates remain the same. An approximation of the variance is given as:

$$\operatorname{var}(\hat{y}_t) \approx \left(\frac{\alpha}{2-\alpha}\right)\left(\frac{\rho - 1}{\rho^2}\left(\mu^2 + \sigma^2\right) + \frac{\sigma^2}{\rho}\right)$$

With Croston's method observed to over-forecast, the other two modifications incorporate a deflator to remove the bias.

### 8.1.2 Bias Reduction Method

With the bias associated with Croston's method known, the bias reduction method subtracts the expected bias from Croston's calculation of the mean demand per period:

$$\hat{y}_t = \frac{z_t}{p_t} - \left(\frac{\alpha}{2-\alpha} z_t \frac{(p_t - 1)}{p_t^2}\right)$$

### 8.1.3 Approximation Method

The third method takes the smoothing parameter into consideration and is reported to provide a reasonable approximation of the actual demand per period, especially for very low values of $\alpha$ and large intervals between transactions. Under this method the estimates of size and interval are combined as:

$$\hat{y}_t = \left(1 - \frac{\alpha}{2}\right)\frac{z_t}{p_t}$$

## 8.2 Forecasting Performance

This section examines the forecasting performance of the methods put forward as suitable when considering erratic demand, including Croston's method, the revised

Croston's method, the bias reduction method and the approximation method. Comparisons across a range of forecasting measures are presented in Appendix H where the results were obtained from the previously utilised sample of 18,750 line items. The results are directly comparable to those presented for the more traditional forecasting methods in Appendix E. The same smoothing constant values are used for each method, previously derived as optimal for Croston's method from the hold-out sample of 500 line items. In the case of the revised Croston's method, constant $c$ is given the value of 100 and the ratio $\left(1/\rho c^{\rho-1}\right)$ is updated using the demand interval smoothing constant. For both the bias reduction method and the approximation method the value of the smoothing constant applying to the bias is taken as the average of the demand size constant and the demand interval constant. Johnston *et al*. [41] have recently suggested the smoothing constant should be taken as that applying to the demand interval.

The overall pattern is remarkably consistent whether considering quarterly, monthly or weekly data series, or whether comparing the forecast with the one-period ahead demand, or the lead-time demand, either in all periods or at times of positive demand only. In the vast majority of cases the approximation method provides the best results, with only the MAD- result being consistently bettered, where for the most part Croston's method is best.

In considering the specifics of the one-period ahead demand, while the approximation method performs the best with respect to most measures, Croston's method provides the lowest MdAPE observation for the monthly data, with the revised Croston's method providing the lowest MdAPE observation for the weekly data. The bias reduction method does not provide the best result for any measure, although it frequently provides the second best result after the approximation method.

A clear pattern is displayed when comparing the forecast value with the lead-time demand for all periods. In this case the approximation method provides the best results throughout with the exception of MAD- where Croston's method is best for all demand aggregations. The bias reduction method again comes second in the majority of cases.

A similar pattern arises when comparing the forecast value with the lead-time demand only in periods with positive demand. Again, the approximation method dominates the results with the revised Croston's method providing the best results for MAD- in all cases. However, in the case of weekly data the bias reduction method provides the best results for both MAPE and MdAPE, with the approximation method now coming second.

In general, both the bias reduction method and the approximation method provide an improvement over Croston's method, with the approximation method providing the greatest improvement. Alternatively, the revised Croston's method rarely outperforms Croston's method. Comparative results between the variations on Croston's method and the more traditional forecasting methods indicate that the approximation method often provides the best overall result, particularly in the case of MAPE and MdAPE. However, the method only performs best when considering the one-period ahead demand or lead-time demand comparisons in periods of positive demand. In the case of lead-time demand comparisons in all periods, ES maintains complete dominance and it is still only in the case of MAD- for monthly and weekly data that another method provides the best result.

Attention is now given to Croston's method and the variations on this method across the range of previously identified demand patterns. The following three tables present comparative MAPE results for quarterly, monthly and weekly series respectively.

Again, a total of 18,750 equally represented line items are considered and the results are comparable to those presented for the more traditional forecasting methods in Section 7.4.3. Comparisons are made for the one-period ahead demand, as well as for the lead-time demand over all periods and at times of positive demand only.

Considering each demand aggregation period in turn, Table 8.1 presents average MAPE results for quarterly data. The approximation method dominates the results by providing the lowest MAPE in nearly all cases. It is only under the one-period ahead demand comparison, in the case of the slow-moving demand pattern, that another method provides the best result, and in this case it is the revised Croston's method that does so.

**Table 8.1: MAPE by Demand Pattern - Quarterly Data (Croston's and Variants).**

| Type of Forecast | Demand Pattern | Croston's Method | Revised Croston's | Bias Reduction | Approxi-mation | Friedman Statistic |
|---|---|---|---|---|---|---|
| One-Period Ahead Demand - All Periods | **Smooth** | 87.68 | 87.94 | 87.25 | **75.81** | 3,115.88 |
| | **Irregular** | 207.63 | 207.49 | 201.79 | **173.52** | 5,275.87 |
| | **Slow-moving** | 54.22 | **53.34** | 56.57 | 56.20 | 1,999.50 |
| | **Mildly Erratic** | 146.60 | 154.71 | 141.19 | **127.21** | 214.50 |
| | **Highly Erratic** | 138.33 | 140.76 | 133.16 | **119.18** | 708.91 |
| Lead-Time Demand - All Periods | **Smooth** | 106.51 | 106.22 | 100.32 | **77.30** | 3,345.63 |
| | **Irregular** | 286.56 | 274.02 | 261.46 | **196.66** | 6,359.50 |
| | **Slow-moving** | 201.26 | 205.85 | 170.47 | **122.03** | 6,168.81 |
| | **Mildly Erratic** | 478.74 | 491.88 | 408.17 | **312.95** | 5,648.07 |
| | **Highly Erratic** | 449.38 | 450.07 | 388.52 | **293.49** | 6,195.49 |
| Lead-Time Demand - Demand Only | **Smooth** | 96.33 | 96.33 | 90.37 | **70.67** | 2,910.24 |
| | **Irregular** | 221.29 | 209.22 | 195.54 | **151.19** | 5,467.44 |
| | **Slow-moving** | 125.56 | 136.08 | 97.97 | **80.08** | 2,097.44 |
| | **Mildly Erratic** | 330.87 | 350.89 | 252.97 | **215.89** | 2,170.23 |
| | **Highly Erratic** | 330.85 | 326.96 | 264.38 | **215.42** | 2,807.97 |

In comparing these results with those of Table 7.11 for the traditional forecasting methods, it is observed that the approximation method provides the best overall results for the smooth and irregular demand patterns under all forecast situations. However, when it comes to the two erratic demand patterns it is only when comparing the lead-time demand at periods of demand only in which the approximation method provides better results than ES. In the case of slow-moving demand the revised Croston's method is the best when comparing with the one-period ahead demand, ES is best for lead-time demand compared in all periods, and the approximation method is best for lead-time demand compared in periods of demand only.

Results for the erratic forecasting methods with monthly data, as presented in Table 8.2, show a similar pattern. The approximation method again dominates by providing the lowest MAPE in all cases, except for the one-period ahead demand comparison where the revised Croston's method provides the best result for slow-moving demand.

**Table 8.2: MAPE by Demand Pattern - Monthly Data (Croston's and Variants).**

| Type of Forecast | Demand Pattern | Croston's Method | Revised Croston's | Bias Reduction | Approxi-mation | Friedman Statistic |
|---|---|---|---|---|---|---|
| One-Period Ahead Demand - All Periods | **Smooth** | 94.93 | 95.08 | 94.91 | **91.58** | 140.41 |
| | **Irregular** | 142.97 | 149.30 | 140.45 | **135.71** | 1,577.68 |
| | **Slow-moving** | 69.79 | **67.75** | 71.01 | 71.40 | 6,915.85 |
| | **Mildly Erratic** | 96.80 | 103.28 | 95.83 | **94.79** | 987.06 |
| | **Highly Erratic** | 87.56 | 92.30 | 86.94 | **85.99** | 732.03 |
| Lead-Time Demand - All Periods | **Smooth** | 117.79 | 125.36 | 111.18 | **103.14** | 3,871.61 |
| | **Irregular** | 295.14 | 305.49 | 277.19 | **260.34** | 5,011.68 |
| | **Slow-moving** | 237.11 | 266.88 | 215.02 | **204.61** | 5,639.68 |
| | **Mildly Erratic** | 566.66 | 628.25 | 519.45 | **499.50** | 4,759.90 |
| | **Highly Erratic** | 496.15 | 546.33 | 456.15 | **436.50** | 4,853.49 |
| Lead-Time Demand - Demand Only | **Smooth** | 105.87 | 117.18 | 93.84 | **84.01** | 4,488.58 |
| | **Irregular** | 230.76 | 246.56 | 196.77 | **181.05** | 5,880.10 |
| | **Slow-moving** | 133.41 | 186.97 | 101.48 | **98.61** | 2,328.59 |
| | **Mildly Erratic** | 349.63 | 455.15 | 274.28 | **269.52** | 2,385.05 |
| | **Highly Erratic** | 311.69 | 392.82 | 246.15 | **239.37** | 2,742.31 |

In comparing the results with the traditional forecasting methods it is observed that the approximation method provides the best overall results for the smooth, irregular and highly erratic demand patterns when comparing the one-period ahead demand, while the revised Croston's method is best for slow-moving demand, and ES is best for mildly erratic demand. When it comes to comparisons with the lead-time demand at all periods, it is ES which provides the best results for all demand patterns, except for smooth demand where the moving average method marginally provides the best result. Alternatively, when the lead-time demand is compared in periods of demand only it is the approximation method which provides the best results for all patterns.

The approximation method no longer dominates the results when considering weekly data, as presented in Table 8.3.

**Table 8.3: MAPE by Demand Pattern - Weekly Data (Croston's and Variants).**

| Type of Forecast | Demand Pattern | Croston's Method | Revised Croston's | Bias Reduction | Approxi-mation | Friedman Statistic |
|---|---|---|---|---|---|---|
| One-Period Ahead Demand - All Periods | **Smooth** | 94.38 | 94.49 | 94.41 | **93.93** | 1,903.06 |
| | **Irregular** | 94.03 | 95.53 | 93.80 | **93.50** | 913.74 |
| | **Slow-moving** | 89.62 | **89.16** | 89.86 | 89.89 | 9,298.51 |
| | **Mildly Erratic** | **88.47** | 88.97 | 88.56 | 88.54 | 6,004.24 |
| | **Highly Erratic** | **85.57** | 85.64 | 85.75 | 85.75 | 6,071.34 |
| Lead-Time Demand - All Periods | **Smooth** | 119.51 | 160.39 | 112.14 | **110.05** | 5,266.77 |
| | **Irregular** | 299.75 | 361.32 | 281.43 | **278.00** | 4,376.23 |
| | **Slow-moving** | 239.26 | 325.24 | 220.55 | **219.02** | 4,472.17 |
| | **Mildly Erratic** | 577.27 | 777.08 | 537.93 | **535.40** | 3,601.69 |
| | **Highly Erratic** | 507.36 | 667.67 | 472.51 | **470.02** | 3,655.85 |
| Lead-Time Demand - Demand Only | **Smooth** | 108.01 | 143.92 | 86.47 | **84.89** | 6,331.13 |
| | **Irregular** | 252.33 | 315.84 | **194.18** | 197.18 | 5,144.60 |
| | **Slow-moving** | 133.00 | 264.61 | **92.26** | 97.33 | 2,815.01 |
| | **Mildly Erratic** | 354.73 | 611.64 | **258.42** | 271.93 | 2,854.26 |
| | **Highly Erratic** | 295.73 | 494.28 | **214.85** | 225.93 | 2,959.48 |

Although the approximation method provides the best results for the smooth and irregular demand patterns when comparing with the one-period ahead demand, Croston's method provides the best results for the two erratic demand patterns, and the revised Croston's method is best once again for the slow-moving demand. The approximation method still provides the best results for all five demand patterns when comparing the lead-time demand in all periods, while it is only with smooth demand that this method provides the best result with the lead-time demand comparison in periods of demand only. The rarely top-performing bias reduction method provides the best results for the remaining four demand patterns under this situation.

In comparing the erratic demand forecasting methods with the traditional methods under weekly data, Croston's method, the revised Croston's method and the approximation method between them provide the best overall results when considering with the one-period ahead demand. However, in the case of lead-time demand compared at all periods, it is ES that maintains the best results across all demand patterns. When it comes to comparing lead-time demand in periods of demand only, the approximation method, along with the bias reduction method, provide the best results.

Confirmation of the forecasting performance of the variations on Croston's method is sought through an examination of the median absolute percentage error (MdAPE) by demand pattern, as presented in Appendix I. The MdAPE results for the most part mirror the corresponding MAPE results, with some exceptions. The revised Croston's method provides the best MdAPE result for the smooth demand pattern when comparing the one-period ahead demand using monthly data, whereas the approximation method provides the best MAPE result. In addition, the revised Croston's method provides the best MdAPE results for all demand patterns when comparing the one-period ahead demand using weekly data rather than just for slow-moving demand, as was the case with MAPE.

In comparing the MdAPE results for the variations on Croston's method with those of the more traditional methods, previously presented in Appendix F, it is observed that, when considering the one-period ahead demand, the approximation method and the revised Croston's method alternately provide the majority of the best results. However, ES provides the best MdAPE results for the two erratic demand patterns with quarterly data. ES is consistently the best method when comparing lead-time demand in all periods by providing the best results for each of the smooth, slow-moving and two erratic demand patterns for all aggregation periods. The moving average method provides the best results for the irregular demand pattern. When it comes to comparing lead-time demand in periods of positive demand only, the approximation method provides the best results for quarterly and monthly aggregations for all demand patterns, and for the smooth demand pattern for weekly data. The bias reduction method provides the best MdAPE results for the remaining demand patterns for weekly data.

This analysis has examined whether particular forecasting methods are suitable when demand follows a specified pattern. It was observed that the approximation method for the most part was the best of the methods put forward as suitable for erratic demand and consistently bettered the performance of Croston's method. Overall, however, it could be argued that it is the manner in which the forecasts are compared with the actual demand that, for the most part, determines which method performs the best. ES clearly dominates the results when the lead-time demand is compared at all periods and the approximation method similarly dominates the results when the lead-time demand is compared at periods of demand only. In the case of the one-period ahead demand, it would appear that the demand pattern bears greater influence, with the approximation method best for smooth and irregular demand, the revised method best for the slow-moving demand, and either ES or Croston's method best for the erratic demand.

Further comments on the performance of the selected forecasting methods when faced by particular demand patterns will be put forward in the concluding remarks to this chapter, but first it is worthwhile to examine the effect of autocorrelation on forecasting performance and also re-examine the effect of the smoothing constant by considering the impact for each demand pattern.

## 8.3 Effect of Autocorrelation and Crosscorrelation

This section examines the effect of autocorrelation and crosscorrelation on the forecasting performance of two of the previously considered forecasting methods, namely ES and Croston's method. Although most research on erratic demand assumes independence between successive demand intervals and successive demand sizes, it was observed in Section 4.5.6 that up to a quarter of the demand profiles are significantly autocorrelated and/or crosscorrelated. The analysis undertaken in this section determines whether the forecasting performance is affected by the absence of such independence.

Autocorrelation amongst a particular demand pattern indicates that peaks of demand may last for more than one time period and it is generally expected that such an effect would improve the forecasting performance. The presence of significant autocorrelation in demand data has been tested for, with classifications of nil, positive or negative given to each of the 18,750 sample line items. MAPE and MdAPE results for the line items within these autocorrelation classifications for each of demand size autocorrelation, transaction interval autocorrelation and size and interval crosscorrelation are presented in Appendix J. In each case the results are given for the one-period ahead demand, the lead-time demand compared at all periods, as well as in periods of demand only, for quarterly, monthly and weekly demand aggregations for both ES and Croston's method. On the whole, the MAPE results and the MdAPE results are in agreement as to which forecasting method is best for each autocorrelation classification and therefore the rest of the comments and analysis in this section only refer to MAPE.

It is observed that the forecasting method with the best MAPE result under each autocorrelation classification in all cases precisely matches the method with the best corresponding overall MAPE result, as presented in Appendix E. For example, ES has a better overall MAPE result when comparing the one-period ahead demand for quarterly data, and similarly the corresponding ES has the best result for MAPE regardless of whether there is nil, negative or positive autocorrelation in any of the demand size, transaction interval or size and interval. In this manner it is observed that the two forecasting methods do not change their comparative rankings with respect to the presence or absence of autocorrelation. This leaves the sole consideration as to whether the individual performance of either method is affected by autocorrelation.

An assessment of whether autocorrelation has an effect on forecasting performance is provided by an analysis of variance (ANOVA) test. As previously described in Section 5.2.1, ANOVA tests the hypothesis that the means of three or more populations are equal. In this instance the test is whether the mean of the selected performance measure is the same for nil, negative or positive autocorrelation. An illustration of the ANOVA test now follows.

Table 8.4 provides sample MAPE results, taken from Appendix J, for the one-period ahead demand comparison using quarterly data, with consideration given to autocorrelation in the demand size. Some of the 18,750 line items are lost as the correlations were significant as a whole although, with no individually significant correlations, *no sign* could be given. It is observed that for both ES and Croston's method the lowest average MAPE occurs when there is negative autocorrelation (shown in bold type) and the highest average MAPE occurs when there is positive autocorrelation. However, in all cases the standard deviation of the MAPE values are extraordinarily large, indicating considerable overlap between each autocorrelation classification.

**Table 8.4: Sample Demand Size Autocorrelation Comparison.**

| Statistic | Autocorrelation Classification | | |
|---|---|---|---|
| | Negative | Nil | Positive |
| **Sample Size (*n*)** | 993 | 15,730 | 1,359 |
| **Exponential Smoothing** | | | |
| **- Average MAPE** | **110.90** | 116.94 | 125.99 |
| **- Standard Deviation of MAPE** | 243.81 | 260.89 | 227.65 |
| **Croston's Method** | | | |
| **- Average MAPE** | **117.13** | 126.75 | 135.20 |
| **- Standard Deviation of MAPE** | 264.79 | 325.57 | 254.03 |

The null hypothesis under examination is that the three average MAPEs under each forecasting method are not significantly different. The SAS GLM (General Linear Models) procedure is used as the different number of observations in each category leads to unbalanced data. Results from an ANOVA test, as presented in Table 8.5, indicate the calculated $F$ values of 1.10 and 0.94 for ES and Croston's Method respectively, are both less than the tabulated values at the 1 percent and 5 percent significance levels. Thus, the null hypothesis is not rejected and the average MAPEs are the same for each autocorrelation classification within each forecasting method.

**Table 8.5: Sample Demand Size Autocorrelation ANOVA Test.**

| Forecasting Method | Test Statistics | | | $F_{0.01}$ Value | $F_{0.05}$ Value | Model Significant |
|---|---|---|---|---|---|---|
| | *F* Ratio | $df_n$ | $df_d$ | | | |
| **Exponential Smoothing** | 1.10 | 2 | 18,079 | 4.61 | 3.00 | No |
| **Croston's Method** | 0.94 | 2 | 18,079 | 4.61 | 3.00 | No |

Conducting ANOVA tests in this manner, for all MAPE comparisons with quarterly, monthly and weekly data, provides a series of $F$ values as presented in Table 8.6, where it is seen that the values range between 0.06 and 3.63. With no values greater than the tabulated value of 4.61 at the 1 percent significance level, the null hypothesis is not rejected. However, two values (shown in bold type) are greater than the tabulated value of 3.00 at the 5 percent significance level, both of which occur with ES with quarterly data.

**Table 8.6: Autocorrelation ANOVA Results - *F* Values.**

| Demand Aggreg'n | One-Period Ahead Demand - All Periods | | Lead-Time Demand - All Periods | | Lead-Time Demand - Demand Only | |
|---|---|---|---|---|---|---|
| | Expon'l Smooth. | Croston's Method | Expon'l Smooth. | Croston's Method | Expon'l Smooth. | Croston's Method |
| **Demand Size Autocorrelation** | | | | | | |
| **Quarterly** | 1.10 | 0.94 | **3.63** | 0.40 | 2.09 | 1.79 |
| **Monthly** | 1.44 | 1.45 | 0.26 | 0.19 | 1.36 | 0.81 |
| **Weekly** | 0.81 | 1.00 | 0.11 | 0.12 | 1.46 | 0.84 |
| **Transaction Interval Autocorrelation** | | | | | | |
| **Quarterly** | 1.72 | 2.36 | **3.60** | 1.01 | 0.86 | 2.35 |
| **Monthly** | 1.52 | 1.24 | 0.97 | 0.78 | 0.52 | 0.35 |
| **Weekly** | 1.83 | 2.96 | 0.73 | 0.91 | 1.06 | 0.14 |
| **Size and Interval Crosscorrelation** | | | | | | |
| **Quarterly** | 0.31 | 1.05 | 1.32 | 0.43 | 0.88 | 1.58 |
| **Monthly** | 1.79 | 0.33 | 0.77 | 0.65 | 2.54 | 2.36 |
| **Weekly** | 0.43 | 0.06 | 0.43 | 0.63 | 1.44 | 1.37 |

In determining whether autocorrelation affects forecasting performance the results are somewhat inconclusive. With only two significant $F$ values at the 5 percent level there is only a marginal suggestion that autocorrelation has an effect. There is so much variation in the MAPE values across the autocorrelation classifications that it is not possible to conclude there is an effect using ANOVA.

Perhaps what is more compelling is that when considering autocorrelation in the transaction interval, in 15 out of 18 cases for both ES and Croston's method across all demand aggregations and all forecast comparisons, the lowest MAPE occurs when negative autocorrelation has been identified (from Table J.2, Appendix J). On the other hand, in 12 out of 18 cases, when considering demand size autocorrelation, the lowest MAPE occurs when there is no autocorrelation (from Table J.1). When considering size and interval crosscorrelation the lowest MAPE similarly occurs in 10 out of 18 cases when there is no crosscorrelation (Table J.3).

Overall, any effect of autocorrelation is difficult to quantify. The presence of statistically significant autocorrelation can itself be difficult to identify with the methods presented in Appendix C not being in full agreement. Furthermore, with no clear distinction as to what constitutes negative, positive or nil autocorrelation for a line item there is a danger that miss-classification occurs, leading to contamination of the results. If autocorrelation is present, it would suggest forecast improvements could be obtained by introducing model terms which take advantage of this aspect of the data. As it stands, however, autocorrelation is difficult to identify and its effect is mostly unknown.

## 8.4 Effect of Smoothing Parameters

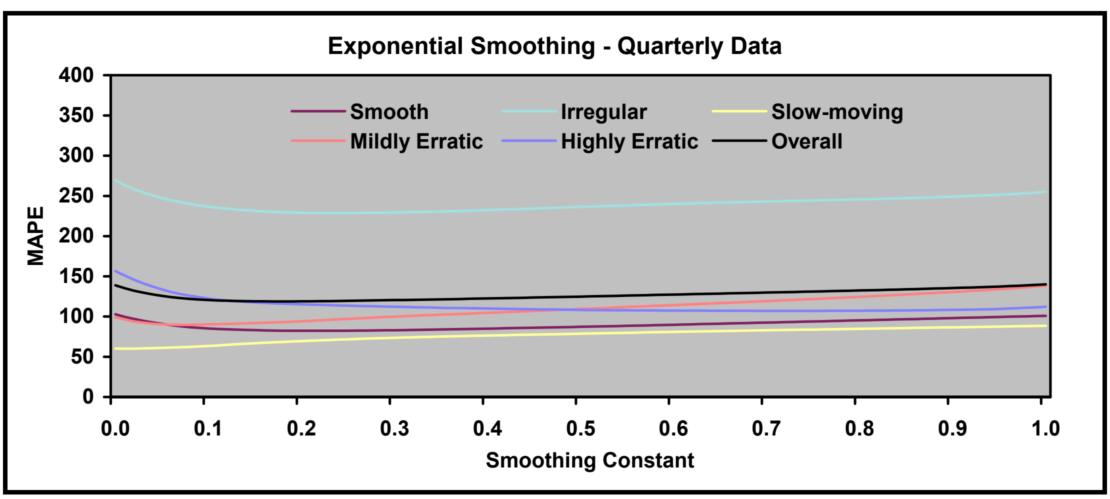
Consideration is given to the effect of smoothing parameters on forecasting performance by comparing optimal values across the demand patterns. Forecasting results for ES and

Croston's method presented previously were produced using optimal smoothing parameters from a hold-out sample of 500 line items. Although the sample was composed of equally represented demand patterns, the optimal values were produced for the group as a whole and did not take the demand patterns into consideration. This section explores the effect of the smoothing parameters by comparing the optimal values across the demand patterns, firstly for ES and then for Croston's method.

### 8.4.1 Smoothing Parameters by Demand Pattern - Exponential Smoothing

As an illustrative example, the forecasting performance by demand pattern, when comparing the one-period ahead demand using quarterly data, is presented in Figure 8.1, where MAPE has been recorded for increments of 0.01 in the smoothing constant. The overall MAPE from the full set of observations is also shown.

**Figure 8.1: Effect of Smoothing Constant - One-Period Ahead (All Periods).**




It is observed that the irregular demand pattern produces the highest MAPE in general and the slow-moving demand produces the lowest. In addition, the optimal smoothing constant for the slow-moving demand pattern is close to zero (0.01), while the optimal smoothing constant for the highly erratic demand pattern (0.74) is substantially further along the axis.

In a similar manner, Figure 8.2 presents the resultant MAPEs when lead-time demand is compared at all periods. In this instance it is the highly erratic demand pattern that produces the highest MAPE overall, while the smooth demand pattern produces the lowest. Once again, the slow-moving demand pattern has the lowest value for the optimal smoothing constant, while the mildly erratic demand pattern has the highest.

**Figure 8.2: Effect of Smoothing Constant - Lead-Time Demand (All Periods).**

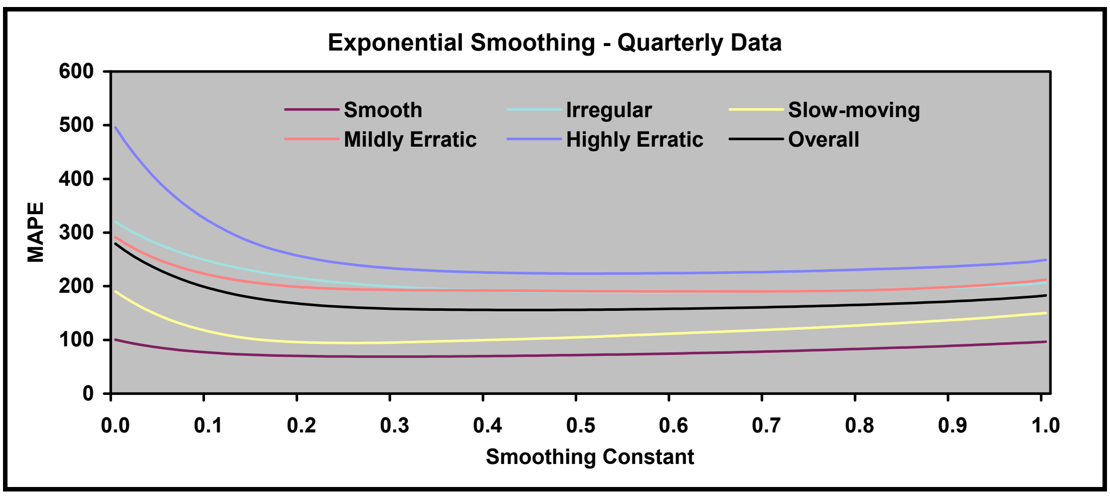
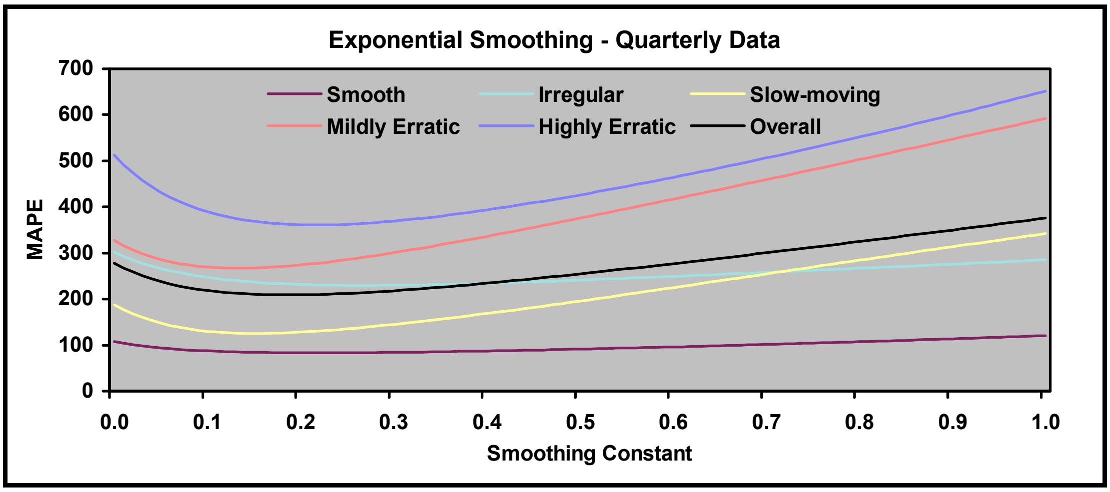



Figure 8.3 presents comparative MAPEs by demand pattern when the lead-time demand is compared in periods of demand only. Once again, the highly erratic demand pattern produces the highest MAPE and the smooth demand pattern produces the lowest. The optimal smoothing constant values are more similar in this case, ranging from 0.13 for the mildly erratic demand pattern to 0.27 for the irregular demand pattern.

**Figure 8.3: Effect of Smoothing Constant - Lead-Time Demand (Demand Only).**

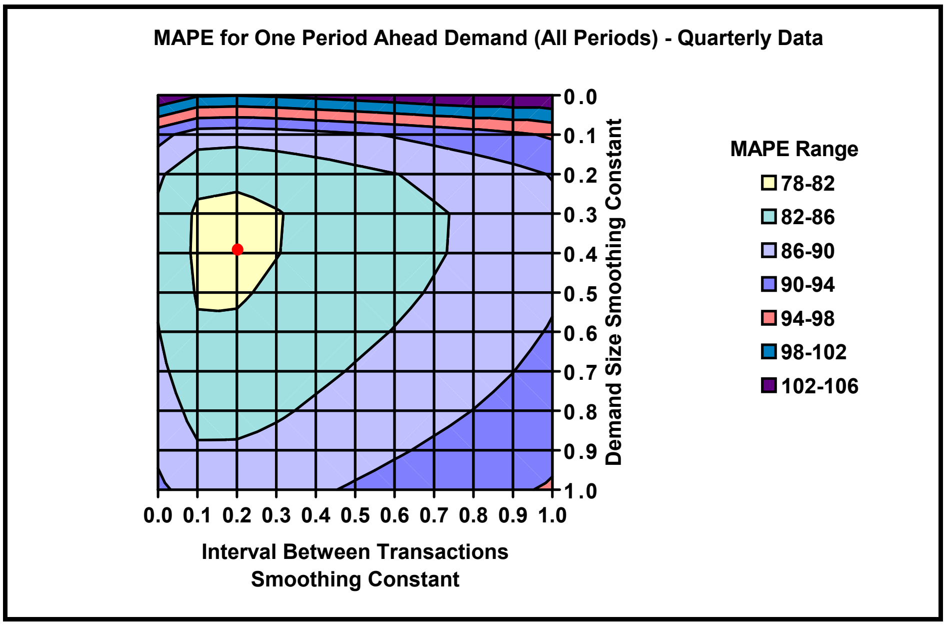



The optimal smoothing constant values by demand pattern are presented in Appendix K, where the resultant MAPEs are compared with those derived from optimal values as a whole. It is observed that the overall MAPE improves when the smoothing constants are allowed to vary by demand pattern, with the greatest improvements occurring with quarterly data, and particularly in the case of slow-moving demand. In general, the optimal values for the slow-moving demand are the lowest and it is this demand pattern that experiences the greatest benefit when the smoothing constants are allowed to vary.

### 8.4.2 Smoothing Parameters by Demand Pattern - Croston's Method

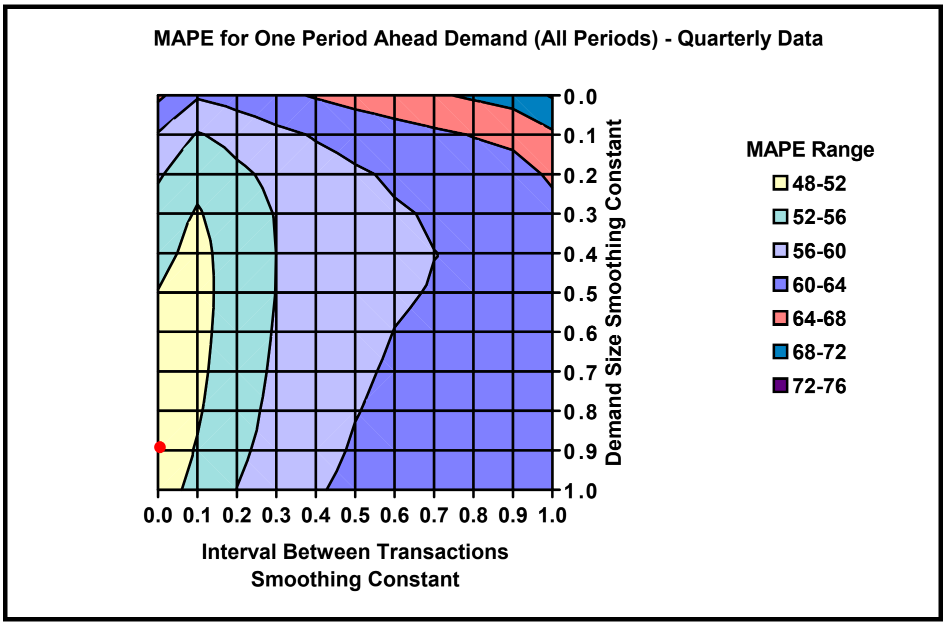
The two-dimensional surface maps presented in the following figures illustrate the variation in MAPE that occurs between demand patterns when the smoothing constants are allowed to vary under Croston's method. Figure 8.4 illustrates the results for the smooth demand pattern, with the red dot indicating the minimum MAPE. The optimal smoothing constants for both the demand size and the interval between transactions have moderate values of 0.4 and 0.2 respectively.

**Figure 8.4: Effect of Smoothing Constants - Croston's Method (Smooth Demand).**



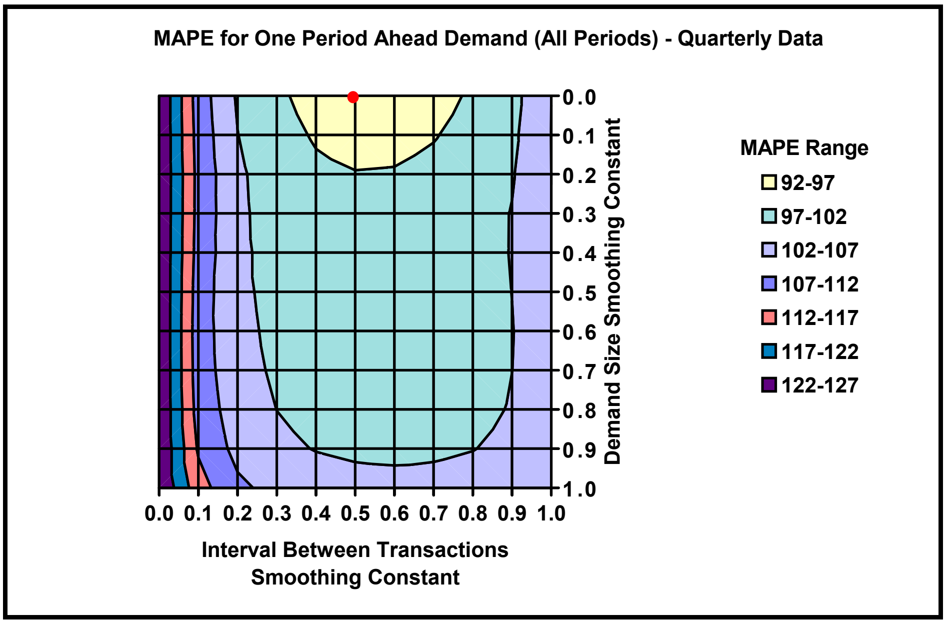

A different pattern arises for slow-moving demand, as illustrated in Figure 8.5. Here the optimal value for the demand size smoothing constant is very high at 0.9 while the optimal value for the interval between transactions is very low at 0.0.

**Figure 8.5: Effect of Smoothing Const. - Croston's Method (Slow-Moving Demand).**



A further variation occurs with the mildly erratic demand as illustrated in Figure 8.6.

**Figure 8.6: Effect of Smoothing Const. - Croston's (Mildly Erratic Demand).**




Optimal smoothing constant values by demand pattern for Croston's method are also presented in Appendix K. In this case the greatest overall improvements are not necessarily experienced with quarterly data, but rather that quarterly data improves the most when considering one-period ahead demand, monthly data improves the most when considering lead-time demand in all periods, and weekly data improves the most when considering lead-time demand in periods of demand only.

For the most part, the slow-moving demand pattern experiences the greatest improvement when considering the one-period ahead demand and the smooth demand pattern experiences the greatest improvement when considering the lead-time demand, both in all periods and in periods of demand only.

## 8.5 Concluding Remarks

A mathematical error in the manner in which Croston calculates the expected demand per period contributes to his forecasting method providing only modest improvements in performance over simpler methods. Croston's method reduces the forecast bias of exponential smoothing, although it does not completely eliminate it. Consequently, modifications to Croston's method have themselves recently been put forward in the literature as alternatives. Three modified methods, identified as the revised Croston's method, the bias reduction method and the approximation method, have been developed in order to remove the bias.

Of the proposed alternatives to Croston's method, the approximation method was observed to provide the best results overall and consistently bettered the results of the original method. The bias reduction method also betters Croston's method in most cases, albeit to a lesser extent than the approximation method, while the revised Croston's method rarely performs better than the original.

Less sophisticated forecasting methods were still found to provide the best results under specific conditions, as shown by the summary results for MAPE presented in Table 8.7. It is observed that the approximation method is the best forecasting method overall when considering the one-period ahead demand, although other methods provide the best results for various individual demand patterns. ES still very much dominates the results when comparing against lead-time demand in all periods, while the approximation method is best for quarterly and monthly data when comparing against lead-time demand in periods of demand only, and the bias reduction method is best for weekly data under this situation.

**Table 8.7: Best Forecasting Method by Demand Pattern (Using MAPE).**

| Demand Aggregation | Demand Pattern | Type of Forecast | | | | | |
|---|---|---|---|---|---|---|---|
| | | One-Period Ahead Demand - All Periods | | Lead-Time Demand - All Periods | | Lead-Time Demand - Demand Only | |
| | | Method | MAPE | Method | MAPE | Method | MAPE |
| Quarterly | Smooth | Approx | 75.81 | Approx | 77.30 | Approx | 70.67 |
| | Irregular | Approx | 173.52 | Approx | 196.66 | Approx | 151.19 |
| | Slow-moving | Revised | 53.34 | ES | 109.92 | Approx | 80.08 |
| | Mildly Erratic | MA | 111.83 | ES | 232.93 | Approx | 215.89 |
| | Highly Erratic | ES | 111.34 | MA | 239.24 | Approx | 215.42 |
| | **Overall** | **Approx** | **110.68** | **ES** | **173.29** | **Approx** | **144.40** |
| Monthly | Smooth | Approx | 91.58 | MA | 94.26 | Approx | 84.01 |
| | Irregular | Approx | 135.71 | ES | 206.49 | Approx | 181.05 |
| | Slow-moving | Revised | 67.75 | ES | 110.89 | Approx | 98.61 |
| | Mildly Erratic | ES | 90.90 | ES | 239.64 | Approx | 269.52 |
| | Highly Erratic | Approx | 85.99 | ES | 222.21 | Approx | 239.37 |
| | **Overall** | **Approx** | **95.94** | **ES** | **174.76** | **Approx** | **172.05** |
| Weekly | Smooth | Approx | 93.93 | ES | 93.14 | Approx | 84.89 |
| | Irregular | Approx | 93.50 | ES | 209.06 | Bias Red | 194.18 |
| | Slow-moving | Revised | 89.16 | ES | 106.38 | Bias Red | 92.26 |
| | Mildly Erratic | Croston’s | 88.47 | ES | 232.31 | Bias Red | 258.42 |
| | Highly Erratic | Croston’s | 85.57 | ES | 215.55 | Bias Red | 214.85 |
| | **Overall** | **Approx** | **90.32** | **ES** | **171.29** | **Bias Red** | **168.03** |

Comparing the performance of forecasting methods through the use of measures of accuracy, as demonstrated in this chapter, is not considered ideal. The measures themselves are open to questions of validity and different conclusions arise depending on which measure is utilised. An alternative method for assessing forecasting performance is to compare the implied stock reprovisioning performance resulting from each method, as suggested in the next chapter.

# 9. FORECAST PERFORMANCE BY IMPLIED STOCK-HOLDING

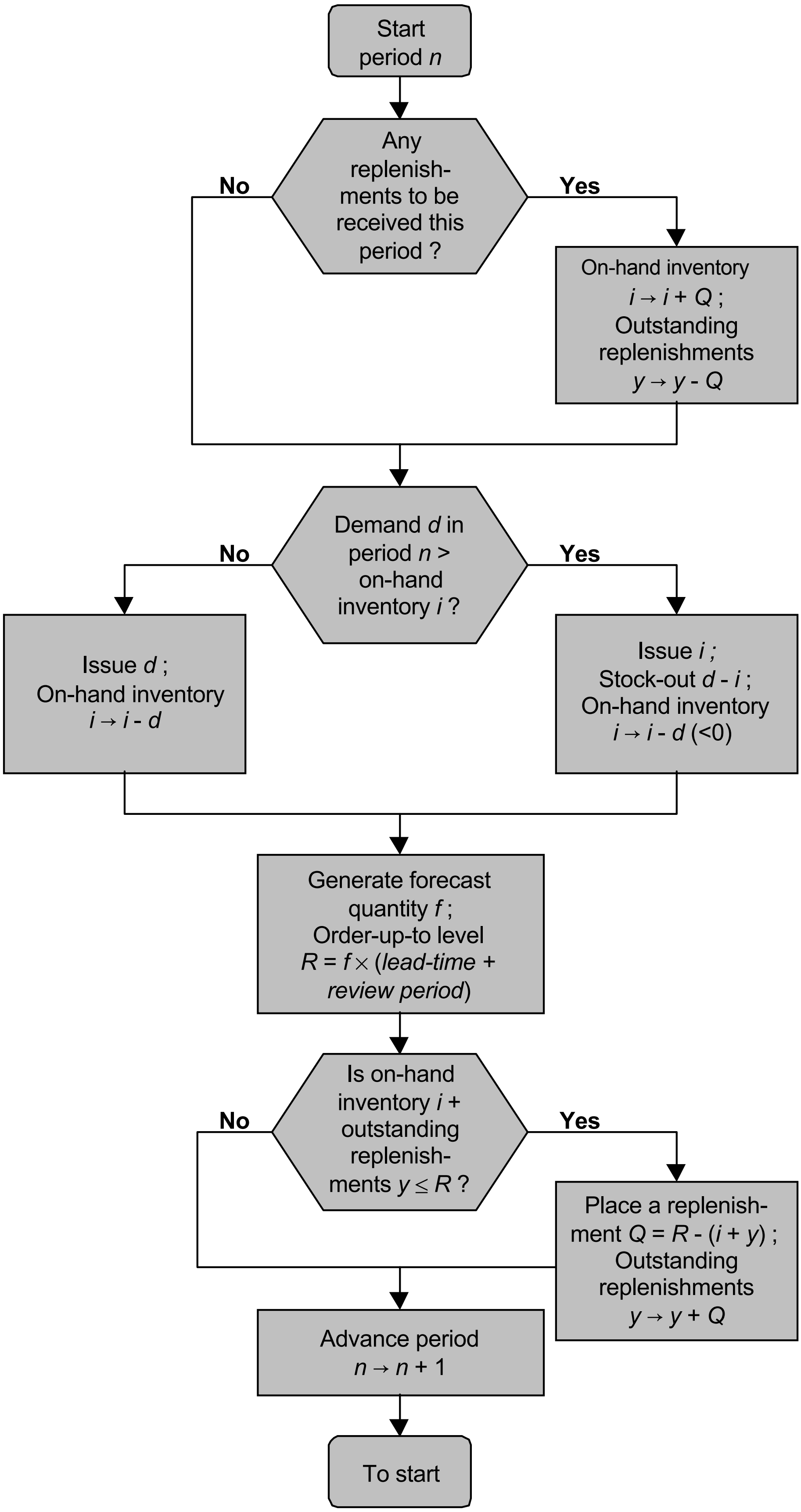
A practical solution to problems arising from using the measures of forecasting accuracy, such as MAPE and MdAPE, is to measure the forecast performance according to the implied stock-holdings. In this chapter, stock reprovisioning performance is monitored for each forecasting method using an extension to the FORESTOC model. This extension, based on a method employed by Wemmerlöv [84], allows a comparison of the implied stock-holdings by calculating the exact safety margin that provides a maximum stock-out quantity of zero. The safety margin is calculated by iteratively adding the maximum stock-out quantity to the order-up-to level until no further stock-outs occur. In this manner the average stock-holdings for each method can be calculated and compared using a common service level of 100 percent.

## 9.1 Forecasting and Stock Reprovisioning

Stock-outs naturally occur when lead-times are non-zero and forecasts are incorrect. In considering the impact of forecast errors on the performance of a reprovisioning system, Wemmerlöv observes that, as the errors increase in size, greater stock-outs occur and service levels decline, while inventories increase and reprovisioning activities also increase. This leads to a paradoxical result where the service level declines while, at the same time, the average inventory increases. In order to avoid estimating the cost of stock-outs, Wemmerlöv introduced safety stocks so that the service levels for all reprovisioning methods were set at 100 percent. The performance of the methods was compared in terms of the sum of the resultant holding and ordering costs only.

A methodology similar to Wemmerlöv's is utilised in this research, ultimately using the previous sample of 18,750 RAF line items. An inventory system with continuous review is simulated in this chapter based on the flow diagram presented in Figure 9.1.

**Figure 9.1: Flow Diagram for Inventory System.**



Any stock replenishments arrive at the start of a period, and are therefore available to satisfy demand during that period. A demand will be satisfied in full if there is sufficient stock on-hand, otherwise the demand will be partially fulfilled with the remainder back-ordered. Forecasted demand per period is calculated at the end of a period and an order-up-to level $R$ is derived as the product of the forecast and lead-time, plus the review period rounded up to a whole unit. A replenishment quantity is ordered if the closing inventory position, including any outstanding orders, is at or below the order-up-to level. Such a replenishment quantity is calculated as the difference between $R$ and the closing inventory level, plus any outstanding orders. The replenishment quantity is rounded up to become a multiple of the primary packaged quantity (PPQ) and if this quantity is less than the contractor's minimum batch quantity (CMBQ) then no order is placed.

A simulation of a periodic review inventory system, with a review every period, is illustrated using a sample line item in Table 9.1. In this instance, the system utilises an exponential smoothing (ES) forecasting model using a quarterly demand series, where the smoothing constant takes the optimal value (0.18) for comparisons with the one-period ahead demand from Table 7.5. System parameters are initialised using values from the first four quarters or one complete year. The simulation starts with the current stock and any outstanding orders are assumed to arrive in the first simulation period for ease of modelling, while any subsequent orders are delivered in a full lead-time.

With one complete year used for initialising the model, the simulation starts in quarter 5 with an opening stock balance of 7 units and a delivery of 2 units. The minimum size and multiple for the replenishment are both set at one unit as specified by the CMBQ and the PPQ respectively.

**Table 9.1: Example Simulation of a Periodic Review Inventory System.**

| Qtr<br>(*n*) | Stock On-hand Open | Delivery Qty<br>(*Q*) | Demand Qty<br>(*d*) | Stock On-hand Close<br>(*i*) | Forecast Qty<br>(*f*) | Stock On Order<br>(*y*) | Order-up-to Level<br>(*R*) | Order Qty |
|---|---|---|---|---|---|---|---|---|
| 1 | | | 3 | | | | | |
| 2 | | | 3 | | | | | |
| 3 | | | 2 | | | | | |
| 4 | | | 4 | 7 | 3.000 | 2 | | |
| 5 | 7 | 2 | 1 | 8 | 2.640 | | 12 | 3 |
| 6 | 8 | | 1 | 7 | 2.345 | 3 | 11 | |
| 7 | 7 | | 2 | 5 | 2.283 | 3 | 10 | |
| 8 | 5 | 3 | | 8 | 1.872 | | 10 | 2 |
| 9 | 8 | | | 8 | 1.535 | 2 | 8 | |
| 10 | 8 | | 3 | 5 | 1.799 | 2 | 7 | |
| 11 | 5 | 2 | 3 | 4 | 2.015 | | 8 | 1 |
| 12 | 4 | | 6 | -2 | 2.732 | 1 | 9 | 4 |
| 13 | -2 | | 4 | -6 | 2.960 | 5 | 11 | 8 |
| 14 | -6 | 1 | 1 | -6 | 2.608 | 12 | 12 | 5 |
| 15 | -6 | 4 | | -2 | 2.138 | 13 | 11 | |
| 16 | -2 | 8 | 3 | 3 | 2.293 | 5 | 9 | |
| 17 | 3 | 5 | 1 | 7 | 2.061 | | 10 | 2 |
| 18 | 7 | | 3 | 4 | 2.230 | 2 | 9 | |
| 19 | 4 | | | 4 | 1.828 | 2 | 9 | 3 |
| 20 | 4 | 2 | 1 | 5 | 1.679 | 3 | 8 | |
| 21 | 5 | | 1 | 4 | 1.557 | 3 | 7 | |
| 22 | 4 | 3 | 3 | 4 | 1.817 | | 7 | |
| 23 | 4 | | | 4 | 1.490 | | 8 | 4 |
| 24 | 4 | | 1 | 3 | 1.402 | 4 | 6 | |

As a delivery is assumed to occur at the start of a period, any stock replenishments are available to satisfy demand during that period, while the forecast is updated at the end of the period, after the demand. The order-up-to level $R$ is calculated as the forecast quantity at the end of the previous period multiplied by the lead-time of 3 quarters, plus the review period of one quarter in this instance. Any replenishment quantity is calculated as $R$ minus the sum of the opening stock, any deliveries and stock on order.

In the example, the delivery of the initial outstanding order of 2 units occurs at the start of quarter 5, in effect making it part of the opening stock balance. Demand is satisfied from stock with a new order for 3 units placed immediately, and this is subsequently delivered after a full lead-time in quarter 8. New orders are placed when the opening stock level, including outstanding orders, drops below the order-up-to level generated from the forecast quantity. The first demand that cannot be satisfied from stock on-hand occurs in quarter 12. At this time the stockout is back-ordered until delivery of a replenishment order is made. At the completion of the simulation the minimum *closing stock level* is -6 units in quarters 13 and 14 jointly, thus the maximum stock-out is 6 units.

The methodology employed in this chapter for comparing the stock reprovisioning performance for each forecasting method, iteratively adds this minimum closing stock level to the order-up-to level until no stock-outs occur. The implied stock-holdings are calculated as the average of the opening stock plus deliveries and the closing stock.

In the analysis that follows Wemmerlöv's methodology is used as an alternative means of assessing forecast accuracy. The usage of such a methodology is deemed suitable in an inventory context in order to alleviate problems which arise with more traditional measures, including MAPE and MdAPE. The method directly provides a key performance measure, namely an assessment of the implied stock-holdings which arise through using various forecasting methods. As it stands, the methodology is not prone to problems in deciding whether to forecast one-period ahead or over a lead-time, or whether to measure in all periods or only after a demand occurs, and likewise there are no problems in making a calculation when the actual demand is zero.

The methodology as implemented does not give guidance on stocks to hold. The backward-looking simulation requires perfect information and is therefore not put forward as a model of reality. Service levels of 100 percent are nigh on impossible to achieve although in this instance they are utilised in order to give an impartial means of comparing forecast accuracy. A major disadvantage of this methodology is the additional processing that is required to obtain the results. After calculating forecast values, an average of ten iterations are required for determining the safety stock levels at which no stock-outs occur.

The next section illustrates the methodology by comparing the implied performance of ES with Croston's method using the example simulation. The principle behind the methodology is to provide an alternative forecasting measure which can be utilised in an inventory context.

## 9.2 Inventory Management

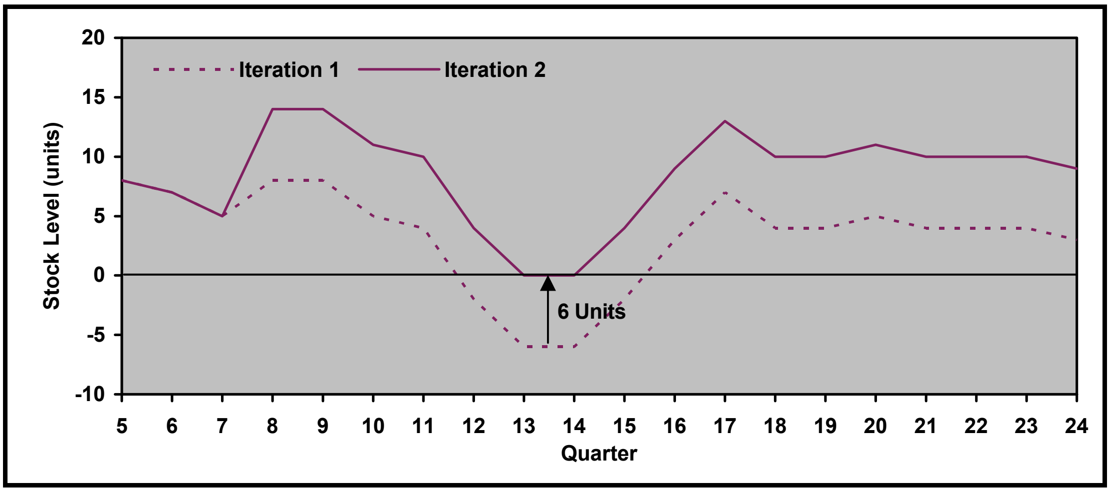
Continuing with the example line item from the previous section, an analysis of the reprovisioning performance is illustrated in the following two tables. Table 9.2 presents calculations for an ES forecasting model, while Table 9.3 presents comparable calculations for Croston's method. In each case the inventory system is modelled according to the methodology introduced previously, although the tables only show the principal calculations, firstly for Iteration 1, where there is no safety stock, and secondly for Iteration 2, where safety stock is added to ensure no stock-outs.

**Table 9.2: Inventory Management - Exponential Smoothing (Quarterly Data).**

| One-Period Ahead Forecast | | | Iteration 1 (No Safety Stock) | | | | Iteration 2 (With Safety Stock) | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Qtr | Demand Qty | Forecast Qty | Closing Stock | *R* | Order Qty | Delivery Qty | Closing Stock | *R* | Order Qty | Delivery Qty |
| 5 | 1 | 2.640 | 8 | 12 | 3 | 2 | 8 | 18 | 9 | 2 |
| 6 | 1 | 2.345 | 7 | 11 | | | 7 | 17 | | |
| 7 | 2 | 2.283 | 5 | 10 | | | 5 | 16 | | |
| 8 | | 1.872 | 8 | 10 | 2 | 3 | 14 | 16 | 2 | 9 |
| 9 | | 1.535 | 8 | 8 | | | 14 | 14 | | |
| 10 | 3 | 1.799 | 5 | 7 | | | 11 | 13 | | |
| 11 | 3 | 2.015 | 4 | 8 | 1 | 2 | 10 | 14 | 1 | 2 |
| 12 | 6 | 2.732 | -2 | 9 | 4 | | 4 | 15 | 4 | |
| 13 | 4 | 2.960 | -6 | 11 | 8 | | 0 | 17 | 8 | |
| 14 | 1 | 2.608 | -6 | 12 | 5 | 1 | 0 | 18 | 5 | 1 |
| 15 | | 2.138 | -2 | 11 | | 4 | 4 | 17 | | 4 |
| 16 | 3 | 2.293 | 3 | 9 | | 8 | 9 | 15 | | 8 |
| 17 | 1 | 2.061 | 7 | 10 | 2 | 5 | 13 | 16 | 2 | 5 |
| 18 | 3 | 2.230 | 4 | 9 | | | 10 | 15 | | |
| 19 | | 1.828 | 4 | 9 | 3 | | 10 | 15 | 3 | |
| 20 | 1 | 1.679 | 5 | 8 | | 2 | 11 | 14 | | 2 |
| 21 | 1 | 1.557 | 4 | 7 | | | 10 | 13 | | |
| 22 | 3 | 1.817 | 4 | 7 | | 3 | 10 | 13 | | 3 |
| 23 | | 1.490 | 4 | 8 | 4 | | 10 | 14 | 4 | |
| 24 | 1 | 1.402 | 3 | 6 | | | 9 | 12 | | |
| **Minimum Stock** | | | **-6** | | | | **0** | | | |
| **Total Order Quantity** | | | | | **32** | **30** | | | **38** | **36** |
| **Number of Orders** | | | | | **9** | | | | **9** | |

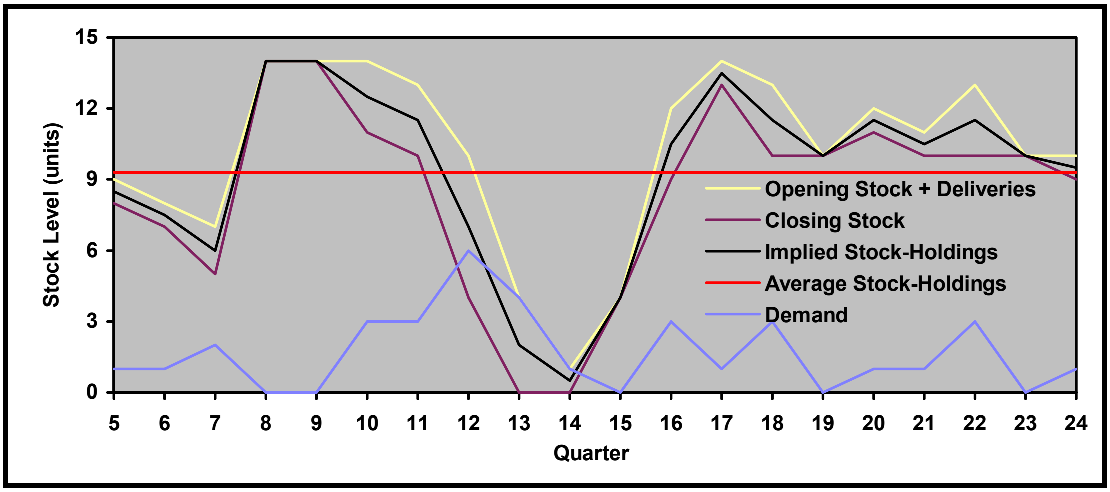
Under Iteration 1 in Table 9.2, as previously determined, the minimum closing stock level is -6 units, thus the maximum stock-out is 6 units. The order-up-to level is increased to include a safety stock margin of 6 units in Iteration 2. In effect, this leads to a larger initial order than was previously the case and the later stock-out is averted. This situation is illustrated by Figure 9.2, which compares the closing stock levels for the two iterations. No stock-outs occur after the second iteration, leading to a service level of 100 percent.

**Figure 9.2: Closing Stock-Holdings - Exponential Smoothing.**




Implied stock-holdings are calculated as the average of the opening stock plus deliveries and the closing stock. The average stock-holdings for the sample line item using ES is 9.30 units, as illustrated by the horizontal line in Figure 9.3.

**Figure 9.3: Average Stock-Holdings - Exponential Smoothing.**




The implied stock-holdings are calculated as the average of the opening stock plus deliveries and the closing stock, and the closing stock itself is determined as the opening stock plus deliveries minus the demand for each period. Thus, in periods where demand is equal to zero, the closing stock equals the opening stock plus deliveries, so the stock-holding is simply the closing stock. Average stock-holdings are taken as the overall average across all periods.

**Table 9.3: Inventory Management - Croston's Method (Quarterly Data).**

| One-Period Ahead Forecast | | | Iteration 1 (No Safety Stock) | | | | Iteration 2 (With Safety Stock) | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Qtr | Demand Qty | Forecast Qty | Closing Stock | *R* | Order Qty | Delivery Qty | Closing Stock | *R* | Order Qty | Delivery Qty |
| 5 | 1 | 2.220 | 8 | 12 | 3 | 2 | 8 | 20 | 11 | 2 |
| 6 | 1 | 1.744 | 7 | 9 | | | 7 | 17 | | |
| 7 | 2 | 1.844 | 5 | 7 | | | 5 | 15 | | |
| 8 | | 1.844 | 8 | 8 | | 3 | 16 | 16 | | 11 |
| 9 | | 1.844 | 8 | 8 | | | 16 | 16 | | |
| 10 | 3 | 1.471 | 5 | 8 | | | 13 | 16 | | |
| 11 | 3 | 1.831 | 2 | 6 | 1 | | 10 | 14 | 1 | |
| 12 | 6 | 3.028 | -4 | 8 | 5 | | 4 | 16 | 5 | |
| 13 | 4 | 3.262 | -8 | 13 | 11 | | 0 | 21 | 11 | |
| 14 | 1 | 2.430 | -8 | 14 | 5 | 1 | 0 | 22 | 5 | 1 |
| 15 | | 2.430 | -3 | 10 | | 5 | 5 | 18 | | 5 |
| 16 | 3 | 2.071 | 5 | 10 | | 11 | 13 | 18 | | 11 |
| 17 | 1 | 1.675 | 9 | 9 | | 5 | 17 | 17 | | 5 |
| 18 | 3 | 2.063 | 6 | 7 | | | 14 | 15 | | |
| 19 | | 2.063 | 6 | 9 | 3 | | 14 | 17 | 3 | |
| 20 | 1 | 1.334 | 5 | 9 | | | 13 | 17 | | |
| 21 | 1 | 1.187 | 4 | 6 | | | 12 | 14 | | |
| 22 | 3 | 1.734 | 4 | 5 | | 3 | 12 | 13 | | 3 |
| 23 | | 1.734 | 4 | 7 | 3 | | 12 | 15 | 3 | |
| 24 | 1 | 1.168 | 3 | 7 | | | 11 | 15 | | |
| **Minimum Stock** | | | **-8** | | | | **0** | | | |
| **Total Order Quantity** | | | | | **31** | **30** | | | **39** | **38** |
| **Number of Orders** | | | | | **7** | | | | **7** | |

Table 9.3 presents comparative calculations for an inventory management system based on Croston's method, where the smoothing constants take the optimal values (0.39 and 0.28) for comparisons with the one-period ahead demand from Table 7.6. In this instance the maximum stockout is 8 units in quarters 13 and 14 of Iteration 1, thus the

safety margin is set to 8 units for the next iteration. Increasing the order-up-to level for the second iteration leads to average stock-holdings of 10.95 units, some 1.65 units more than required by ES.

Once the safety stocks necessary to completely eliminate stock-outs are determined, the simulations are re-run across the sample line items and the performance of each forecasting method can be compared in terms of average stock-holdings. First, however, it is necessary to define the modelling parameters used in this analysis.

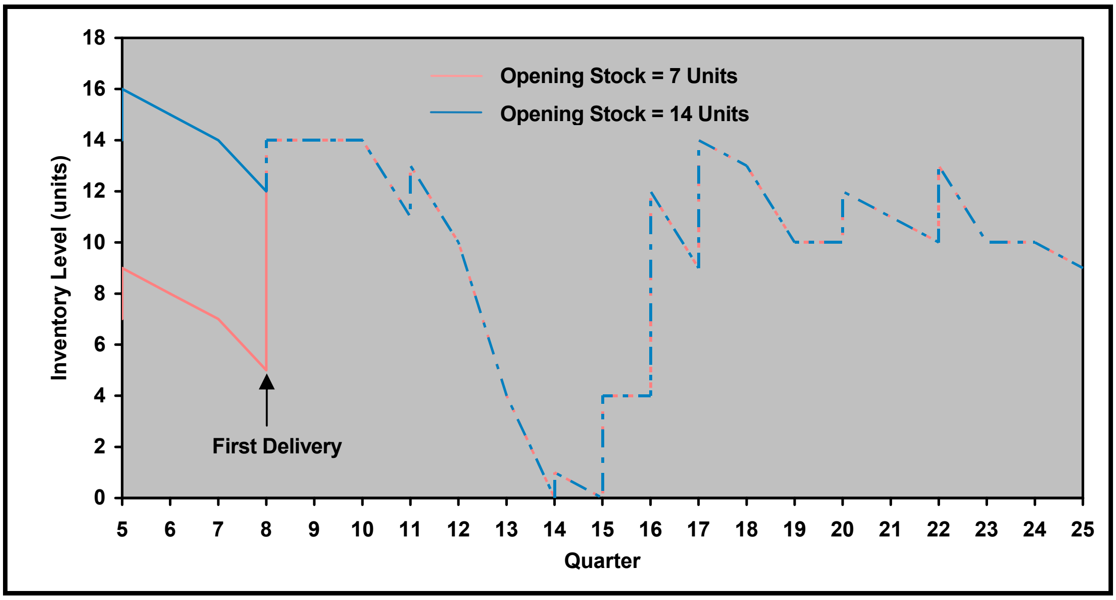
## 9.3 Modelling Parameters

There are factors outside of the forecasting methods which affect the average stock-holding calculations, including the simulation period, the demand aggregation, the measurement, forecast and reorder intervals, and the smoothing parameters. Each of these factors are defined, and their effects commented upon, in this section.

### 9.3.1 Simulation Period Selection

Comparisons between stock-holdings should only be made after the first replenishment delivery has occurred. Prior to this time the stock-holdings are governed by the opening stock level and outstanding orders and, as a result, do not reflect the actual demand forecasting. Continuing with the same sample line item and exponential smoothing, observations that arise before the first delivery occurrence, as shown in Figure 9.4, should be discarded. The figure shows the comparative stock-levels for quarterly data when the opening stock is 7 units and when it is 14 units.

**Figure 9.4: Quarterly Inventory Levels (Simulation Period Effect).**




The data covers a 5-year simulation period, once safety stocks have been introduced in order to avoid stock-outs. As orders are delivered, the stock replenishments increase the inventory level, as depicted by the vertical lines. Subsequent demands are satisfied from stock and the inventory level decreases until a new replenishment is received. The lead-time for this line item is 3 quarters and the first *calculated* delivery is seen to occur in quarter 8, once an initial lead-time period has elapsed.

When the opening stock is 7 units the first delivery is for 9 units and the average stock-holdings are 9.30 units, whereas when the opening stock is 14 units the first delivery is for 2 units and the average stock-holdings are 10.35 units. In both cases, the inventory level is 14 units after the delivery, and they remain equivalent beyond this point. Discarding the observations that arise before the first delivery leads to average stock-holdings of 9.65 units in both cases.

The stock-holding calculations for the remaining analyses in this chapter all utilise a simulation period which excludes observations occurring before the first delivery. This will ensure the calculated values are not affected by the initialisation, therefore wholly reflecting the model under consideration.

Furthermore, in the actual modelling, the opening stock is set to equal the first lead-time demand plus one unit and the initial stock on order is set to zero. This has the effect of ensuring a delivery is made at the earliest opportunity for the vast majority of line items irrespective of the forecast. The next section examines the effect that the various demand aggregations have on the comparative stock-holdings.

### 9.3.2 Demand Aggregation

Demand aggregation refers to the bucketing of individual demand observations into time periods. The data used in this chapter, and the previous two chapters, comprises quarterly, monthly and weekly demand aggregations. A quarter contains three calendar months, a month is taken as a calendar month, and a week is a seven-day period starting on a Sunday and ending on a Saturday. The periods are not entirely compatible for dissection; a calendar month contains a non-integer number of weeks and while a quarter is equivalent to 13 weeks, the periods of coverage are not exactly the same. For example, a demand occurrence on Tuesday the 28$^{th}$ of September, 1999 falls in the third quarter of the year but occurs in week 40. This can lead to minor variations in the comparative stock-holdings, with all other factors held constant. An actual demand of 5 units falling on the example date, with total demand of 247 units over a five-year simulation period, was observed to lead to a stock-holding discrepancy of 0.52 percent between quarterly and weekly data using exponential smoothing.

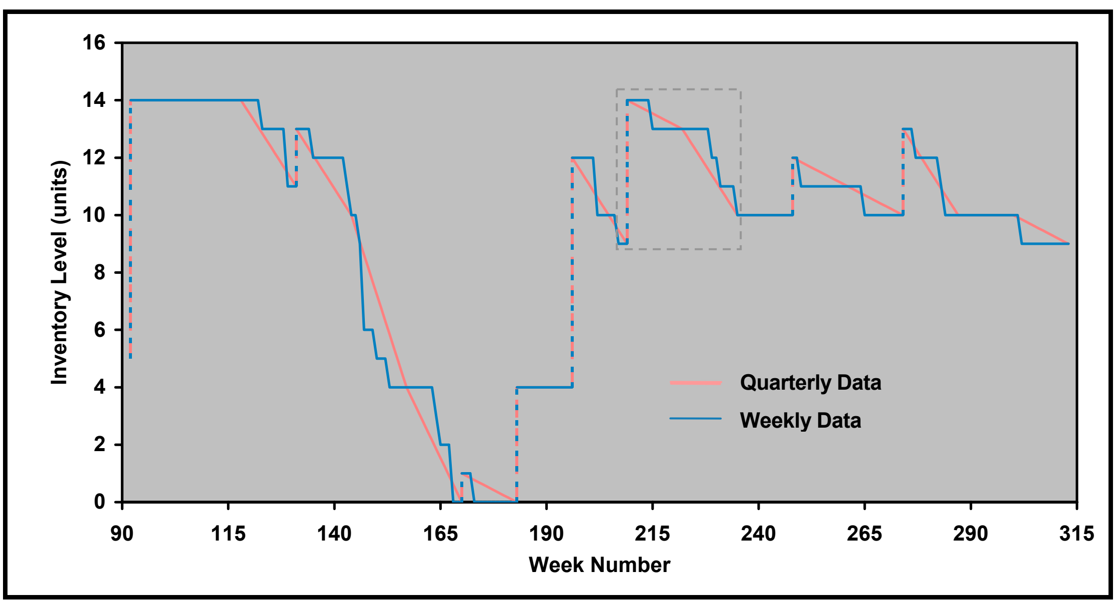
Discrepancies through differences in bucket coverage arise infrequently and tend to cancel out over a large number of series. A potentially more serious issue with regards to demand aggregation, referred to as aggregation bias, is examined in the next section.

### 9.3.3 Aggregation Bias

The purpose of aggregating demand in this instance is to allow comparison of the implied stock-holding performance between the different aggregations and forecasting methods. However, in comparing the average stock-holdings for quarterly, monthly and weekly data, a bias is introduced, termed *aggregation bias*, which can either increase or decrease the relative average performance.

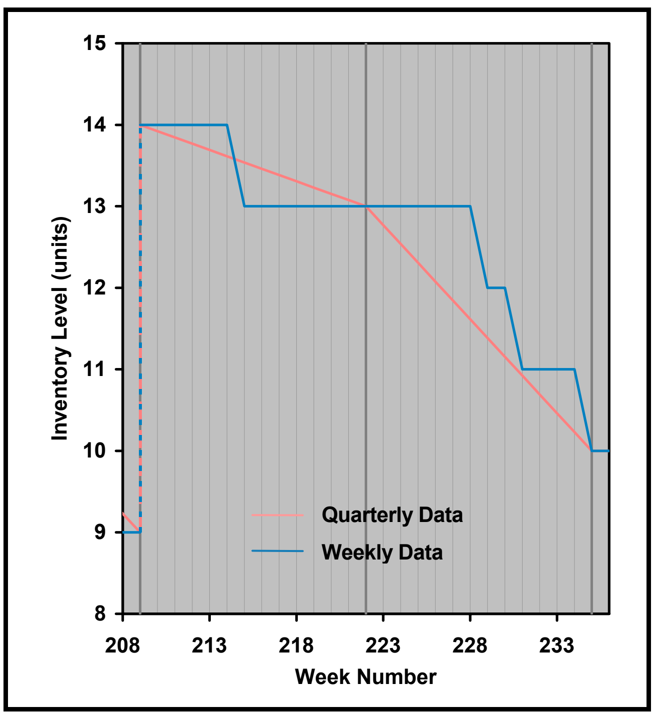
The circumstance in which bias occurs is illustrated in Figure 9.5, which compares inventory levels resulting from quarterly and weekly demand data. Inventory levels are presented on a weekly basis, with the quarterly levels linearly depleted over 13-week intervals. All forecasts are made at quarterly intervals and there is a quarterly review period in this instance, allowing both series to have the same order-up-to level, leading to replenishments of the same size arriving at the same time.

**Figure 9.5: Comparative Inventory Levels (Aggregation Bias Effect).**




The inventory levels display differences over time between the demand aggregations; sometimes the quarterly data has the lowest inventory while at other times the weekly data has the lowest. This is clearly observed in Figure 9.6, which resizes the dashed area of the previous figure. The extract covers a 28-week period divided into weekly intervals, and includes a stock replenishment in week 209. A demand of one unit occurs in week 214 and reduces the inventory level from 14 to 13 units. This reduction occurs immediately for the weekly data, although the reduction must be averaged over the appropriate weeks for the quarterly data.

**Figure 9.6: Comparative Inventory Levels (Resized).**




The result is that the average stock-holdings vary in each case, and the aggregation bias can be positive or negative. The bias is at a minimum when weekly demands are evenly distributed across a quarterly period. Although this is not likely to be the case when demand is erratic, as there will be many zero demands, it is expected that the bias would cancel out over a simulation period, and even more so over many series.

In the case of the sample line item considered in this section, the average stock-holdings are 9.65, 9.61 and 9.65 for quarterly, monthly and weekly data respectively. Therefore, the maximum deviation of 0.04 provides a bias of 0.41 percent. The maximum bias calculated from a sample of five line items is presented in Table 9.4, where the sample comprises one line item from each demand pattern. Demand patterns for each of these line items were previously illustrated on page 131.

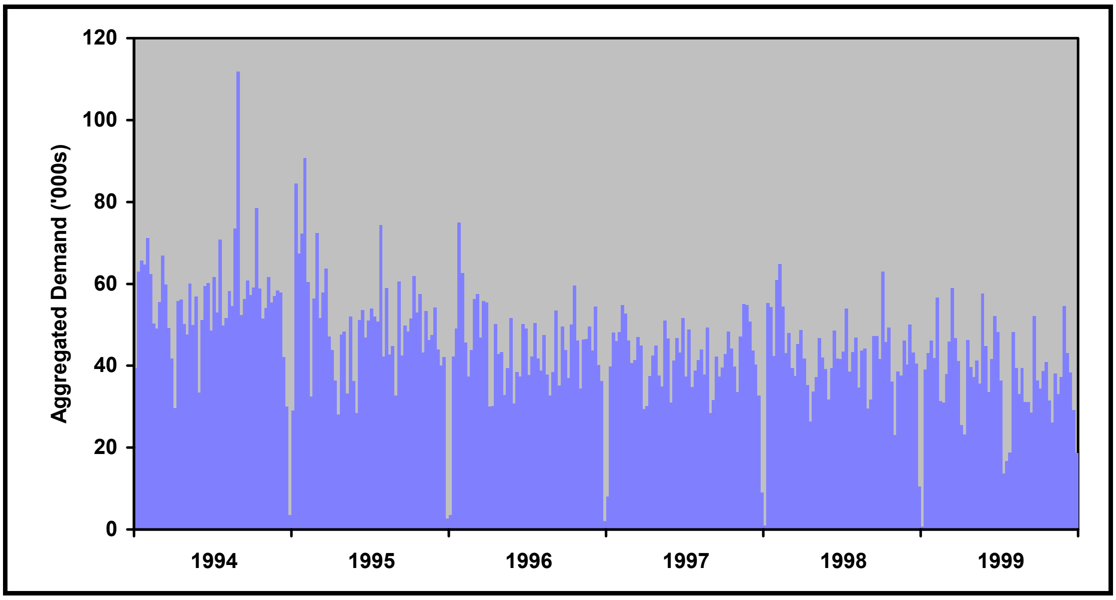
**Table 9.4: Sample Aggregation Bias.**

| Sample Item | Demand Pattern | Average Stock-Holdings | | | Maximum Bias |
|---|---|---|---|---|---|
| | | Quarterly | Monthly | Weekly | |
| 1 | Smooth | **44.67** | 44.91 | 45.50 | 1.86% |
| 2 | Irregular | 20.05 | **19.02** | 19.35 | 5.42% |
| 3 | Slow-moving | **2.74** | 2.75 | 2.76 | 0.73% |
| 4 | Mildly Erratic | **30.10** | **30.10** | 32.65 | 8.47% |
| 5 | Highly Erratic | 942.47 | 940.68 | **931.19** | 1.21% |
| **Overall Average** | | 208.01 | 207.49 | **206.29** | 0.83% |

The average stock-holdings are seen to vary across the three demand aggregations and the aggregation with the lowest stock-holdings also varies (as shown in bold type). This suggests the bias for the most part would be cancelled out over many series. The maximum bias over the five sample line items is seen to range between 0.73 and 8.47 percent, while the overall average is relatively low at 0.83 percent.

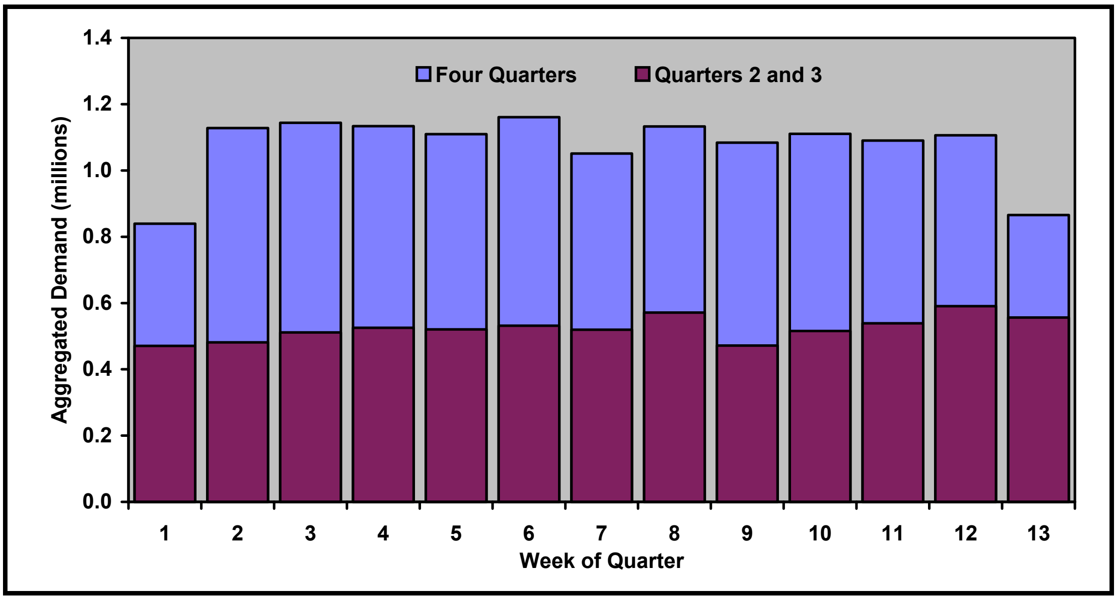
As previously noted, the aggregation bias is minimised for a large sample when the weekly demands are evenly distributed across the corresponding quarterly periods. Consideration is given to this aspect using the previously utilised sample of 18,750 line items. Figure 9.7 presents the total demand aggregated by week over a six year period. Apart from the decrease in demand over the years, a key feature is the low demand occurrences in the first and last week of each year. The decrease in demand over the years is attributed to a general decrease in RAF activity, while the latter observation is attributed to reduced activity over the Christmas / New Year period.

**Figure 9.7: Aggregated Weekly Demand for 18,750 Line Items.**




Of particular interest, however, is the manner in which weekly demand behaves over the course of the quarterly intervals. Figure 9.8 presents the quarterly aggregated demand divided into the 13 week periods that constitute each quarter. The full length bars show the aggregated demand from all four quarters, while the shorter, darker-coloured bars show the aggregated demand from periods 2 and 3 only. For the most part the frequencies are relatively consistent across a quarter, with the exception of the first and last weeks for the four quarters' aggregation. Demand in weeks 1 and 13 is considerably lower as a consequence of the reduced activity over the Christmas / New Year period. When this period is removed, by considering quarters two and three only, the first and last weeks are no longer lower than the rest, although there is greater variation between the weeks overall.

**Figure 9.8: Quarterly Aggregated Demand by Week.**




In considering the four quarters aggregation, the weekly demand is *balanced* over a quarter and the bias between weekly and quarterly aggregations is minimised; lower demand in the week at the beginning of the first quarter (and therefore higher average weekly stock-holdings) is cancelled out by lower demand in the week at the end of the last quarter (demand is higher during the first part of this quarter and therefore lower average weekly stock-holdings result).

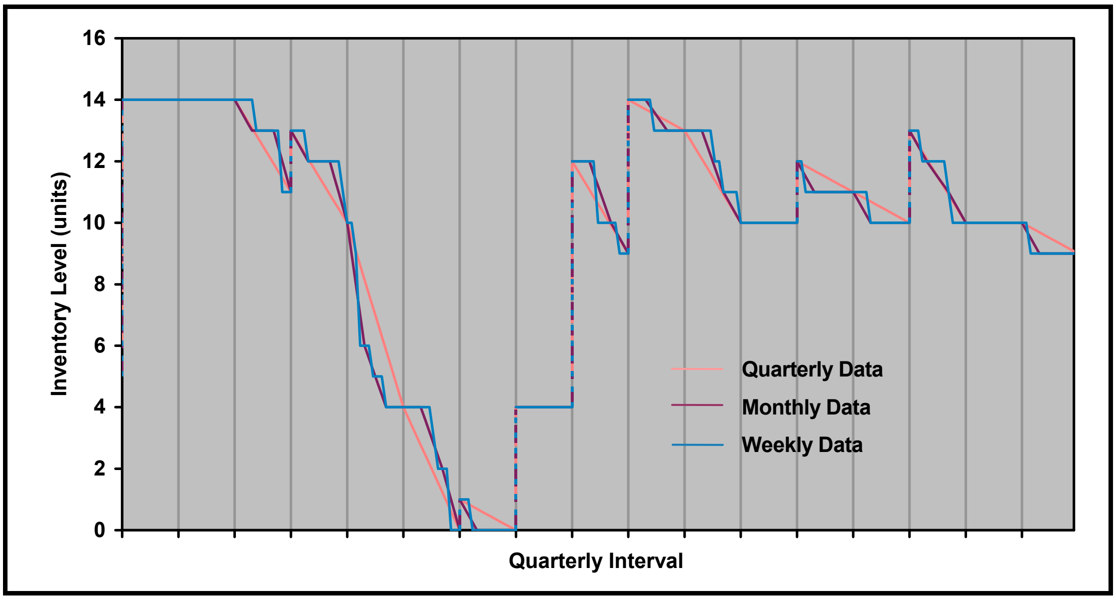
The analysis from this section raises an awareness of the inherent bias in average stock-holdings due to demand aggregation, although the effect is minimal when considering the large sample sizes of this research. Apart from reduced activity at the start and end of each year, there is no evidence to suggest spare parts are withdrawn at particular times during a quarter.

A method of completely removing the aggregation bias is to measure the stock-holdings at a common time interval, which would be quarterly in this case as it provides the lowest common denominator. The effect of the measurement interval on the stock-holding calculations is investigated in the next section.

### 9.3.4 Measurement Interval

Only measuring the stock-holdings at quarterly intervals removes the incremental steps of the monthly and weekly data and no averaging across time periods is required. This has the effect of eliminating the aggregation bias, as illustrated by Figure 9.9, which compares inventory levels resulting from using quarterly, monthly and weekly aggregated demand data for the sample line item. At the quarterly intervals the stock-holdings are all equal under each aggregation.

**Figure 9.9: Comparative Inventory Levels (Measurement Interval Effect).**



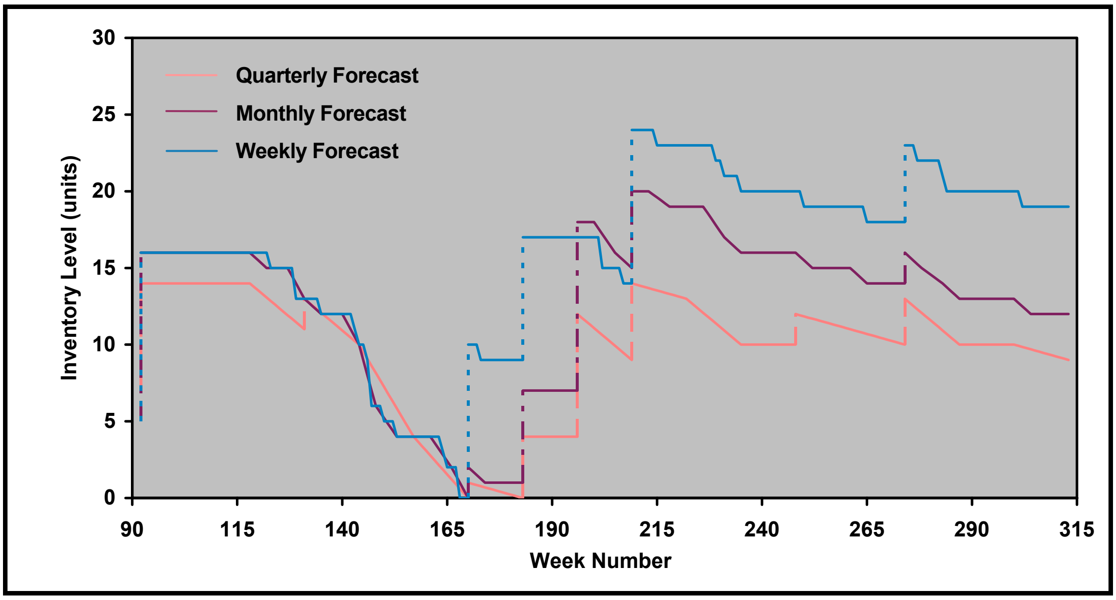

Inventory levels are reviewed weekly with linear apportionment for the two non-weekly series (allowances have to be made for illustrative purposes as one month equals a non-integer 4.3 weeks in reality; a quarter, however, is equal to exactly 13 weeks). In this case, all forecast updates and reordering occurs on a quarterly basis leading to replenishments of the same size arriving at the same time for each demand aggregation.

With forecast updates and reordering occurring quarterly, the inventory levels are equal at the end of each quarter. With a quarterly measurement interval, all demand aggregations have average stock-holdings of 9.65 units and there is no aggregation bias. Alternatively, with stock-holdings measured every period, the average values are 9.65, 9.61 and 9.65 for quarterly, monthly and weekly data respectively, giving a maximum bias of 0.41 percent as observed previously in Section 9.3.3. It is purely a matter of coincidence that the stock-holdings for quarterly and weekly data are the same when measured every period.

### 9.3.5 Forecast Interval

Another factor which affects the stock-holdings is the forecast interval, or periodicity of forecasting. The example inventory simulations considered thus far have all used quarterly forecasting. When demand is captured and formatted more frequently, such as monthly or weekly, the forecast interval can likewise be more frequent. This situation is illustrated in Figure 9.10, where forecasts are made every period for each demand aggregation, although reordering is still undertaken quarterly in all cases.

**Figure 9.10: Comparative Inventory Levels (Forecast Interval Effect).**



As the forecast quantities differ between the demand aggregations, the order-up-to levels also differ leading to differing stock-holdings. With stock-holdings measured every quarter the averages are 9.65, 12.65 and 16.06 for quarterly, monthly and weekly forecasting respectively. Alternatively, with stock-holdings measured every period the corresponding averages are 9.65, 12.61 and 16.06 respectively. The quarterly result is of course the same in both cases although the fact that the weekly data provides the same stock-holdings in each case is a matter of coincidence.

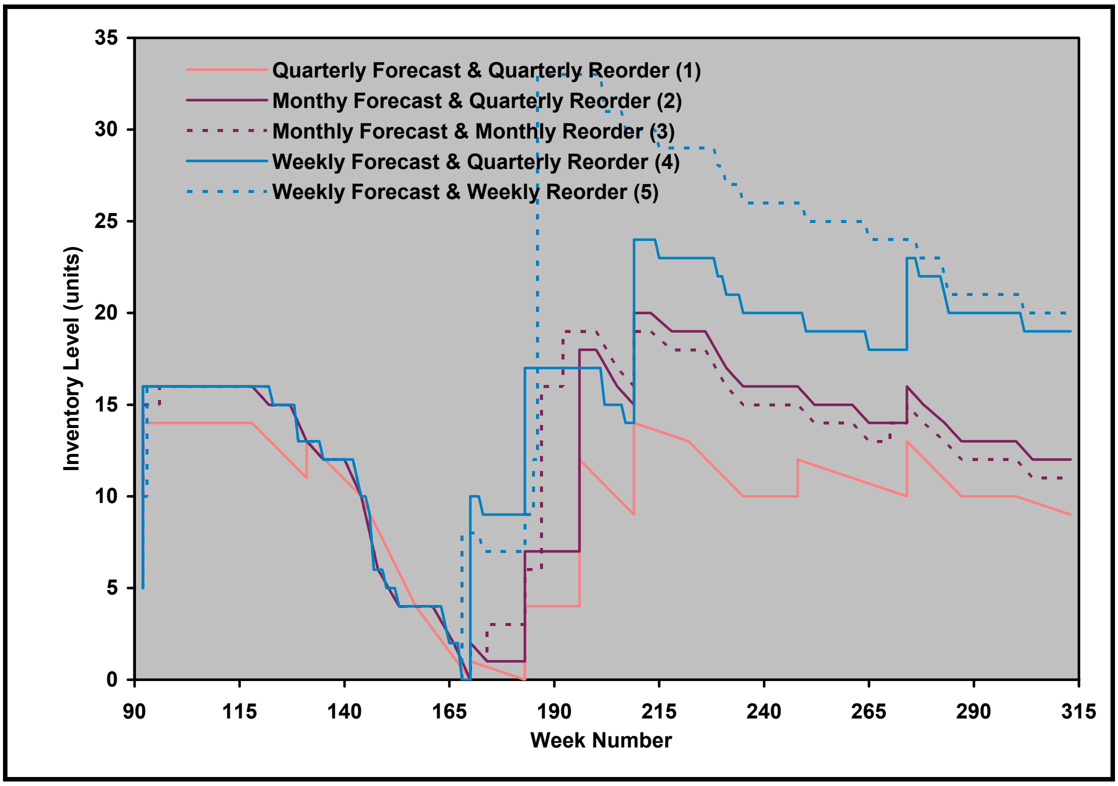
Surprisingly, the comparative results for this line item illustrate an effect of the forecast update interval that is not immediately intuitive. The calculated average stock-holdings suggest the more frequent forecasting provided by the weekly data is inferior to that provided by the quarterly data. This issue is examined further in the next section, once the reorder interval has been considered.

### 9.3.6 Reorder Interval

In a similar manner to the forecast interval, the reorder interval can also match the frequency of the demand data in practice. Thus far, the example simulations have all used a quarterly replenishment cycle, whereby orders are only placed on a quarterly basis. This section considers the effect of the selected reorder interval.

The reorder interval has an impact on the calculated stock-holdings, as demonstrated in Figure 9.11. This figure presents comparative inventory levels for a range of combinations of the forecast interval and reorder interval. The solid lines representing quarterly, monthly and weekly forecast intervals, each with quarterly reorder intervals, were observed in the previous figure. At that time it was noted that the quarterly forecast provided the best performance.

**Figure 9.11: Comparative Inventory Levels (Reorder Interval Effect).**




Also shown in this figure are the results for monthly forecasting with monthly reordering, and weekly forecasting with weekly reordering. In the case of the monthly data, the inventory level is seen to moderately change, sometimes higher and sometimes lower. However, in the case of the weekly data, the inventory level increases substantially throughout the latter periods. Under periodic reordering the average stock-holdings measured every quarter are 9.65, 12.62 and 19.12 for quarterly, monthly and weekly forecasting respectively. Alternatively, with stock-holdings measured every period the corresponding averages are 9.65, 12.67 and 19.50 respectively.

Henceforth, where the reorder interval is the same as the forecast interval, this will be referred to as the review period. Thus, monthly forecasting with monthly reordering will more conveniently be referred to as monthly review.

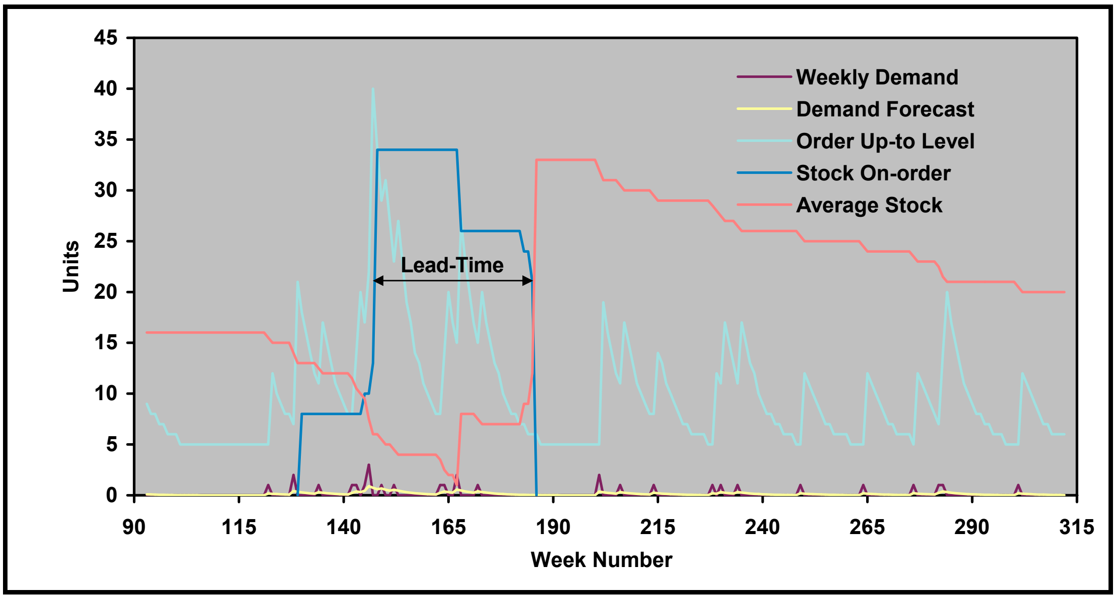
The calculated average stock-holdings from this section are summarised in Table 9.5, where the observation numbers refer to the previous figure. It is observed that quarterly review provides the best performance overall, while weekly review provides the worst. Furthermore, quarterly reordering with both monthly and weekly forecasting tends to provide better results than the corresponding monthly and weekly reordering.

**Table 9.5: Average Stock-Holdings by Update Interval.**

| Obs | Forecast Interval | Reorder Interval | Safety Stock | Measurement Interval | |
|---|---|---|---|---|---|
| | | | | Every Quarter | Every Period |
| (1) | Quarterly | Quarterly | 6 | 9.65 | 9.65 |
| (2) | Monthly | Quarterly | 8 | 12.65 | 12.61 |
| (3) | Monthly | Monthly | 9 | 12.62 | 12.67 |
| (4) | Weekly | Quarterly | 8 | 16.06 | 16.06 |
| (5) | Weekly | Weekly | 4 | 19.12 | 19.50 |

Examining the weekly data, in order to determine the reasoning behind the poor performance, indicates a large delivery in week 186 and hence a high inventory level thereafter, as illustrated in Figure 9.12.

**Figure 9.12: Example Order Levels and Average Stock - Weekly Data.**



A relatively high demand at the time of ordering raised the order-up-to level substantially. At the time of ordering the demand forecast was 0.897 per week whereas after the delivery arrived the actual demand was only 0.102 per week, therefore leaving a large quantity of stock in the system. As it happens, when reordering is performed on a quarterly basis the timing of the replenishment cycle avoids the highest order-up-to level by chance and a number of smaller replenishments are initiated instead. Allowing an opportunity to reorder every week increases the risk of encountering the highest order-up-to level to 100 percent.

Prior to examining the final factor which affects the stock-holding calculations, namely the smoothing parameters, it is useful to review the combined effects of the forecast and reorder intervals.

### 9.3.7 Forecast and Reorder Interval Review

The results obtained from the sample line item indicate that the lowest implied stock-holdings arise under a quarterly review model, as opposed to monthly or weekly review. It is worth examining whether this is the case more generally. Table 9.6 presents the average stock-holdings for the sample line item across a range of demand aggregations, forecast intervals and reorder intervals. Also shown are similar results for five further line items, one from each demand pattern, and the average stock-holdings from these items. Stock-holdings are presented when measured every quarter as well as every period and, in the case of the weekly aggregation, also as the average of 4 and 5 weeks to approximate a monthly measurement interval. Under each demand aggregation, a quarterly review with a quarterly measurement interval, as shown by rows 1, 2 and 8, produces the same results, as required by definition.

**Table 9.6: Average Stock-Holdings for Selected Individual Line Items.**

| Observation | Demand Aggregation | Forecast Interval | Reorder Interval | Measurement Interval | Implied Stock-Holdings | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Sample Demand Pattern Items | | | | | |
| | | | | | Sample Line Item | Smooth | Irregular | Slow-Moving | Mildly Erratic | Highly Erratic | Demand Pattern Average |
| 1 | Q | Q | Q | Q | 9.65 | 44.67 | 20.05 | 2.74 | 30.10 | 942.47 | 208.01 |
| 2 | M | Q | Q | Q | 9.65 | 44.67 | 20.05 | 2.74 | 30.10 | 942.47 | 208.01 |
| 3 | M | Q | Q | M | 9.61 | 44.91 | 19.02 | 2.75 | 30.10 | 940.68 | 207.49 |
| 4 | M | M | Q | Q | 12.65 | 45.93 | 21.68 | 3.15 | 15.70 | 1,004.28 | 218.15 |
| 5 | M | M | Q | M | 12.61 | 46.18 | 20.65 | 3.17 | 15.70 | 1,002.49 | 217.64 |
| 6 | M | M | M | Q | 12.62 | 42.89 | 18.50 | 3.18 | 31.70 | 1,070.81 | 233.42 |
| 7 | M | M | M | M | 12.67 | 43.45 | 18.56 | 3.21 | 29.10 | 1,037.41 | 226.35 |
| 8 | W | Q | Q | Q | 9.65 | 44.67 | 20.05 | 2.74 | 30.10 | 942.47 | 208.01 |
| 9 | W | Q | Q | 4/5w | 9.67 | 45.75 | 18.89 | 2.76 | 29.79 | 921.80 | 203.80 |
| 10 | W | Q | Q | W | 9.65 | 45.50 | 19.35 | 2.76 | 32.65 | 931.19 | 206.29 |
| 11 | W | W | Q | Q | 16.06 | 51.87 | 15.86 | 4.50 | 8.30 | 1,056.78 | 227.46 |
| 12 | W | W | Q | 4/5w | 16.02 | 53.44 | 14.72 | 4.51 | 8.21 | 1,031.20 | 222.42 |
| 13 | W | W | Q | W | 16.06 | 52.70 | 15.17 | 4.52 | 8.30 | 1,045.50 | 225.24 |
| 14 | W | W | W | Q | 19.12 | 66.47 | 24.77 | 5.85 | 10.85 | 2,067.27 | 435.04 |
| 15 | W | W | W | 4/5w | 19.51 | 68.58 | 24.55 | 5.89 | 11.27 | 2,049.57 | 431.97 |
| 16 | W | W | W | W | 19.50 | 68.87 | 25.08 | 5.90 | 11.95 | 2,097.98 | 441.96 |

Q = quarterly, M = monthly, W = weekly and 4/5w = average of 4 and 5 weeks (monthly approximation).

For the most part the stock-holdings, when measured every quarter, are approximately the same as when measured every period, with any differences due to the aggregation bias. In considering the overall results, the following observations are made without reference to the measurement interval:

(i) Quarterly review on the whole produces the lowest stock-holdings and weekly review produces the highest.

(ii) In terms of monthly data, monthly forecasting, with quarterly reordering, results in lower stock-holdings than monthly reordering.

(iii) Similarly, in terms of weekly data, weekly forecasting with quarterly reordering results in lower stock-holdings than weekly reordering.

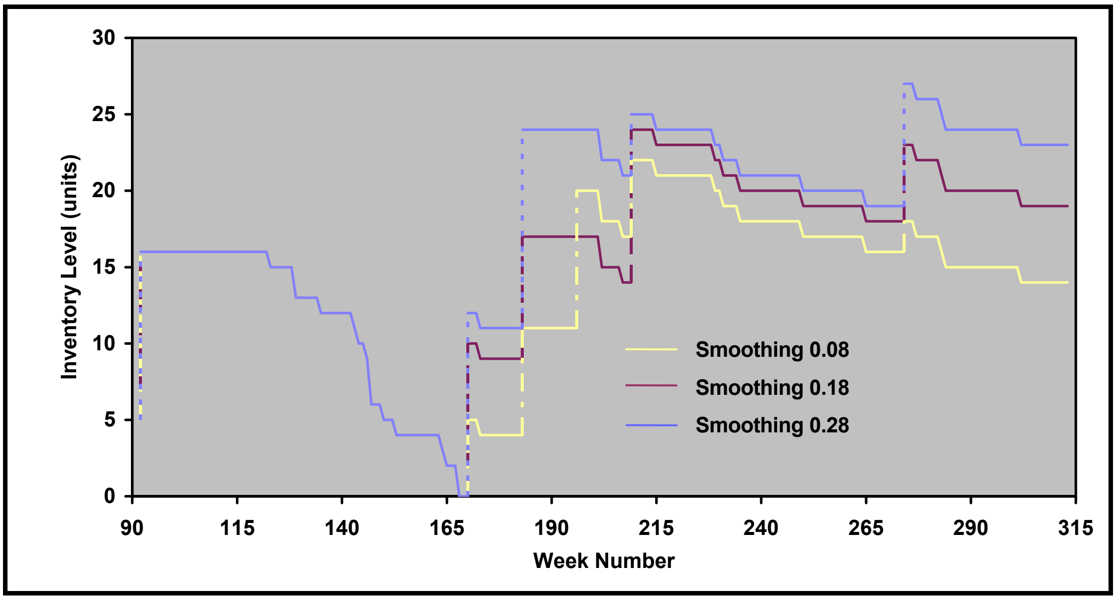
These observations mostly hold true for all the sample line items except for the mildly erratic item where weekly forecasting clearly provides the best results. In this case, the opening stock was so high that the first delivery was not received until a substantial portion of the simulation had elapsed, leaving only a small sample of periods for generating results. As previously mentioned, this situation is avoided in the actual study by setting the opening stock to equal the demand over the first lead-time plus one unit.

The results of this section illustrate a weakness of ES for establishing stock levels when demand is erratic and reviews occur every period. As ES places most weight on the more recent data, the forecast estimates are highest just after a demand. This leads to over-estimation in the periods in which replenishment orders are placed, and therefore there is a tendency to hold unnecessarily high stocks. The problem is exacerbated by the large proportion of zero demands in the weekly data as compared with the monthly and quarterly data. With reviews every period the chance of encountering an unnecessarily high order-up-to level is maximised. On the other hand, the bucketing of demand into quarterly aggregations pre-smoothes the forecast series and tends to lead to a better stock-holding performance. The performance of other forecasting methods under these conditions will be examined in a later section.

### 9.3.8 Smoothing Parameters

In the case of smoothing methods, such as ES and Croston's method, different smoothing constants will provide differing demand forecasts and therefore differing implied stock-holdings. This situation is illustrated by the comparative inventory levels presented in Figure 9.13, where three different constants are utilised under ES.

**Figure 9.13: Comparative Inventory Levels (Smoothing Parameter Effect).**




Weekly forecasting is modelled with a quarterly reordering interval and the average stock-holdings are measured in all periods. The previously utilised smoothing constant of 0.18 provides average stock-holdings of 16.06 units. In comparison, a smoothing constant of 0.1 below this provides the lowest average stock-holdings of 14.12 units, while a smoothing constant of 0.1 above provides the highest at 18.00 units.

Attention is turned to a wider sample of line items in order to determine what constitutes suitable smoothing parameter values in terms of minimum average stock-holdings for both ES and Croston's method. As calculated from the hold-out sample of 500 line items, the following two tables present average implied stock-holdings using the optimal smoothing values according to MAPE, as generated in Chapter 7. The first table considers the effect of updating both the forecast and the order quantity every period, while the second table considers the effect of allowing a reorder only every quarter.

Table 9.7 presents implied stock-holdings where the forecast interval is equal to one and the reorder interval is also equal to one for each demand aggregation. Thus, in each case, the forecasts, the order-up-to levels and the orders themselves are updated each and every period, whether that be quarterly, monthly or weekly. For both ES and Croston's method, the implied stock-holdings are seen to differ across the range of selected smoothing constant values.

**Table 9.7: Implied Stock-Holdings from MAPE Constants (Updating Every Period).**

| Demand Aggregation | Type of Forecast Providing Smoothing Constants | Expon. Smoothing | | Croston's Method | | |
|---|---|---|---|---|---|---|
| | | Smoothing Constant | Implied Stock-Holdings | Smooth. Constants: Demand Size | Smooth. Constants: Demand Interval | Implied Stock-Holdings |
| **Measuring Every Period** | | | | | | |
| **Quarterly** | **One-Period Ahead Demand - All Periods** | 0.18 | **37.46** | 0.39 | 0.28 | **41.37** |
| **Monthly** | | 0.05 | **35.22** | 0.18 | 0.08 | 38.40 |
| **Weekly** | | 0.01 | **34.48** | 0.10 | 0.01 | **36.29** |
| **Quarterly** | **Lead-Time Demand - All Periods** | 0.43 | 46.11 | 0.92 | 0.40 | 54.29 |
| **Monthly** | | 0.16 | 45.60 | 0.10 | 0.34 | **37.04** |
| **Weekly** | | 0.04 | 45.94 | 0.02 | 0.25 | 38.78 |
| **Quarterly** | **Lead-Time Demand - Demand Only** | 0.19 | 37.81 | 0.92 | 0.46 | 54.58 |
| **Monthly** | | 0.06 | 36.17 | 0.50 | 0.36 | 53.26 |
| **Weekly** | | 0.01 | **34.48** | 0.58 | 0.30 | 80.09 |
| **Measuring Every Quarter** | | | | | | |
| **Quarterly** | **One-Period Ahead Demand - All Periods** | 0.18 | **37.46** | 0.39 | 0.28 | **41.37** |
| **Monthly** | | 0.05 | **35.26** | 0.18 | 0.08 | 38.42 |
| **Weekly** | | 0.01 | **34.71** | 0.10 | 0.01 | **36.48** |
| **Quarterly** | **Lead-Time Demand - All Periods** | 0.43 | 46.11 | 0.92 | 0.40 | 54.29 |
| **Monthly** | | 0.16 | 45.57 | 0.10 | 0.34 | **37.11** |
| **Weekly** | | 0.04 | 46.20 | 0.02 | 0.25 | 39.01 |
| **Quarterly** | **Lead-Time Demand - Demand Only** | 0.19 | 37.81 | 0.92 | 0.46 | 54.58 |
| **Monthly** | | 0.06 | 36.21 | 0.50 | 0.36 | 53.17 |
| **Weekly** | | 0.01 | **34.71** | 0.58 | 0.30 | 79.84 |

With the minimum stock-holdings shown in bold type for each demand aggregation, it is observed that ES provides the best results for quarterly, monthly and weekly data, in this instance. The same general patterns emerge regardless of whether the stock-holdings are measured every period or every quarter. Stock-holdings for monthly and weekly data for the most part are higher when measured on a quarterly basis, and obviously the results for quarterly data remain the same.

In a similar manner, Table 9.8 presents the implied stock-holdings when the reorder interval is quarterly. Forecast quantities and order-up-to levels are updated each period, although an order can only be placed each quarter, or alternatively every 3 months or every 13 weeks. Stock-holdings are again measured each period as well as each quarter.

**Table 9.8: Implied Stock-Holdings from MAPE Constants (Quarterly Reordering).**

| Demand Aggregation | Type of Forecast Providing Smoothing Constants | Expon. Smoothing | | Croston's Method | | |
|---|---|---|---|---|---|---|
| | | Smoothing Constant | Implied Stock-Holdings | Smooth. Constants: Demand Size | Smooth. Constants: Demand Interval | Implied Stock-Holdings |
| **Measuring Every Period** | | | | | | |
| **Quarterly** | **One-Period Ahead Demand - All Periods** | 0.18 | **37.46** | 0.39 | 0.28 | **41.37** |
| **Monthly** | | 0.05 | **35.87** | 0.18 | 0.08 | 38.56 |
| **Weekly** | | 0.01 | **36.93** | 0.10 | 0.01 | **37.81** |
| **Quarterly** | **Lead-Time Demand - All Periods** | 0.43 | 46.11 | 0.92 | 0.40 | 54.29 |
| **Monthly** | | 0.16 | 44.50 | 0.10 | 0.34 | **38.24** |
| **Weekly** | | 0.04 | 45.34 | 0.02 | 0.25 | 40.79 |
| **Quarterly** | **Lead-Time Demand - Demand Only** | 0.19 | 37.81 | 0.92 | 0.46 | 54.58 |
| **Monthly** | | 0.06 | 36.74 | 0.50 | 0.36 | 49.76 |
| **Weekly** | | 0.01 | **36.93** | 0.58 | 0.30 | 65.30 |
| **Measuring Every Quarter** | | | | | | |
| **Quarterly** | **One-Period Ahead Demand - All Periods** | 0.18 | **37.46** | 0.39 | 0.28 | **41.37** |
| **Monthly** | | 0.05 | **36.03** | 0.18 | 0.08 | 38.71 |
| **Weekly** | | 0.01 | **37.06** | 0.10 | 0.01 | **37.94** |
| **Quarterly** | **Lead-Time Demand - All Periods** | 0.43 | 46.11 | 0.92 | 0.40 | 54.29 |
| **Monthly** | | 0.16 | 44.65 | 0.10 | 0.34 | **38.40** |
| **Weekly** | | 0.04 | 45.47 | 0.02 | 0.25 | 40.92 |
| **Quarterly** | **Lead-Time Demand - Demand Only** | 0.19 | 37.81 | 0.92 | 0.46 | 54.58 |
| **Monthly** | | 0.06 | 36.89 | 0.50 | 0.36 | 49.91 |
| **Weekly** | | 0.01 | **37.06** | 0.58 | 0.30 | 65.43 |

Once again, the stock-holdings are generally higher when measured every quarter rather than every period. In comparing the results between updating every period and quarterly reordering, the results are mixed. For the corresponding smoothing constant values, on some occasions updating every period provides the lowest stock-holdings while on other occasions quarterly reordering provides the lowest.

In practice, implied stock-holdings cannot actually be calculated for all line items for a number of reasons, such as there not being enough initial stock in the system to satisfy demand during the first lead-time; an issue considered further in Section 9.5. Differing forecasts, either from using different forecast methods or different smoothing parameters with the same method, will not allow the calculation of implied stock-holdings for precisely the same line items from a particular sample. Therefore, the results presented in this section were only obtained from line items where the implied stock-holdings could be resolved in all cases, which amounts to 274 of the 500 line items under consideration.

Optimal values, in terms of MAPE, are not the same as those which provide minimum stock-holdings overall. For all intents and purposes, the current smoothing constants are practically random when applied in this manner. It is therefore not reasonable to compare the results between the forecasting methods in this instance. The next section seeks to determine optimal smoothing constant values for ES and Croston's method to allow robust comparisons between the methods.

## 9.4 Optimal Smoothing Parameters

Optimal smoothing constant values have been calculated for minimising the stock-holdings using the same hold-out sample, with the results presented in Table 9.9. Only one set of smoothing values are calculated for each demand aggregation in this instance, and these were optimised for updating every period with measurements taken every period. Stock-holdings are calculated for reordering every quarter using the same set of optimal values. Average stock-holdings are considered provisional at this stage and will be recalculated in a later section using a significantly larger sample size.

**Table 9.9: Optimal Smoothing Values for Implied Stock-Holdings.**

| Demand Aggreg'n | Exponential Smoothing | | | Croston's Method | | | |
|---|---|---|---|---|---|---|---|
| | Smooth. Constant | Provisional Stock-Holdings | | Smoothing Constants | | Provisional Stock-Holdings | |
| | | Updating Every Period | Quarterly Reorder | Demand Size | Demand Interval | Updating Every Period | Quarterly Reorder |
| **Measuring Every Period** | | | | | | | |
| **Quarterly** | 0.08 | 32.35 | 32.35 | 0.04 | 0.13 | 31.87 | 31.87 |
| **Monthly** | 0.04 | 34.29 | 34.34 | 0.03 | 0.09 | 32.88 | 32.96 |
| **Weekly** | 0.01 | 34.48 | 34.71 | 0.01 | 0.05 | 32.92 | 33.14 |
| **Measuring Every Quarter** | | | | | | | |
| **Quarterly** | 0.08 | 32.35 | 32.35 | 0.04 | 0.13 | 31.87 | 31.87 |
| **Monthly** | 0.04 | 35.05 | 35.20 | 0.03 | 0.09 | 33.62 | 33.78 |
| **Weekly** | 0.01 | 36.93 | 37.06 | 0.01 | 0.05 | 35.02 | 35.15 |

The results from Table 9.9 show an improvement in the stock-holdings from those that occurred when using the optimal values according to MAPE. In this instance it is Croston's method that provides the best results in all cases. The provisional stock-holdings indicate that the best results occur with quarterly data, followed by monthly data, and finally weekly data, for both updates every period and reordering every quarter. The optimal values are highest for quarterly data and lowest for weekly data. This results in greater dampening of the weekly series, the logical reaction given the greater variation in the data.

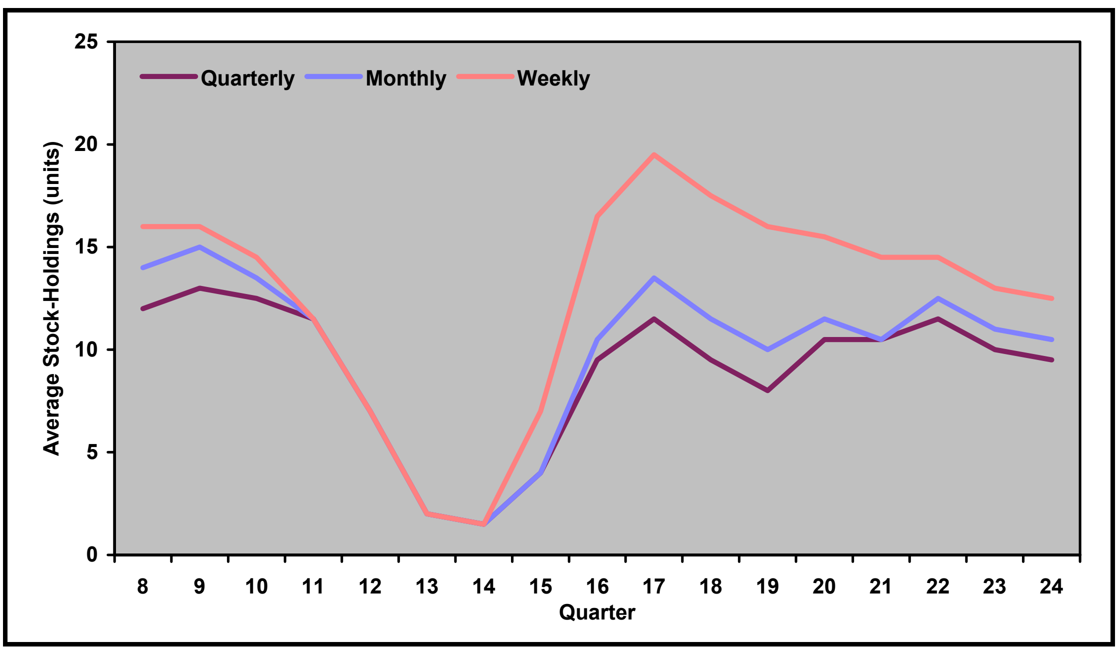
The variation in the results between the demand aggregations may be due to the differing smoothing constant values. In examining this issue, representative smoothing constant values were selected for each demand aggregation, as the average of the optimal values from Table 9.9; that is, 0.04 for ES, and 0.03 for the demand size and 0.09 for the demand interval for Croston's Method. The implied stock-holdings for these values are presented in Table 9.10, where there are substantial changes to the implied stock-holdings. Weekly data is the most affected, while quarterly data continues to provide the lowest stock-holdings. The implication from these results is that you should not use the same smoothing values across the range of demand aggregations. Quarterly data is pre-smoothed by the aggregation procedure, whereas weekly data requires additional dampening through the use of lower value smoothing constants.

**Table 9.10: Implied Stock-Holdings for Averaged Optimal Smoothing Values.**

| Demand Aggreg'n | Exponential Smoothing | | | Croston's Method | | | |
|---|---|---|---|---|---|---|---|
| | Smooth. Constant | Provisional Stock-Holdings | | Smoothing Constants | | Provisional Stock-Holdings | |
| | | Updating Every Period | Quarterly Reorder | Demand Size | Demand Interval | Updating Every Period | Quarterly Reorder |
| **Measuring Every Period** | | | | | | | |
| **Quarterly** | 0.04 | 33.95 | 33.95 | 0.03 | 0.09 | 32.25 | 32.25 |
| **Monthly** | 0.04 | 34.29 | 34.34 | 0.03 | 0.09 | 32.88 | 32.96 |
| **Weekly** | 0.04 | 45.94 | 46.20 | 0.03 | 0.09 | 35.03 | 35.26 |
| **Measuring Every Quarter** | | | | | | | |
| **Quarterly** | 0.04 | 33.95 | 33.95 | 0.03 | 0.09 | 32.25 | 32.25 |
| **Monthly** | 0.04 | 35.05 | 35.20 | 0.03 | 0.09 | 33.62 | 33.78 |
| **Weekly** | 0.04 | 45.34 | 45.47 | 0.03 | 0.09 | 37.45 | 37.58 |

Overall, the provisional stock-holdings indicate that the best results occur with quarterly data, followed by monthly data, and finally weekly data. This observation is graphically illustrated by the comparative stock-holdings presented in Figure 9.14 for the sample line item considered previously. In this example, ES has been used with the same smoothing constant for each demand aggregation (taken as the average of the optimal values) and reordering occurs each quarter. With the results shown after the first delivery, it is seen that the quarterly data has the lowest stock-holdings in the vast majority of periods. Average stock-holdings measured each quarter are 9.06, 10.00 and 12.65 units for quarterly, monthly and weekly forecasting respectively.

**Figure 9.14: Comparative Stock-Holdings by Demand Aggregation.**




The next section examines reasons why the implied stock-holdings cannot be calculated for all line items, and an analysis of the previously utilised sample of 18,750 line items is undertaken using the optimal smoothing constant values determined in this section.

## 9.5 Determining Safety Margins

Very infrequently can the safety margin that leads to a zero stock-out be calculated in just two iterations. In fact, the average number of iterations for obtaining the safety margin, when it is possible to do so, is 3.4 for quarterly data, 3.5 for monthly data and 4.4 for weekly data. While the methodology for calculating the implied safety margins for each forecasting method is described in Section 9.2, the example is too simplistic, as there are a number of situations where it is not possible to calculate the safety margin:

(i) **Situation A**: With high stock-holdings maintained by the RAF, it is very often the case that the initial stock, including outstanding orders, is in excess of the total demand over the entire simulation period and no further orders are required. In fact, this situation arises for 58.2 percent of line items using a quarterly series, 57.6 percent using a monthly series, and 56.8 percent using a weekly series.

(ii) **Situation B**: It can also be the case that there is not enough initial stock to satisfy demand. A stock-out will occur if the combined initial stock and outstanding orders are not enough to satisfy demand over the first lead-time period before any new orders can be delivered. From the remaining line items not affected by Situation A, this situation arises for 16.3 percent of the total using a quarterly series, 15.8 percent using a monthly series, and 15.1 percent using a weekly series.

(iii) **Situation C**: There may not be enough iterations available for resolution. Although 99 iterations have been allowed for each line item, the safety margin must be calculated across all forecasting methods to ensure that fair and accurate comparisons are made. In some circumstances it is not possible to obtain safety margins for all methods within the allotted number of iterations.

(iv) **Situation D**: The restrictions imposed by a non-unitary CMBQ or PPQ may prevent the demand from being met exactly. In fact, this situation arises for 8.9 percent of line items using a quarterly series, 7.8 percent using a monthly series, and 9.8 percent using a weekly series.

Each of the identified situations are not mutually exclusive and a particular line item may be affected by more than one of them. The percentage of line-items that do not fall within the four situations, and are therefore available for comparison, is only 16.6, 18.8 and 18.3 percent for quarterly, monthly and weekly series respectively. This means that the sample sizes are somewhat inadequate for a detailed analysis and adjustments are required to increase the number of line items available.

It is possible to guarantee there will neither be too much stock, nor too little, by setting the opening stock balance to match the total demand exactly over the first lead-time duration and by setting the outstanding orders to zero. This means that the stock balance will reach zero at the end of the first lead-time and, since a reorder placed in the first period will arrive at this time, the stock balance will subsequently increase again.

However, in this situation a realistic safety margin cannot be determined, as the stock-outs are artificially set at zero and no safety stock will be required. This problem is overcome by setting the opening stock to equal the total demand plus one extra unit over the first lead-time period. As a trade-off, adding this one unit also means there may be too much stock in the system and it may not be possible to calculate a safety margin.

Overall, a total of 11,203 of the 18,750 line items, or 59.7 percent, have been resolved for all forecasting methods across all demand aggregations. The vast majority of cases where implied stock-holdings could not be resolved are due to non-unitary CMBQ or

PPQ values. As a point of interest, the average number of iterations required for resolution has increased to around ten. The next section compares the implied stock-holdings from each forecasting method, where a common service level of 100 percent has been determined.

## 9.6 Forecasting Performance by Implied Stock-Holdings

The average implied stock-holdings for Croston's method and the simple forecasting methods (including ES, a one year moving average and a previous year average) are compared in the following tables. Table 9.11 presents results where forecast updating and reordering occurs every period, while Table 9.12 presents results where reordering occurs quarterly. An individual line item is only included if a service level of 100 percent could be obtained for each and every forecasting method, including the variations on Croston's method, where results are presented in subsequent tables.

**Table 9.11: Average Implied Stock-Holdings - Updating Every Period.**

| Demand Aggregation | Nomen-clature | Expon'l Smooth. | Croston's Method | Moving Average | Prev Year Average | Friedman Statistic by Method |
|---|---|---|---|---|---|---|
| Quarterly | (Q,Q,Q) | 50.41 | 49.88 | 63.31 | 59.42 | 4,168.62 |
| Monthly | (M,M,M) | 52.36 | 49.63 | 60.98 | 57.54 | 3,858.87 |
| Weekly | (W,W,W) | 51.83 | 48.62 | 59.94 | 56.20 | 4,020.91 |
| Friedman Statistic by Aggregation | | 2,627.18 | 353.84 | 840.10 | 903.88 | - |
| Quarterly | (Q,Q,Q) | 50.41 | 49.88 | 63.31 | 59.42 | 4,168.62 |
| Monthly | (M,M,Q) | 52.53 | 49.80 | 61.27 | 57.31 | 4,292.43 |
| Weekly | (W,W,Q) | 51.90 | 48.72 | 60.08 | 55.26 | 4,646.17 |
| Friedman Statistic by Aggregation | | 2,210.82 | 398.43 | 529.20 | 105.63 | - |

A nomenclature is used to signify the demand aggregation, update/reorder interval, and measurement interval combinations. These take the form (*D*,*U*,*M*) where *D* signifies the demand aggregation, *U* signifies the update interval and *M* signifies the measurement interval. Thus, (M,M,Q) refers to a monthly demand aggregation, with a monthly update interval and a quarterly measurement interval.

The results for updating every period indicate that Croston's method consistently provides the lowest stock-holdings regardless of whether measuring occurs every period or every quarter. However, unlike the provisional results for ES and Croston's method from the hold-out sample of 500 line items presented in Section 9.4, the results from this larger sample of 11,203 line items do not show a clear pattern of quarterly updating providing the best results and weekly updating the worst. The best results for ES still occur with quarterly updating, while the best results for Croston's method now occur with weekly updating. The best results for both the moving average method and the previous year average method also occur with weekly updating, although these results are substantially worse than any of those from ES and Croston's method.

The percentage improvement of Croston's method over ES is observed to increase as the data moves from quarterly to monthly and on to weekly aggregation. With the stock-holdings measured every period, the improvements are 1.05, 5.21 and 6.19 percent for quarterly, monthly and weekly data respectively. The results are very similar when measuring every period and when measuring every quarter.

Two sets of Friedman statistics are presented. These test whether a series of related samples have been drawn from the same population, such that test values less than a tabulated value indicate little difference between the samples, while large test values indicate significant differences. Friedman statistics are shown firstly by forecasting method in the vertical direction and secondly by demand aggregation in the horizontal direction. In the first instance, all the Friedman statistics are substantially greater than the tabulated value of 7.81 at the 5 percent significance level with 3 degrees of freedom, indicating that there are significant differences in the average stock-holdings between the methods. In the second instance, all the Friedman statistics are substantially greater than the tabulated value of 5.99 at the 5 percent significance level with 2 degrees of freedom, indicating that there are significant differences between the average stock-holdings for quarterly, monthly and weekly data.

The three Friedman statistics by method are all of a similar magnitude, indicating that the differences in stock-holdings are similar, irrespective of the demand aggregation. Alternatively, the Friedman statistics by aggregation show greater variation. The high value for ES suggests that the differences between demand aggregations are greater for this method than they are for Croston's method, which has a comparatively low Friedman statistic. The implication is that ES is more affected by the choice of demand aggregation, while results from Croston's method are less dependent on the aggregation.

Similar results are presented in Table 9.12 for quarterly reordering, where it is observed that the implied stock-holdings for monthly and weekly data tend to increase. As a result, quarterly data generally provides the best results for each method. The Friedman statistics are once again all significant.

**Table 9.12: Average Implied Stock-Holdings - Quarterly Reordering.**

| Demand Aggregation | Nomen-clature | Expon'l Smooth. | Croston's Method | Moving Average | Prev Year Average | Friedman Statistic by Method |
|---|---|---|---|---|---|---|
| Quarterly | (Q,Q,Q) | 50.41 | 49.88 | 63.31 | 59.42 | 4,168.62 |
| Monthly | (M,Q,M) | 54.20 | 51.39 | 64.84 | 59.20 | 4,433.69 |
| Weekly | (W,Q,W) | 54.71 | 51.18 | 64.80 | 59.31 | 5,081.62 |
| Friedman Statistic by Aggregation | | 3,804.13 | 290.52 | 2,902.26 | 1,777.98 | - |

| Demand Aggregation | Nomen-clature | Expon'l Smooth. | Croston's Method | Moving Average | Prev Year Average | Friedman Statistic by Method |
|---|---|---|---|---|---|---|
| **Quarterly** | (Q,Q,Q) | 50.41 | 49.88 | 63.31 | 59.42 | 4,168.62 |
| **Monthly** | (M,Q,Q) | 54.42 | 51.61 | 65.06 | 59.42 | 4,430.52 |
| **Weekly** | (W,Q,Q) | 54.82 | 51.29 | 64.90 | 59.41 | 5,079.26 |
| **Friedman Statistic by Aggregation** | | 3,907.43 | 519.26 | 3,092.87 | 2,171.02 | - |

With little difference between measuring the stock-holdings every period and measuring every quarter, only results for measuring every period are presented in the remaining analysis.

Attention is now turned to the variations on Croston's method, with comparative results presented in the following two tables. In this analysis all the forecasting methods use the two smoothing constant values determined as optimal for Croston's method. Where a single value for $\alpha$ is applied to the bias, as required by the bias reduction method and the approximation method, this is taken as the average of the demand size constant and the demand interval constant, previously presented in Table 9.9.

Table 9.13 presents implied stock-holdings with forecasting and reordering every period. The approximation method consistently provides the lowest stock-holdings for each demand aggregation. The bias reduction method also shows an improvement over Croston's method, while the revised Croston's method is consistently worse. Weekly data generally provides the best results, although quarterly data does so under the revised Croston's method. Despite each of the methods appearing to give very similar results, the Friedman statistics indicate that there are significant differences in the average stock-holdings between the methods, as well as significant differences between the demand aggregations.

**Table 9.13: Average Stock-Holdings - Updating Every Period (Crost. and Variants).**

| Demand Aggregation | Nomen-clature | Croston's Method | Revised Croston's | Bias Reduction | Approxi-mation | Friedman Statistic by Method |
|---|---|---|---|---|---|---|
| Quarterly | (Q,Q,Q) | 49.88 | 49.89 | 49.87 | 49.80 | 588.77 |
| Monthly | (M,M,M) | 49.63 | 50.15 | 49.61 | 49.50 | 999.62 |
| Weekly | (W,W,W) | 48.62 | 50.84 | 48.59 | 48.56 | 1,400.82 |
| Friedman Statistic by Aggregation | | 353.84 | 700.61 | 316.32 | 353.65 | - |

The Friedman statistics are substantially reduced from those shown previously, indicating less variation in the stock-holdings between these four methods. With the performance of the revised Croston's method deteriorating as the updating interval moves from quarterly to weekly while the other methods improve, the Friedman statistic between the methods is relatively high for weekly data in this case.

Similar results emerge when reordering occurs every quarter as presented in Table 9.14, although in this case the stock-holdings from monthly and weekly updating tend to increase under each method, allowing the quarterly updating to provide the lowest stock-holdings in all cases.

**Table 9.14: Average Stock-Holdings - Quarterly Reordering (Croston and Variants).**

| Demand Aggregation | Nomen-clature | Croston's Method | Revised Croston's | Bias Reduction | Approxi-mation | Friedman Statistic by Method |
|---|---|---|---|---|---|---|
| Quarterly | (Q,Q,Q) | 49.88 | 49.89 | 49.87 | 49.80 | 588.77 |
| Monthly | (M,Q,M) | 51.39 | 51.95 | 51.37 | 51.25 | 837.23 |
| Weekly | (W,Q,W) | 51.18 | 53.55 | 51.16 | 51.12 | 1,013.34 |
| Friedman Statistic by Aggregation | | 290.52 | 603.69 | 300.91 | 308.70 | - |

With the approximation method providing an improvement on Croston's method, it is this method which gives the best results overall. A later section will examine whether this is the case across all demand patterns, but firstly consideration is given to the value of the implied stock-holdings from each forecasting method.

Table 9.15 presents the value of the additional stock-holdings, where the figures represent the additional value (in millions of pounds) from a base value provided by the best system. The minimum investment was obtained by the approximation method using weekly updating, where the value of the stock-holdings was £63.90m for the 11,203 line items when measuring every period. The same base value is used with both updating every period and quarterly updating. It should be borne in mind that these savings are obtained from using a backward simulation with perfect information. The same savings may not be achieved with a standard prescriptive stock control method based on the mean and variance of a lead-time forecast.

**Table 9.15: Additional Investment in Stock-Holdings Above the Best System.**

| Demand Aggregation | Expon'l Smooth. | Croston's Method | Moving Average | Prev Year Average | Revised Croston's | Bias Reduction | Approxi-mation |
|---|---|---|---|---|---|---|---|
| **Updating Every Period (Base = £63.90m; 11,203 Line Items)** | | | | | | | |
| **Quarterly** | £2.40m | £1.12m | £23.32m | £17.42m | £1.65m | £1.07m | £0.97m |
| **Monthly** | £5.37m | £0.38m | £20.14m | £13.67m | £1.87m | £0.42m | £0.23m |
| **Weekly** | £4.67m | £0.10m | £19.10m | £11.86m | £5.34m | £0.03m | **£0.00m** |
| **Quarterly Reordering (Base = £63.90m; 11,203 Line Items)** | | | | | | | |
| **Quarterly** | £2.40m | £1.12m | £23.32m | £17.42m | £1.65m | £1.07m | £0.97m |
| **Monthly** | £8.31m | £3.75m | £24.46m | £17.15m | £5.21m | £3.68m | £3.50m |
| **Weekly** | £9.00m | £4.25m | £24.72m | £17.39m | £10.13m | £4.18m | £4.08m |

Differences in the values of the stock-holdings between the methods and demand aggregations can be quite substantial. For example, ES leads to an additional £4.67m investment over the approximation method for the 11,203 line items when considering weekly updating. When translated to the entire consumable inventory of 684,000 line items, this represents an additional investment of £285m or 13.6 percent of the total value of the consumable inventory.

When measuring the stock-holding value every quarter, the base figure is £64.15m for the approximation method with weekly updating. This difference in base figures of approximately £0.25m defines the range of significance, such that values within ± £0.25m are not significantly different. Thus, for example, the implied investments from using Croston's method, the bias reduction method and the approximation method with weekly updating do not differ significantly.

Overall, Croston's method provides a favourable comparison with ES when stock levels are assessed by the additional investment. However, it is the approximation method which provides the best performance, with lower investment than ES and Croston's method on all occasions. The moving average and previous year average methods are observed to perform particularly poorly in comparison.

This section has considered the implied stock-holdings for all line items combined. Once again it is of interest to compare results by demand pattern and ascertain whether individual methods are better suited to particular demand patterns.

## 9.7 Implied Stock-Holdings by Demand Pattern

Average stock-holdings by demand pattern for Croston's method and the traditional forecasting methods are compared in Table 9.16. These results portray the almost complete dominance of Croston's method over the simpler forecasting methods for all demand patterns. It is only with the mildly erratic demand pattern under a weekly aggregation that another method gives lower implied stock-holdings and, in this instance, it is ES that provides the better result. ES tends to provide lower stock-holdings through quarterly updating and it is only the smooth demand pattern that has a better result with weekly updating. Alternatively, Croston's method and the moving average and previous year average methods all have lower stock-holdings from weekly updating across all demand patterns.

**Table 9.16: Average Stock-Holdings by Demand Pattern.**

| Demand Aggregation | Demand Pattern | Expon'l Smooth. | Croston's Method | Moving Average | Prev Year Average | Friedman Statistic |
|---|---|---|---|---|---|---|
| **Updating Every Period and Measuring Every Period** | | | | | | |
| **Quarterly (Q,Q,Q)** | **Smooth** | 68.81 | **68.20** | 88.88 | 78.68 | 2,032.16 |
| | **Irregular** | 128.72 | **127.59** | 157.30 | 154.42 | 1,583.13 |
| | **Slow-moving** | 5.76 | **5.69** | 7.35 | 6.65 | 390.16 |
| | **Mildly Erratic** | **23.75** | 24.01 | 31.35 | 28.88 | 366.61 |
| | **Highly Erratic** | 24.51 | **23.48** | 30.45 | 28.64 | 440.91 |
| **Monthly (M,M,M)** | **Smooth** | 68.66 | **66.42** | 82.11 | 74.47 | 1,795.36 |
| | **Irregular** | 135.73 | **130.49** | 154.50 | 150.79 | 1,275.60 |
| | **Slow-moving** | 6.06 | **5.63** | 7.22 | 6.49 | 336.69 |
| | **Mildly Erratic** | 26.14 | **22.94** | 31.41 | 28.83 | 417.46 |
| | **Highly Erratic** | 25.55 | **22.64** | 29.51 | 27.81 | 540.06 |
| **Weekly (W,W,W)** | **Smooth** | 66.61 | **65.48** | 78.90 | 72.73 | 1,604.61 |
| | **Irregular** | 135.59 | **127.46** | 153.40 | 146.28 | 1,429.14 |
| | **Slow-moving** | 6.04 | **5.53** | 7.08 | 6.44 | 331.87 |
| | **Mildly Erratic** | 26.01 | **22.18** | 31.14 | 28.56 | 531.75 |
| | **Highly Erratic** | 25.61 | **22.28** | 29.53 | 27.65 | 578.03 |

Percentage improvements of Croston's method over ES by demand pattern are presented in Table 9.17. In the first instance, *like with like* comparisons are made where the demand aggregations for each method are the same. Alternatively, in the last column *best with best* comparisons are made where the best result from ES is compared with the best result from Croston's method irrespective of the demand aggregation. It is observed that, in general, the greatest improvements occur with the two erratic demand patterns where the improvements are as high as 14.7 percent for the mildly erratic demand under a weekly aggregation. The slow-moving demand pattern also experiences a large improvement through the use of Croston's method. Weekly data experiences the greatest improvements while the quarterly data experiences the least, such that in the case of the mildly erratic demand pattern the improvement is negative for the quarterly data.

**Table 9.17: Improvement in Stock-Holdings of Croston's Method Over ES.**

| Demand Pattern | Comparing Like with Like | | | Comparing Best with Best |
|---|---|---|---|---|
| | Quarterly | Monthly | Weekly | |
| **Smooth** | 0.89% | 3.26% | 1.70% | 1.70% |
| **Irregular** | 0.88% | 3.86% | 6.00% | 0.98% |
| **Slow-moving** | 1.22% | 7.10% | 8.44% | 3.99% |
| **Mildly Erratic** | -1.09% | 12.24% | 14.73% | 6.61% |
| **Highly Erratic** | 4.20% | 11.39% | 13.00% | 9.10% |
| **Overall** | **1.05%** | **5.21%** | **6.19%** | **3.55%** |

In considering the results for the variations on Croston's method, Table 9.18 presents average stock-holdings by demand pattern. It is observed that the approximation method provides the best results in all cases, though sometimes equalled by Croston's method and the bias reduction method. The differences between Croston's method and the approximation method are reduced for weekly data as the selected smoothing constant value, which acts to remove the bias, is itself very small at only 0.03.

**Table 9.18: Average Stock-Holdings (Croston's and Variants).**

| Demand Aggregation | Demand Pattern | Croston's Method | Revised Croston's | Bias Reduction | Approxi-mation | Friedman Statistic |
|---|---|---|---|---|---|---|
| Quarterly (Q,Q,Q) | **Smooth** | 68.20 | 68.17 | 68.21 | **68.13** | 233.21 |
| | **Irregular** | 127.59 | 127.54 | 127.58 | **127.38** | 162.20 |
| | **Slow-moving** | 5.69 | 5.67 | 5.68 | **5.66** | 91.78 |
| | **Mildly Erratic** | 24.01 | 24.26 | 23.98 | **23.93** | 114.70 |
| | **Highly Erratic** | 23.48 | 23.42 | 23.45 | **23.46** | 131.33 |

| Demand Aggregation | Demand Pattern | Croston's Method | Revised Croston's | Bias Reduction | Approxi-mation | Friedman Statistic |
|---|---|---|---|---|---|---|
| Monthly (M,M,M) | **Smooth** | 66.42 | 66.61 | 66.43 | **66.20** | 447.10 |
| | **Irregular** | 130.49 | 131.49 | 130.45 | **130.19** | 213.58 |
| | **Slow-moving** | 5.63 | 5.73 | 5.63 | **5.62** | 111.94 |
| | **Mildly Erratic** | 22.94 | 23.62 | 22.90 | **22.89** | 124.92 |
| | **Highly Erratic** | 22.64 | 23.44 | **22.60** | **22.60** | 200.96 |
| Weekly (W,W,W) | **Smooth** | 65.48 | 68.28 | 65.45 | **65.33** | 805.40 |
| | **Irregular** | 127.46 | 131.86 | 127.39 | **127.34** | 319.68 |
| | **Slow-moving** | **5.53** | 5.96 | **5.53** | **5.53** | 102.16 |
| | **Mildly Erratic** | **22.18** | 24.31 | **22.18** | **22.18** | 134.28 |
| | **Highly Erratic** | 22.28 | 23.68 | 22.27 | **22.26** | 193.41 |

The improvements in reduced stock-holdings from the approximation method compared to ES and Croston's method are presented in Table 9.19. Again, comparisons are made for *like with like* demand aggregations as well as *best with best*, irrespective of the aggregation. In virtually all cases the approximation method improves substantially upon the results of ES, with ES only providing a better performance for the mildly erratic demand pattern under quarterly updating. The approximation method performs at least as well or better than Croston's method for all demand patterns.

**Table 9.19: Improvement in Stock-Holdings of Approximation Method.**

| Demand Pattern | Comparing Like with Like | | | | | | Comparing Best with Best | |
|---|---|---|---|---|---|---|---|---|
| | Quarterly | | Monthly | | Weekly | | | |
| | Method Improved Upon | | | | | | | |
| | ES | Croston | ES | Croston | ES | Croston | ES | Croston |
| **Smooth** | 0.99% | 0.10% | 3.58% | 0.33% | 1.92% | 0.23% | 1.92% | 0.23% |
| **Irregular** | 1.04% | 0.16% | 4.08% | 0.23% | 6.08% | 0.09% | 1.07% | 0.09% |
| **Slow-moving** | 1.74% | 0.53% | 7.26% | 0.18% | 8.44% | 0.00% | 3.99% | 0.00% |
| **Mildly Erratic** | -0.76% | 0.33% | 12.43% | 0.22% | 14.73% | 0.00% | 6.61% | 0.00% |
| **Highly Erratic** | 4.28% | 0.09% | 11.55% | 0.18% | 13.04% | 0.04% | 9.14% | 0.04% |
| **Overall** | **1.21%** | **0.16%** | **5.46%** | **0.26%** | **6.31%** | **0.12%** | **3.67%** | **0.12%** |

Implied stock-holdings by demand pattern are presented in greater detail along with additional Friedman statistics in Appendix M. Comparisons are made between ES, Croston's method, the previous year average method as the best of the two averaging methods, and the approximation method as the best method overall. With a smooth demand pattern all four methods are observed to perform better with weekly updating. However, for the remaining demand patterns ES provides better results with quarterly updating while the other three methods continue to perform better with weekly updating.

With reference to the Friedman statistics by demand aggregation, it is observed that ES provides significantly larger values than Croston's method and the approximation method for all demand patterns. This indicates that ES is more affected by the choice of the demand aggregation, irrespective of the demand pattern.

The investment in additional stock-holdings for each demand pattern is presented in Table 9.20, where updating occurs every period and measuring also occurs every period. As determined in Section 6.2.4, the smooth demand pattern has the highest average unit value and this is reflected in the comparatively high investment base.

**Table 9.20: Additional Investment in Stock-Holdings by Demand Pattern.**

| Demand Aggregation | Expon'l Smooth. | Croston's Method | Moving Average | Prev Year Average | Revised Croston's | Bias Reduction | Approxi-mation |
|---|---|---|---|---|---|---|---|
| **Smooth Demand (Base = £34.27m; 2,662 Line Items)** | | | | | | | |
| **Quarterly** | £0.67m | £0.21m | £12.70m | £9.79m | £0.19m | £0.21m | £0.17m |
| **Monthly** | £1.60m | £0.11m | £10.70m | £7.08m | £0.51m | £0.14m | **£0.00m** |
| **Weekly** | £1.14m | £0.15m | £9.69m | £5.76m | £2.65m | £0.11m | £0.07m |
| **Irregular Demand (Base = £14.26m; 2,077 Line Items)** | | | | | | | |
| **Quarterly** | £0.45m | £0.23m | £4.19m | £3.24m | £0.26m | £0.21m | £0.20m |
| **Monthly** | £1.23m | £0.25m | £3.70m | £2.57m | £0.68m | £0.24m | £0.21m |
| **Weekly** | £1.12m | £0.03m | £3.80m | £2.42m | £1.37m | **£0.00m** | **£0.00m** |

| Demand Aggregation | Expon'l Smooth. | Croston's Method | Moving Average | Prev Year Average | Revised Croston's | Bias Reduction | Approxi-mation |
|---|---|---|---|---|---|---|---|
| **Slow-Moving Demand (Base = £7.67m; 2,281 Line Items)** | | | | | | | |
| **Quarterly** | £0.91m | £0.61m | £4.13m | £2.68m | £1.07m | £0.58m | £0.54m |
| **Monthly** | £1.64m | £0.02m | £3.40m | £2.46m | £0.39m | £0.04m | £0.04m |
| **Weekly** | £1.44m | £0.01m | £3.19m | £2.10m | £0.89m | **£0.00m** | **£0.00m** |
| **Mildly Erratic Demand (Base = £4.28m; 1,975 Line Items)** | | | | | | | |
| **Quarterly** | £0.20m | £0.04m | £1.49m | £1.10m | £0.07m | £0.03m | £0.03m |
| **Monthly** | £0.64m | £0.07m | £1.58m | £1.11m | £0.22m | £0.06m | £0.05m |
| **Weekly** | £0.70m | **£0.00m** | £1.65m | £1.13m | £0.32m | £0.01m | £0.02m |
| **Highly Erratic Demand (Base = £3.32m; 2,208 Line Items)** | | | | | | | |
| **Quarterly** | £0.27m | £0.14m | £0.91m | £0.72m | £0.16m | £0.14m | £0.13m |
| **Monthly** | £0.37m | £0.04m | £0.86m | £0.56m | £0.18m | £0.04m | £0.04m |
| **Weekly** | £0.36m | **£0.00m** | £0.86m | £0.55m | £0.22m | **£0.00m** | £0.01m |

An examination of the additional investment in stock-holdings by demand pattern reveals results which remain consistent with previous observations. ES tends to provide better results with quarterly updating for all demand patterns while Croston's method, and the variations on this method, tend to provide better results with weekly updating throughout. The approximation method gives the best results in virtually all cases.

## 9.8 Concluding Remarks

Calculating average implied stock-holdings to assess forecasting performance in an inventory control environment is a reasonable alternative to the traditional measures of accuracy. This measure avoids the validity issues of the other measures whilst preventing the generation of conflicting results. Calculating the implied stock-holdings also allows the attribution of monetary costs to differences in accuracy between the methods. A disadvantage of this measure is the substantial additional processing, as 10 iterations are required on average.

While it is clear that the approximation method provides an improvement over and above Croston's method, which is itself an improvement upon ES, it is unclear which review period provides the best results. Quarterly updating consistently provides the best results when using ES. Quarterly updating also provides the best results when using Croston's method and variations on this method, with the optimal smoothing constants determined from the same dataset. However, results from the larger independent sample indicate weekly updating generally provides the best results when using these latter methods. This apparent change could be the result of using smoothing constants that were not optimal across the larger sample. As Croston's method and the variations on this method require the independent optimisation of two smoothing constants, the processing would prove prohibitive across a large sample and would not be justified within this particular study.

The RAF currently uses a monthly review period with demand forecasting provided by an ES methodology which is unlikely to be using optimal parameters. Results from a large sample of line items indicate that the approximation method provides an improvement in stock-holdings of 5.5 percent over ES under monthly review with both methods using optimal parameters. The improvement is higher for slow-moving and erratic demand items, ranging between 7.0 and 12.4 percent. Croston's method and the variations on this method require additional processing over ES as two series require smoothing, but this is hardly a concern with the computing power now available. The improvement of the approximation method over Croston's method may only be 0.3 percent with monthly updating but, as the additional processing is inconsequential, this method is superior overall.

Results from this chapter provide a more conclusive assessment of the performance of the various forecasting methods than was provided by the performance measures in the previous chapter. On that occasion none of the methods were viewed as having overall superiority. However, by considering the implied stock-holdings as an alternative measure, the approximation method stands out as having a clear advantage over the other methods.

An interesting phenomenon arising from this research is that less frequent reordering may not suffer from spikes in the forecasts over the short term. With reordering every period the system is susceptible to unnecessarily high order-up-to levels. This was observed when weekly forecasting with quarterly reordering led to lower stock-holdings than weekly forecasting with weekly reordering. Over an infinite time horizon the results would tend to be the same, but for an inventory system time is not infinite and the methods are properly assessed over a shorter horizon. This observation supports the use of longer review intervals. However, shortening the time span has the advantage of reducing the reaction time when a sudden change in the demand pattern occurs.

With the research into forecasting for the ordering and stock-holding of consumable spare parts now complete, the final chapter presents the main conclusions.

# 10. CONCLUSIONS

This chapter presents the main conclusions from this research. The properties of a spare parts inventory are initially examined using the Royal Air Force (RAF) consumable inventory as an example. A methodology for demand pattern classification using RAF demand and replenishment lead-time data is reviewed in the second section. The third section reviews the forecasting performance of Croston's method, as well as three recently developed modifications to this method, using traditional measures of forecast accuracy, while the fourth section reviews the performance by implied stock-holdings. The fifth and final section describes some future issues for consideration.

The purpose of this research was to identify and assess the usefulness of models put forward in the academic literature for improving the forecasting for the ordering and stock-holding of consumable spare parts. The cost-effective management of spare parts is a problem faced by many organisations across a range of industries, and it is an area that has received increasing attention over the years. New models have not been developed in the course of this research, but instead existing models have been assessed using large volumes of actual data. A key part of the assessment was to examine the properties of a spare parts inventory and determine the applicability of the proposed models in a practical setting.

## 10.1 Properties of a Spare Parts Inventory

The RAF is dependent on readily available spare parts for in-service aircraft and ground systems in order to maximise operational capability. However, as with any organisation, maintaining a large and diverse inventory capable of meeting all customer demands as they arise requires a substantial investment. The RAF manages a large consumable inventory with approximately 700 thousand stock-keeping units with a total value of £2.1 billion. Cut-backs in defence budgets in recent years have led to a requirement for a reasoned and scientific analysis of the RAF inventory as an aid to obtaining cost-efficiencies in the supply environment.

The RAF is fortunate in having long demand transaction histories for all line items in a readily accessible format. Over 375,000 line items have at least six years of individual demand transactions. This extensive demand information, together with a source of accurate replenishment lead-time data, allows a full and extensive analysis of practical models across a range of demand patterns. For a model to be considered useful in this instance, it should measurably improve forecasting and inventory control, and should not be overly complex so as to require excessive processing. This is a key requirement given the large inventory under consideration.

A large proportion of the inventory is described as having an erratic or intermittent demand pattern, which is characterised by infrequent transactions with variable demand sizes. A further large proportion has a slow-moving demand pattern, which is also characterised by infrequent transactions, although in this case demand sizes are always low. Both erratic demand and slow-moving demand can create significant problems as far as forecasting and inventory control are concerned. The management of spare parts is a problem faced by many organisations across a range of industries.

An erratic demand pattern may occur when there is a large number of small customers and a few large customers, or when the frequency of customer requests varies. Small variations in demand magnified along a supply chain can also lead to erratic demand, as can correlation between customer requests, which may itself be due to sympathetic replacement. In large repair facilities erratic demand may arise through the pooling of repair items in order to minimise the number of set-ups.

Spare parts with an apparent lumpy demand history need not always be treated as erratic. Most often overhauls of large items of equipment are scheduled far in advance of the time at which the spare parts are required. An improvement in the flow of information will allow the requirements to be included as scheduled demand, removing the need for forecasting. Forecasting requirements are also reduced if the demands are fixed in size or the transactions occur at fixed intervals.

Slow-moving spare parts are often held as insurance against the high costs that would otherwise be incurred if an item fails and a spare part is not available. The forecasting and control of slow-moving line items is made more difficult by having few historical demand recordings. A further difficulty is their inflexibility, as far as over-stocking is concerned. Excess spares may well become obsolete before they are required. On the other hand, a simplification for slow-moving line items is that rarely is it necessary to hold more than two spares, so the possible decisions are few in number.

Once again, the forecasting of slow-moving spare parts need not always be an issue, if, for example, they are required for an overhaul that is scheduled in advance. Alternatively, they may give adequate warning through wear of the need for future replacement and a spare can be provisioned at such time.

The RAF utilises a classical periodic review inventory management system, although there are a number of factors within the operating environment that combine to form a unique forecasting, ordering and stock-holding system. Large stock-holdings have traditionally been maintained for reasons not normally faced by other industries, such as the necessity to maintain stocks in case of war, the relatively high cost of procurement beyond initial provisioning, and the high costs of stock-out.

The less frequent usage of slow-moving and erratic demand items leads to few replenishment orders being placed. Therefore, there is very little actual lead-time data available to the RAF. Over a six year period, only 19 percent of the current inventory has at least one lead-time observation. Lead-times are comparatively long at 12 months on average, although they can be anything up to 60 months. A replenishment order quantity can be constrained by the supplier, either as a minimum order quantity or as a multiple of the packaged quantity, and 24 percent of line items are restricted in this manner.

A modified chi-square goodness-of-fit test was used to test whether lead-time observations for individual line items fitted a range of probability distributions. Results from 161 line items, with 12 or more lead-time observations, indicated several distributions were candidates. The sample sizes available in the early stages of the analysis were too small for determining the best fitting distribution, with the geometric, negative exponential, negative binomial and gamma distributions all fitting over 85 percent of line items at the 5 percent significance level.

Previous research has not adequately examined and commented upon the existence of autocorrelation in demand data due to a lack of data series suitable for testing. Many of the published models assume independence between successive demand sizes, independence between successive demand intervals and independence between the demand sizes and intervals. However, an analysis of RAF data indicated large positive and negative autocorrelation and crosscorrelation coefficients in the data. With the demand data being severely skewed, the commonly used Pearson's method was found to be inappropriate for measuring correlation and alternative methods for reducing the variation were investigated. Using a natural logarithm transformation, approximately a quarter of the line items were found to be significantly autocorrelated and/or crosscorrelated, suggesting many models in the literature are too simplistic with their assumptions.

Knowledge of the replenishment lead-time is a primary requirement of any inventory management system, however, in reality it is often the case that the lead-time distribution and associated parameter values have to be assumed due to a lack of recorded observations. With few, or more likely zero, lead-time observations for each line item, the usefulness of the data available to the RAF is restricted on an individual item basis. Therefore, line items likely to have similar lead-time patterns were grouped together with the resultant summary statistics applying to the entire group. Several predictors for grouping line items were considered, with ANOVA tests and regression analysis indicating the manufacturer, the range manager and the set lead-time value explained the most variation. Cluster analysis was used to place the lead-time observations into groups suggested by the data. The grouping procedure allows all line items to be assigned lead-time parameter values, regardless of whether or not they have any actual lead-time observations.

The grouping of line items, and the subsequent combining of actual lead-time observations, also allows a more conclusive goodness-of-fit test over the one conducted using individual line items. With the larger sample size it is observed that the normal distribution, which is often used by probability models, provides a poor representation of lead-time demand and a better choice is provided by the log normal distribution. On the other hand, a Poisson arrival process with demand sizes following a geometric distribution, thus forming the commonly used stuttering Poisson distribution, provides a reasonable representation of reality. The logarithmic distribution is also suitable for modelling the demand sizes.

With lead-time parameter values assigned to each line item, demand classifications could then be applied to the observed lead-time demand patterns. A particular forecasting model, for instance, may not perform as well if demand does not adhere to a specific pattern, thus demand classifications are a useful part of the analysis. Assigning demand classifications to the RAF inventory is reviewed in the next section.

## 10.2 Classifying Demand Patterns

An analytical demand classification methodology from the literature has been extended and applied to the RAF inventory. The method decomposes the lead-time demand into the constituent causal parts of demand frequency, demand size and lead-time. In this instance, RAF line items are classified into smooth, irregular, slow-moving, mildly erratic or highly erratic demand patterns. The irregular demand pattern has been added, as the original classifications did not differentiate the observed patterns adequately.

Demand classifications are made on the basis of statistics generated from the data series themselves, with boundaries between classifications set at management's discretion. The selected boundaries in this research led to some 13 percent of the inventory being classed as smooth, 12 percent as irregular, 37 percent as slow-moving, 16 percent as mildly erratic and 22 percent as highly erratic. Boundaries defining a particular pattern apply to the inventory under examination and are not immediately transferable between organisations, such that a demand pattern classed as smooth in this instance may be viewed as slow-moving elsewhere. The classification methodology itself, however, is readily transferable.

The classification methodology requires a long demand history for each line item. Problems obviously arise when a new part is introduced and the demand is initially unknown. Therefore, the classifications were examined to determine whether there were common traits among the line items within each demand pattern, beyond the classification criteria, which could be used to pre-classify new parts. By definition, the two erratic demand patterns are differentiated by lead-time only, with the highly erratic demand pattern having a higher coefficient of variation in the lead-time than the mildly erratic demand pattern. Alternatively, the smooth, irregular and slow-moving demand patterns follow similar lead-time distributions, as the lead-time is not a determining factor for any of these patterns.

The demand patterns were also compared using the three previously identified grouping variables, namely the manufacturer, the range manager and the set lead-time value. A three-variable contingency table tested the hypothesis that the demand classifications were independent of the lead-time groupings within each variable. It was observed that the attainment of a smooth, mildly erratic or highly erratic demand pattern was in fact dependent on the predictors, while the attainment of an irregular or slow-moving demand pattern was not. Although the demand pattern is not wholly independent of the predictors, there tends to be no clear-cut influence that would assist the prior determination of the demand pattern by this means.

With consideration given to the transaction frequency, the slow-moving demand pattern has a low transaction rate with over 50 percent of line items within this category experiencing only one demand over a 72 month period. On average, these line items experience about 3 transactions overall, although such a demand pattern is still observed when there are in excess of 14 transactions. An erratic demand pattern, either mildly erratic or highly erratic, also experiences a low transaction frequency, albeit at a higher rate of 8 transactions on average. The irregular demand pattern has a transaction rate of about 1 per month while the smooth demand pattern has a transaction rate in excess of 2 per month.

Looking at the individual demand sizes, the majority of slow-moving line items experience demands of one unit, although the average demand size for this classification is 4 units. The average demand size for the smooth demand pattern is 7 units, while the highly erratic demand pattern has a higher average demand size of 12 units compared to 9 units for the mildly erratic demand pattern. Overall, it is the irregular demand pattern that has the highest average demand size of 14 units.

For the most part, there is considerable overlap with the demand pattern classifications for both the transaction frequency and the demand size, with few clear boundaries in either case. The same situation arises with their joint consideration. It is only when the demand rate is less than 0.04 units per month that only one demand pattern occurs, in which case it is slow-moving demand. An irregular demand pattern is not observed until the demand rate is in excess of 0.10 units per month, although it is the smooth demand pattern that has the highest average demand rate.

All of these observations suggest that there is no relatively simple way to categorise a line item without considering the demand frequency, demand size and lead-time, thus requiring a long demand history. In cases where a demand history does not exist, it is necessary to match the new part with one it is replacing or one that it is most similar to.

Many of the observed characteristics of the RAF inventory are likely to be found in other spare parts inventories. A large proportion of demand will be slow-moving or erratic and lead-time observations will be scarce or non-existent, leading to similar difficulties in forecasting and inventory control. The particular characteristics which differentiate the RAF inventory from other organisations are the sheer size of the inventory, the large stock-holdings for individual line items and the long lead-times of anything up to 5 years. Despite such differences, the observations from this research are generally applicable to other organisations. The numerical results may differ elsewhere but the methodologies used in this research could still be applied, including the method for assigning lead-time observations when they are unknown and the method for classifying the demand pattern. The results, with regard to the fitting of probability distributions, are also likely to find wider application.

The next section reviews the performance of several forecasting methods specifically put forward as suitable for erratic demand using traditional measures of forecasting accuracy.

## 10.3 Reviewing the Performance of Croston's Method

Forecasting erratic demand is problematical due to variability in the demand size and variability in the transaction interval. Exponential smoothing (ES) is often used for demand forecasting in a spare parts inventory. The inadequacy of ES for handling time series with many periods of zero demand is well documented, although the comparative accuracy of the method depends on the performance measure used and the manner in which it is implemented.

Overall results, in terms of forecasting performance, indicate that all forecasting methods produce very poor results in the traditional sense. The MAD, RMSE and MAPE results are observed to be horrendous in many cases. This is predominantly a reflection of the fact that real data from a spare parts inventory has been used for all analyses.

An important contribution of this research stems from generating results that are meaningful in the real world, such that models are assessed by a means appropriate to their actual implementation. For example, in recognising that the purpose behind demand forecasting is to determine usage over a replenishment lead-time, the forecasting performance is similarly assessed over the lead-time period. Forecast comparisons are made using quarterly, monthly and weekly demand aggregations as some organisations may have a choice in the format of their data and would therefore be interested in the comparative results. In addition, forecast comparisons are made only after a demand has occurred, as well as in every period, as it is only after a demand has occurred that it would be necessary to initiate a new replenishment order.

The comparative performance of four forecasting methods, namely ES, Croston's method, a one year moving average, and a simple previous year average, was initially assessed. Croston's method, which derives a demand forecast by separately smoothing the size of the demands and the interval between demands, has been put forward in the literature as a more suitable alternative to ES when demand is erratic. However, in using real RAF data from 18,750 line items, the results are somewhat mixed and the theoretical superiority of this method does not always show through in practice.

In considering the forecasting performance using the standard measures of accuracy, including MAD, RMSE, MAPE and MdAPE, the results are varied with no forecasting method consistently performing the best. However, patterns do emerge which suggests that some methods are better than others under particular situations. For instance, Croston's method performs well when comparing the forecast value with the one-period ahead demand, particularly in the case of monthly and weekly data. On the other hand, ES completely dominates regardless of the demand aggregation when comparing against lead-time demand and measuring in all periods. When it comes to lead-time demand comparisons in periods of demand only, Croston's method performs well with quarterly data, while the simple previous year average method provides the best results with monthly and weekly data.

ES and the moving average methods are observed to have an advantage over Croston's method when measuring the accuracy in all periods, as these methods are themselves updated every period. On the other hand, Croston's method is updated in periods of demand only and will therefore tend to over-forecast to a greater degree when faced with a series of zero demands. When forecasting demand for spare parts in general, all forecasting methods tend to more frequently over-estimate, rather than under-estimate, the actual demand. With many periods having zero demand, if the predicted demand is not similarly zero then over-forecasting occurs. Smoothing methods tend to over-estimate demand more frequently than moving average methods as the forecasts from smoothing methods decay towards zero over a number of periods of zero demand, whereas averaging methods drop to zero more readily.

The forecasting performance of smoothing methods is dependent on the values of the smoothing parameters, with the best values often obtained by using information drawn from the time series themselves. In accordance with a technique used elsewhere, the smoothing values in this analysis were obtained by determining the optimum values from a representative hold-out sample of 500 line items. The forecasting performance, as measured by MAPE, was observed to vary markedly depending on the value of the smoothing constants. The optimal values tend to decrease as the demand aggregation moves from quarterly to monthly and down to weekly.

In considering the forecast performance by demand pattern, Croston's method performs well with the smooth and slow-moving demand patterns when comparing against the one-period ahead demand at all aggregation levels. It is only with weekly data under this situation that Croston's method performs the best for the erratic demand patterns as well, otherwise ES is the best for erratic demand. ES provides the best performance for all demand patterns when comparing against lead-time demand in all periods. However, when comparing in periods of demand only, ES is not the best method for any pattern. Instead, Croston's method is the best for slow-moving demand at all aggregation levels, and the best for erratic demand with quarterly data, while the previous year average method is best for smooth and irregular demand under all aggregations.

Overall, this research has shown that Croston's method performs well when comparing the forecast against the one-period ahead demand. Support for Croston's method in early studies was obtained from comparisons of this type and therefore tended to show the method in the best light. In this research, as well as recent results from the literature, Croston's method is not always observed to be the best method for erratic demand. Less sophisticated, and in fact very simple, methods can provide better results.

Although Croston's method has been described in the literature as having practical tangible benefit, in common with the findings of this research, the method often produced only modest benefits when using real data. Recent research has identified an error in Croston's mathematical derivation of the demand estimate. The method was observed to reduce the inherent forecast bias of ES but it failed to eliminate it completely. Modifications to Croston's method have been suggested in the literature which seek to further eliminate the bias. These alternative methods are known as the revised Croston's method, the bias reduction method and the approximation method.

This research continued with comparisons between Croston's method and the three alternative methods using the traditional measure of forecasting accuracy across the same 18,750 line items. The results are remarkably consistent with the approximation method providing the best results for most measures irrespective of the demand aggregation or whether comparing the one-period ahead or lead-time demand. The bias reduction method similarly tends to provide better results than Croston's method, while the results from the revised Croston's method tend to be worse than the original method.

In considering the performance of all of the forecasting methods, comparative results using MAPE indicate ES still dominates when comparing against lead-time demand in all periods. The approximation method is the best overall when considering the one-period ahead demand, and also when comparing against lead-time demand in periods of demand only when using quarterly and monthly data, with the bias reduction method providing the best results for weekly data.

An important observation from this research is that identifying the best forecasting method in an inventory control environment is not entirely objective when using the traditional measures of accuracy. The measures themselves are subject to questions of validity and different conclusions arise depending on which measure is utilised. In addition, the appropriate type of forecast comparison, whether it be with the one-period ahead demand, the lead-time demand in all periods or the lead-time demand in periods of demand only, is also a matter open for debate.

## 10.4 Measuring Performance by Implied Stock-Holding

This research has highlighted some of the weaknesses of the traditional measures of forecasting accuracy when applied to inventory control. A more appropriate measure in this case is to calculate the implied stock-holdings resulting from each method and compare the additional investment required. In order to establish a common basis for comparison, exact safety margins are determined which provide service levels of 100 percent under each forecasting method.

The implied stock-holding methodology removes the need to select from conflicting measures of accuracy and the different means of implementation by directly measuring a key performance indicator. This measure allows the stock-holding requirements of each forecasting method to be presented in immediately comparable monetary terms. The disadvantages of this methodology are that the calculations are more complicated and time-consuming, with an average of 10 iterations required for each line item, and not all line items can be resolved in this manner.

A number of factors outside the forecasting methods affect the calculated stock-holdings and these need to be considered to ensure comparable results. Firstly, the simulation period should be selected so as to discard observations prior to the first delivery, otherwise the calculations are affected by the opening stock level and are not wholly dependent on the forecast values. Other factors for consideration arise from bucketing individual demand observations into quarterly, monthly and weekly aggregations. Minor calculation differences occur due to an incomplete overlap between the aggregations. For example, a quarter comprises three months or thirteen weeks, but a week starts on a Sunday while a quarter may not. Fortunately, any differences in period coverage tend to cancel out over many series.

A potentially more serious issue arises when comparing results between the various demand aggregations due to an *aggregation bias*. The bias can either increase or decrease the relative average performance resulting from a particular aggregation. This occurs through greater averaging of the stock-holdings as the updating period lengthens. The bias is at a minimum when weekly demands are evenly distributed over a quarterly period. Although the bias would tend to cancel out over many series, it can be completely eliminated by measuring the stock-holdings at a common time interval. A quarterly interval provides the lowest common denominator in this instance.

Calculated stock-holdings are also obviously affected by the selected smoothing parameters when smoothing methods are used. Optimal parameters for minimising the implied stock-holdings need not be the same as those that provide the lowest MAPE values. A new set of smoothing constant values were determined for ES and Croston's method using the hold-out sample of 500 line items, although it is not possible to resolve the stock-holdings for all line items due to excess stock in the system etc., and a total of 274 line items provided the results. The optimal values were found to decrease as the data moved from quarterly to monthly and on to weekly aggregation, allowing greater damping of the weekly data. Provisional results from this small sample size indicated that quarterly updating provided the best results for both ES and Croston's method, while weekly updating provided the worst.

However, when the sample was extended to 18,750 line items (11,203 resolved), the best results for ES continue to be obtained with quarterly updating, while the best results for Croston's method are now obtained with weekly updating. Croston's method does, however, provide better results than ES for all demand aggregations. There is a possibility that the selected smoothing values are far from optimal when applied to the larger sample and alternative values would lead to different conclusions.

Overall, the best forecasting method for a spare parts inventory is deemed to be the approximation method. This method allows the lowest stock-holdings across all demand patterns, with the greatest improvements occurring with erratic and slow-moving demand. Extrapolating sample results across the whole of the RAF consumable inventory indicates an additional investment in stock-holdings of £314m is required by ES over and above that required by the approximation method when using monthly updating. This represents 15.0 percent of the total value of the inventory. This research, in general, suggests that the RAF tends to carry excessive safety stock. Significant savings can be made by using more accurate forecasting methods and cutting safety stock with no appreciable reduction in service levels.

The analyses undertaken in the course of this research have used data obtained from the RAF. However, it is likely that the findings are applicable in any environment where a spare parts inventory is maintained and, indeed, many of the findings are applicable where demand follows an erratic demand pattern.

## 10.5 Areas for Further Research

This research has provided a practical assessment of a number of models designed to improve the management of a spare parts inventory system. However, other questions have been raised in the course of this research which could be addressed by further analysis. Principal areas in which further research would prove beneficial are explored in this final section of the thesis.

Overall findings indicate that the approximation method provides the best results in terms of the lowest implied stock-holdings. These results were obtained using the two optimal smoothing values derived from a hold-out sample for Croston's method. As the selected smoothing values were found to have a major effect on the forecasting performance of all smoothing methods, it is likely that the results for the approximation method could be improved by deriving values that are optimal for the method itself. Furthermore, the approximation method has a third parameter which acts as a deflator to remove the bias. This value could also be optimised. In this analysis the third parameter was simply the average of the two smoothing constants, although another value may prove more appropriate.

Forecasting methods which remove the bias have been shown to be useful by this research. Croston's method provides only limited improvements in performance over ES, while the modifications to Croston's method provide further improvements. All of these methods continue to utilise a smoothing approach, although improved performance may be possible by using a more complicated model to generate the demand size in unison with the demand interval. The methods which seek to remove additional bias over Croston's method have only recently been developed and their continued development may prove beneficial.

Croston identified four tests for monitoring the performance of his method, as described in Section 7.2. These tests were provided as control indicators to check that the demand pattern had not suddenly changed. An area for further research would be to examine the impact on forecasting performance when the tests were triggered, and in which combination this occurred, for example Test (i), Test (i) and Test (ii), Test (i) and Test (iii), etc. The tests could also be applied when using the variations on Croston's method.

Although the implied stock-holdings provide a better performance measure than the traditional measures of accuracy, there is a substantial increase in the calculations required. Determining optimal values is a major task for Croston's method and the modifications to this method, as two separate values are required. In this research, global optimal values were derived from a hold-out sample. However, it may be beneficial to determine optimal values by demand pattern to enable further improvements in performance. Optimal smoothing values were determined by demand pattern using MAPE and improvements to this measure of up to 15 percent were realised. Taking this a stage further, there may be additional benefit in determining optimal values for each individual line item. Further research is required to evaluate whether onerous simulation runs are worthwhile as they are not easily automated.

MAPE, which has often been reported in this analysis, is perhaps not the most appropriate of the traditional measures of accuracy for making comparisons between forecasting methods when demand is slow-moving or erratic. Percentage errors cannot be calculated when the actual value is zero and therefore MAPE does not fully describe the results. One measure which is receiving widespread acceptance in such cases is the relative geometric root mean square error (RGRMSE), calculated as the ratio of the geometric mean RMSE between one forecasting method and another. Subsequent research in this area may be better served by the use of this alternative measure.

This research has not conclusively determined the best review interval for inventory control. Results differ between the hold-out sample used for setting the optimal smoothing values and the larger independent sample. In some cases quarterly updating provides the lowest stock-holdings, while in other cases the lowest stock-holdings are provided by weekly updating. Very rarely does monthly updating provide the lowest. There is a possibility that the smoothing values are far from optimal when applied to the larger sample, and different values would lead to different conclusions. There are other factors which should also be considered when recommending an update interval, such as the format of the available data and the ease of transforming the aggregation level. Weekly updating might require an excessive work-load, while quarterly updating may not react to changes quickly enough. Further research would be useful in aiding any such recommendations.

An area in which additional research would prove most beneficial lies in the setting of safety stock levels for operational purposes. The safety stock levels that were determined in this study, to give a 100 percent service level when demand is known, may provide the basis of a reasonable approach in determining safety stock for a given service level when demand is uncertain. Safety stock is normally determined by examining demand over a lead-time and relating this to a service level performance by assuming some lead-time demand distribution. This has been a subject of considerable research in the past but to date no satisfactory method has been proposed which is easily implementable and has practical application. A simulation methodology that increments the order-up-to level until a specified service level is reached offers a fresh approach.

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# APPENDICES

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# Appendix A - Data Usage Summary

Given the large quantity of data used in this research, this appendix cross-references the creation and usage of each dataset in terms of the tables and figures in which results are presented. Raw data was gathered up to January 2000.

**Table A.1: RAF Data Usage.**

| Dataset | Dataset Description | Number of Items | Data Usage | |
|---|---|---|---|---|
| | | | Section | Results |
| **1** | All RAF consumable line items. | 684,095 | 4.1<br>4.2<br>4.3 | Figure 4.2<br>Figure 4.3<br>Figure 4.6, Figure 4.7 |
| **2** (created from 1) | Known initial provisioning date. | 492,800 | 4.1 | Figure 4.1 |
| **3** (from 1) | Contractor's minimum batch quantity (CMBQ) greater than one. | 71,260 | 4.2 | Figure 4.4 |
| **4** (from 1) | Primary packaged quantity (PPQ) greater than one. | 120,077 | 4.2 | Figure 4.5 |
| **5** (from 2) | Mature line items with at least six years demand history. | 376,359 | 4.4<br>4.5.2 | Figure 4.8<br>Table 4.2, Figure 4.10 |
| **6** (from 5) | At least one demand occurrence in six years. | 223,746 | 4.5.1 | Table 4.1, Figure 4.9 |
| **7** (from 6) | Sample line item to illustrate customer demand (5310-00-2752000). | 1 | 2.1.1 | Figure 2.1 |
| **8** (from 6) | Between 50 and 99 demand transactions over six years. | 12,644 | 4.5.3<br>4.5.4<br><br>4.5.5 | Figure 4.11<br>Table 4.3, Figure 4.14, Figure 4.16<br>Figure 4.18, Figure 4.20 |
| **9** (from 8) | Non-constant demand size. | 12,251 | 4.5.3<br>4.5.6 | Figure 4.13<br>Table 4.5, Figure 4.21 |
| **10** (from 9) | Sample line item for autocorrelation and crosscorrelation analysis (1005-12-1700095). | 1 | 4.5.3<br>4.5.5<br>Appendix C | Figure 4.12, Figure 4.15<br>Figure 4.17, Figure 4.19<br>Table C.1, Table C.2, Table C.3, Figure C.1, Figure C.2, Figure C.3, Figure C.4 |

| Dataset | Dataset Description | Number of Items | Data Usage | |
|---|---|---|---|---|
| | | | Section | Results |
| **11** (created from 8) | Sample line item to illustrate interval between transactions' calculations (1005-12-1823955). | 1 | 4.5.5 | Table 4.4 |
| **12** | Replenishment contracts placed and completed between Sept 1993 and Oct 1999 (representing 163,452 line items). | 268,330 | 5.1 | Figure 5.1 |
| **13** (from 12) | Line items with 12 or more non-constant lead-time observations. | 161 | 5.1.2 | Table 5.4 |
| **14** (from 5 and 12) | Current line items with at least one lead-time observation. | 72,712 | 5.2.1<br>5.2.2<br><br>5.2.3<br><br>5.2.4<br><br>5.2.6<br>5.2.7<br>5.2.8<br>5.3.1<br><br><br>5.3.2<br><br><br>5.3.3<br><br>5.3.4 | Table 5.5<br>Table 5.6, Table 5.8,<br>Table 5.9, Table 5.10<br>Table 5.11, Table 5.12,<br>Table 5.13<br>Table 5.14, Table 5.15,<br>Table 5.16, Figure 5.3<br>Table 5.18<br>Table 5.19<br>Table 5.20, Figure 5.4<br>Table 5.21, Table 5.22,<br>Figure 5.5, Figure 5.6,<br>Figure 5.7<br>Table 5.23, Table 5.24,<br>Figure 5.8, Figure 5.9,<br>Figure 5.10<br>Table 5.25, Figure 5.11,<br>Figure 5.12, Figure 5.13<br>Table 5.26 |
| **15** (from 14) | Set purchasing lead-time (PLT) up to 20 months. | 71,922 | 5.2.4 | Table 5.17 |
| **16** (same as 6) | Mature line items with positive demand, and lead-time allocated from the lead-time grouping cube and classification by demand pattern. | 223,746 | 6.1<br>6.2.1<br>6.2.2<br><br><br><br>6.2.3<br><br>6.2.4 | Table 6.2, Figure 6.1<br>Table 6.3, Figure 6.4<br>Table 6.4, Table 6.5,<br>Figure 6.5, Figure 6.6,<br>Figure 6.7, Figure 6.8,<br>Figure 6.9<br>Table 6.7, Figure 6.10,<br>Figure 6.11, Figure 6.12<br>Table 6.8 |
| **17** (from 16) | Sample line items to illustrate the identified demand patterns, and test the aggregation bias. (037F-99-6082626, 5306-00-8076329, 1560-99-7978970, 1005-12-1706156, 1560-00-733993). | 5 | 6.1<br>9.3.3<br>9.3.7 | Figure 6.2<br>Table 9.4<br>Table 9.6 |

| Dataset | Dataset Description | Number of Items | Data Usage | |
|---|---|---|---|---|
| | | | Section | Results |
| **18** (created from 16) | Mildly erratic and highly erratic demand items. | 84,308 | 6.2.1 | Figure 6.3 |
| **19** (from 8 and 16) | Comparison of initially selected line items and their later classification. | 12,644 | 6.2.5 | Figure 6.13 |
| **20** (from 14) | Five line items with at least 10 lead-time observations (1650-99-7648229, 1680-99-6546054, 5330-99-8161994, 5985-99-6383805, 5985-99-6462883), plus 231 others within the B-D-E cluster grouping. | 236 | 6.3.3 | Table 6.12, Figure 6.14, Figure 6.15 |
| **21** (from 14) | Line items from groupings with at least 200 lead-time observations. | 82 | 6.3.3 | Table 6.13 |
| **22** (from 16) | Sample line items with equal representation by demand pattern (3,750 of each pattern). | 18,750 | 6.2.3<br>6.2.6<br>7.4.1<br><br>7.4.2<br><br>7.4.3<br><br>8.2<br><br>8.3<br><br>8.5<br>9.3.3<br>Appendix E<br>Appendix F<br>Appendix H<br>Appendix I<br>Appendix J<br><br>Appendix L | Table 6.6<br>Table 6.9<br>Figure 7.9, Figure 7.10,<br>Figure 7.11<br>Table 7.8, Table 7.9,<br>Table 7.10<br>Table 7.11, Table 7.12,<br>Table 7.13<br>Table 8.1, Table 8.2,<br>Table 8.3<br>Table 8.4, Table 8.5,<br>Table 8.6<br>Table 8.7<br>Figure 9.7, Figure 9.8<br>Table E.1<br>Table F.1<br>Table H.1<br>Table I.1<br>Table J.1, Table J.2,<br>Table J.3<br>Table L.1 |
| **23** (from 22) | Line items with at least 20 demand transactions and non-constant demand sizes. | 6,795 | 6.3.1<br>6.3.2 | Table 6.10<br>Table 6.11 |
| **24** (from 22) | Sample line item for illustrating Croston's method (1005-12-1706162). | 1 | 7.1<br>7.2<br>7.3.2 | Table 7.1<br>Table 7.2<br>Table 7.3 |

| Dataset | Dataset Description | Number of Items | Data Usage | |
|---|---|---|---|---|
| | | | Section | Results |
| **25** (created from 22) | Hold-out sample line items with equal representation by demand pattern (100 of each pattern). | 500 | 7.3.3<br>7.3.4<br>8.4.1<br>8.4.2<br>9.3.8<br>9.4<br>Appendix K | Figure 7.1, Figure 7.2, Figure 7.3, Figure 7.4<br>Table 7.5, Table 7.6, Figure 7.6, Figure 7.7<br>Figure 8.1, Figure 8.2, Figure 8.3<br>Figure 8.4, Figure 8.5, Figure 8.6<br>Table 9.7, Table 9.8<br>Table 9.9, Table 9.10<br>Table K.1, Table K.2 |
| **26** (from 25) | Line item with highest MAPE in sample (526MM-14-4312655). | 1 | 7.3.3 | Table 7.4, Figure 7.5 |
| **27** (from 22) | Alternative sample line items with equal representation by demand pattern (includes line items from Dataset 25). | 5 × 500 | 7.3.4 | Table 7.7, Figure 7.8 |
| **28** (from 22) | Sample line item for illustrating inventory system simulation (026LX-99-6294386). | 1 | 9.1<br>9.2<br>9.3.1<br>9.3.3<br>9.3.4<br>9.3.5<br>9.3.6<br>9.3.7<br>9.3.8<br>9.4 | Table 9.1<br>Table 9.2, Table 9.3, Figure 9.2, Figure 9.3<br>Figure 9.4<br>Figure 9.5, Figure 9.6<br>Figure 9.9<br>Figure 9.10<br>Table 9.5, Figure 9.11, Figure 9.12<br>Table 9.6<br>Figure 9.13<br>Figure 9.14 |
| **29** (from 22) | Line items for which it is possible to resolve implied stock-holdings for each and every forecasting method. | 11,203 | 9.6<br>9.7<br>Appendix M | Table 9.11, Table 9.12, Table 9.13, Table 9.14, Table 9.15<br>Table 9.16, Table 9.17, Table 9.18, Table 9.19, Table 9.20<br>Table M.1, Table M.2, Table M.3, Table M.4, Table M.5 |

## Appendix B - Hypothesis Testing

When testing some claim about population parameters at some level of significance the procedure is called hypothesis testing. Hanke *et al*. [33] outline the steps required to conduct a hypothesis test:

(i) State the hypothesis being tested (the null hypothesis - denoted $H_0$) and state the alternative hypothesis (the one accepted if $H_0$ is rejected - denoted $H_1$).

(ii) Collect a random sample from the population and compute the appropriate sample statistic.

(iii) Assume the null hypothesis is true and consult the sampling distribution from which the sample statistic is drawn.

(iv) Compute the probability that such a sample statistic could be drawn from the sampling distribution.

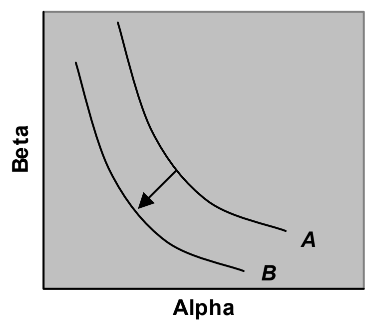
(v) If this probability is high *do not reject* the null hypothesis or if the probability is low *reject* the null hypothesis, with low chance of error.

When hypothesis testing two types of errors can occur, as shown by Table B.1. It is always hoped that the correct decision concerning the null hypothesis will be reached, but there is the possibility of rejecting a true $H_0$ and failing to reject a false $H_0$. The probabilities of these events are *alpha* ($\alpha$) and *beta* ($\beta$) respectively, with alpha also known as the significance level of the test.

**Table B.1: Hypothesis Test Errors.**

| Actual | Do Not Reject $H_0$ | Reject $H_0$ |
|---|---|---|
| $H_0$ True | Correct decision | Type I error probability $\alpha$ |
| $H_0$ False | Type II error probability $\beta$ | Correct decision |

In selecting a significance level in a practical situation, the appropriate question is, what probability of rejecting a true null hypothesis is acceptable? A low probability of committing a Type I error generates a high probability of committing a Type II error, and vice versa, for a given sample size, as shown in the figure below.




From curve $A$ it is apparent that a small choice for alpha results in a large value for beta. When the sample size is increased, the choice moves to curve $B$, where lower risks of both Type I and Type II errors are possible. The appropriate value for alpha depends on the penalties associated with Type I and Type II errors. If rejecting a true $H_0$ is far more costly than accepting a false $H_0$, a small alpha should be chosen; for the reverse, a larger alpha should be chosen so that beta is reduced.

# Appendix C - Investigating Autocorrelation in Demand Data

There are several alternative methods for determining whether there is autocorrelation present in demand data that is more than likely to be skewed. Each method aims to reduce the variation among the observed data and in particular remove the impact of extreme values. The methods considered in this analysis include:

(i) Autocorrelation coefficients as a whole.

(ii) Natural logarithm transformations.

(iii) Spearman rank-order correlation.

(iv) Correlation of successive high-low observations.

(v) Frequency of high-low observations.

The effectiveness of each method will be considered in turn using the observed demand sizes for the same sample line item for illustrative purposes throughout.

## Autocorrelation Coefficients as a Whole

The first method for determining whether a skewed series is autocorrelated involves examining the Pearson correlation coefficients as a whole. Often a few of the coefficients will be significant while the majority are not, in which case an overall test of significance is provided by a modified Box-Pierce $Q$-statistic described by Ljung and Box [48]. Based on the $\chi^2$ (chi-square) distribution of the autocorrelation coefficients, this test is capable of determining whether several coefficients are significantly different from zero. The $Q$-statistic is computed as:

$$Q_K = n(n+2)\sum_{k=1}^{K} \frac{r_k^2}{n-k} \qquad (1)$$

where $K$ is the largest lag included.

If the computed value is less than the tabulated $\chi^2$ value the autocorrelations are deemed not significantly different from zero on the whole, indicating that the data is random, whereas a computed value larger than the tabulated value indicates the existence of some pattern.

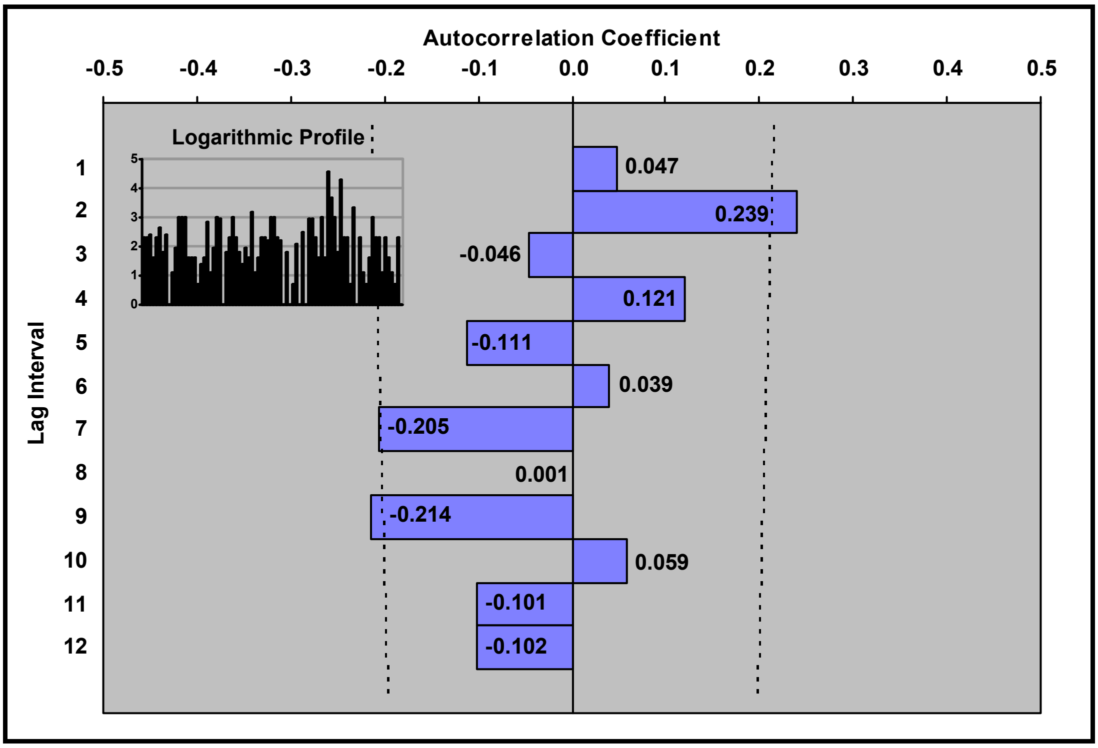
From equation (1), the $Q$-statistic for autocorrelations of the sample line item is $Q_{12} = 25.758$, which is greater than the tabulated $\chi^2$ value of 18.307 at the 5 percent level with 10 degrees of freedom. Therefore, the autocorrelations are significantly different from zero and the demand sizes in this instance are not considered random as a whole.

An analysis of the autocorrelations by this means reveals that 2,320, or 18.9 percent, of the 12,251 selected line items with a non-constant demand size are statistically significant as a whole. If the demand size is constant, the autocorrelation coefficients remain undefined.

## Natural Logarithm Transformations

The second method for examining autocorrelations if the normality assumption is not satisfied is to take natural logarithms (base e) of the data series. Figure C.1 presents the correlogram for the sample line item when this transformation is made. The profile of the transformed data shows considerably less variation than that shown by the original series. In this instance there are three autocorrelation coefficients which are individually significant ($r_2$, $r_7$ and $r_9$), while lag $r_4$ is not amongst these. The non-significance of lag $r_4$ indicates that the previously identified problem of skewed data has been overcome. When considering the significance of the autocorrelations as a whole by using equation (1), the calculated $Q$-statistic with a value of 18.339 is marginally greater than the tabulated $\chi^2$ value of 18.307, which again suggests that the demand sizes are not random as a whole.

**Figure C.1: Sample Correlogram - Demand Size (Log Transform).**




Calculating a $Q$-statistic for all selected line items indicates that 3,129, or 25.5 percent, are statistically significant as a whole when a logarithmic transformation is used.

## Spearman Rank-Order Correlation

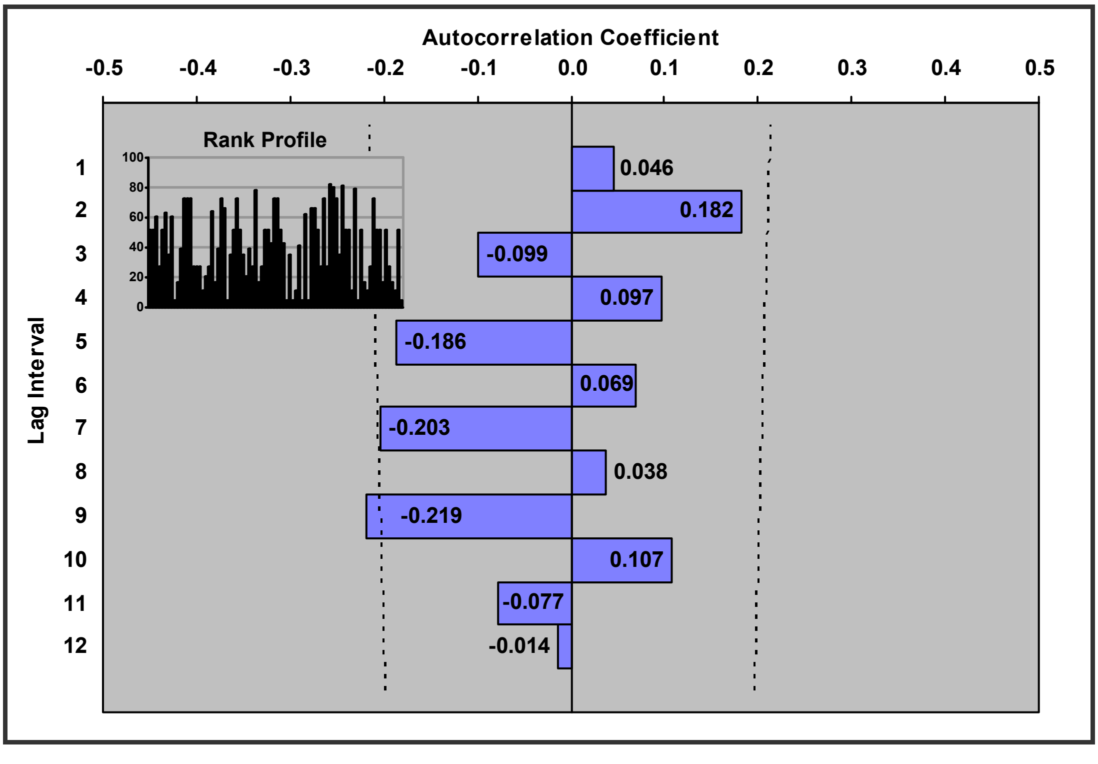
The next method considered is the Spearman rank-order correlation. This is a non-parametric measure of association based on the rank of the data values and is given by:

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} \tag{2}$$

where $x_i$ is the rank of the $i^{th}$ $x$ value, $y_i$ is the rank of the $i^{th}$ $y$ value and, in case of ties, the averaged ranks are used.

The correlogram of rank-order autocorrelation coefficients from equation (2) is shown for the sample line item in Figure C.2. Once again, the variation in the profile is considerably reduced and there are no extreme values. On this occasion only one coefficient ($r_9$) is individually significant with another ($r_7$) very close to significance. Again, $r_4$ is not significant. With a calculated $Q$-statistic of 18.419 marginally greater than the tabulated $\chi^2$ value of 18.307, the demand sizes are again not random as a whole. Computing a $Q$-statistic for all 12,251 line items with a non-constant demand size indicates that 5,042, or 41.2 percent, have statistically significant autocorrelations as a whole using this method.

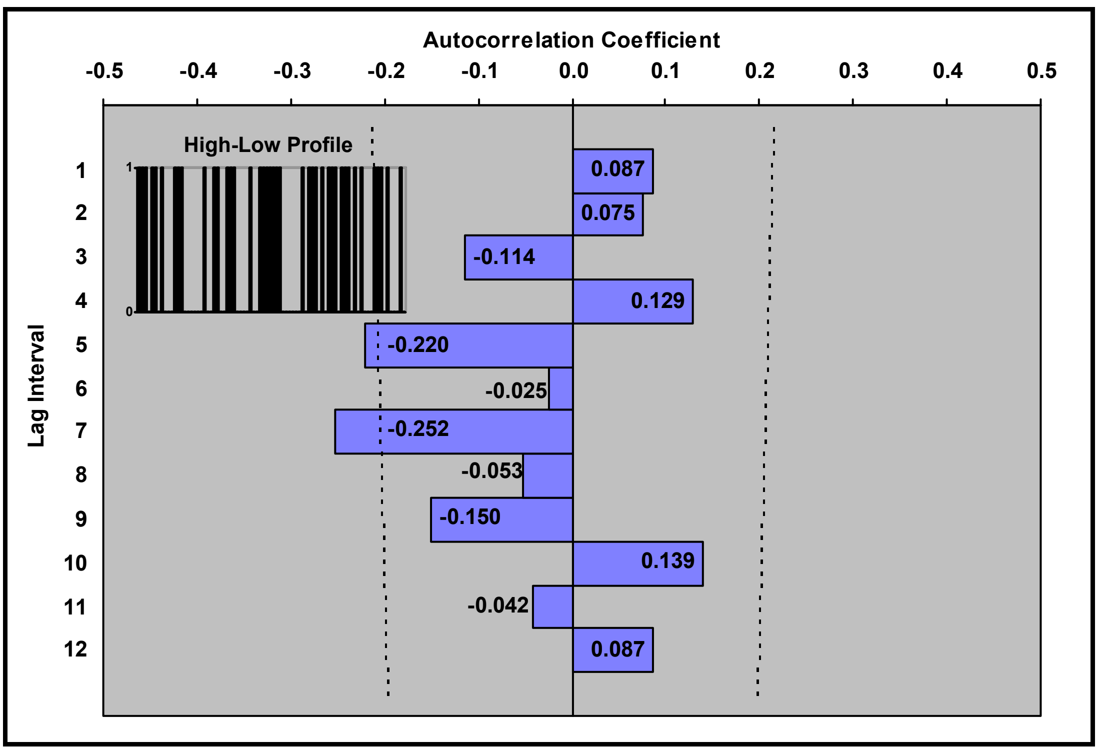
**Figure C.2: Sample Correlogram - Demand Size (Spearman Rank-Order).**



## Correlation of Successive High-Low Observations

A fourth method for determining the significance of autocorrelations in a skewed series is to consider the correlation of successive high and low observations. Under this method, successive observations are categorised as one or zero to signify that the observation is either above or below the median value for the entire series. A Boolean value is assigned to successive demand sizes based on a comparison against the median for the sample line item. The calculated autocorrelations for this transformed series produce the correlogram shown in Figure C.3.

**Figure C.3: Sample Correlogram - Demand Size (High-Low Succession).**




The *bar-code* appearance of the generating profile, where the black bars comprise a sequence of high observations and the grey bars a sequence of low observations, portrays an extreme removal of the variation in the original series. Two autocorrelation coefficients ($r_5$ and $r_7$) are deemed to be individually significant on this occasion, while

$r_4$ is again not significant. The calculated $Q$-statistic of 19.068 is greater than the tabulated $\chi^2$ value of 18.307, which suggests the demand sizes are not random as a whole. An analysis of the 12,251 selected line items by this means reveals that 4,769, or 38.9 percent, have statistically significant autocorrelations as a whole.

The analysis of RAF data using this method suggested that it was preferable to place any observations that were equal to the median in the *below median* category rather than placing them in the *above median* category. This is due to the fact that a series containing a majority of values equal to unity and the remainder greater than unity will have a median value of 1.0, which occurs often in reality. If values equal to the median are placed in the above median category, the transformed series will comprise entirely the same Boolean value and correlation coefficients will therefore remain undefined. If all values are equal to unity or are constant at any level, as occurs with 400 line items, the coefficients will remain undefined regardless.

## Frequency of High-Low Observations

Although the fifth method considered in this analysis uses the same transformed data series as the previous method, this final method differs from the others by the fact that no correlation coefficients are computed. Instead, the method simply measures the observed frequency of pairwise high-low observations from a 2×2 matrix and compares the results with the expected frequency for each lag. Thus, the method considers the frequency with which combinations of high and low values separated by incremental lags are observed. For example, the following matrix presents the observed frequencies of the lag 1 series of demand size for the sample line item:

| | | First Obs | |
|---|---|---|---|
| | | High | Low |
| Second Obs | High | **22** | **19** |
| | Low | **18** | **22** |

The matrix indicates that on 22 occasions a value greater than the median was subsequently followed by a second value greater than the median and, similarly, a value lower then the median was subsequently followed by a second value lower than the median on 22 occasions also. Likewise a low value was followed by a high value on 19 occasions and a high value was followed by a low value on the remaining 18 occasions. Any values equal to the median are once again placed in the *below median* category.

If the series was truly random and did not contain any autocorrelations, the expected frequency for each of the cells of the matrix would closely equal the average value, which is 20.25 in this case. A chi-square test is undertaken to determine whether the observed ($O_i$) and expected ($E_i$) frequencies for each lag interval differ significantly through the calculation of a chi-square statistic:

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \tag{3}$$

where $k$ is the number of cells.

Table C.1 presents the observed pairwise frequencies for each lag interval. A proxy method for determining whether any autocorrelations are positive or negative involves comparing the combined *high-high* and *low-low* frequency (shown as +'ve total in the table) with the combined *high-low* and *low-high* frequency (shown as −'ve total). The autocorrelation is then defined as positive or negative depending on which total is highest. For example, in the case of lag 1, the positive total of 44 is greater than the negative total of 37 so the autocorrelation is defined as positive.

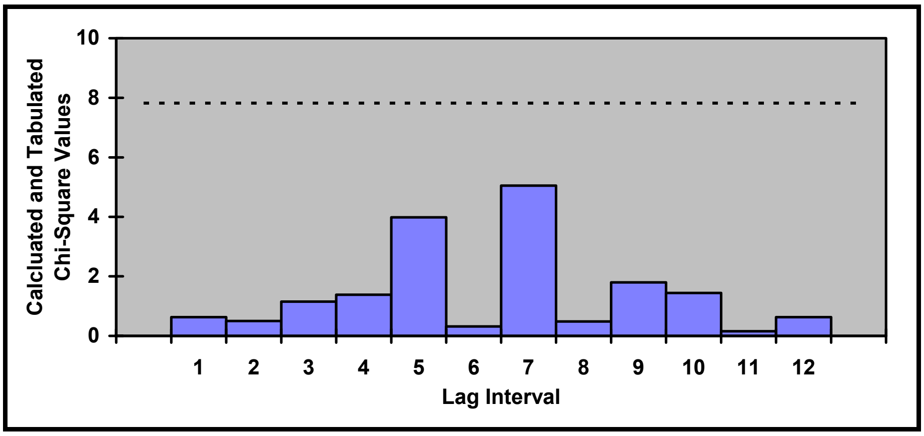
**Table C.1: Demand Size Observed and Expected High-Low Observations.**

| Pairwise Pattern | Lag Interval | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
| High-High | 22 | 21 | 17 | 22 | 15 | 18 | 14 | 17 | 15 | 20 | 17 | 19 |
| Low-Low | 22 | 22 | 18 | 22 | 15 | 19 | 14 | 18 | 16 | 21 | 17 | 19 |
| +'ve total | 44 | 43 | 35 | 44 | 30 | 37 | 28 | 35 | 31 | 41 | 34 | 38 |
| High-Low | 19 | 19 | 23 | 18 | 25 | 21 | 25 | 21 | 22 | 16 | 19 | 17 |
| Low-High | 18 | 18 | 21 | 16 | 22 | 18 | 22 | 18 | 20 | 15 | 18 | 15 |
| −'ve total | 37 | 37 | 44 | 34 | 47 | 39 | 47 | 39 | 42 | 31 | 37 | 32 |
| +'ve / −'ve | + | + | − | + | − | − | − | − | − | + | − | + |
| $E_i$ | 20.25 | 20.00 | 19.75 | 19.50 | 19.25 | 19.00 | 18.75 | 18.50 | 18.25 | 18.00 | 17.75 | 17.50 |
| $\chi^2$ | 0.63 | 0.50 | 1.15 | 1.38 | 3.99 | 0.32 | 5.05 | 0.49 | 1.79 | 1.44 | 0.15 | 0.63 |

Also shown in the table are the expected frequencies ($E_i$), which are simply the average of the four pairwise frequencies for each lag interval. The average decreases as an additional observation is lost with each lag. A chi-square statistic $\chi^2$ is calculated for each lag interval using equation (3), with each pairwise frequency representing an observed frequency.

The results are compared with the tabulated value at $k-1$ degrees of freedom, as presented in Figure C.4 where the bars indicate the calculated chi-square statistics for each lag interval and the horizontal dotted line indicates the tabulated value at the 5 per cent significance level with 3 degrees of freedom. As none of the calculated statistics exceed the tabulated value, this method suggests that there are no significant autocorrelations in the sample data. However, as with the previous method, lags 5 and 7 show the greatest signs of autocorrelation and, as with all the methods in this analysis, lag 4 presents little sign of autocorrelation.

**Figure C.4: Sample Chi-Square - Demand Size (High-Low Frequency).**




This method does not readily lend itself to calculating whether the autocorrelations are significant as a whole, although statistics that make this method comparable with the other methods are presented in the next section.

## Autocorrelation Methods Compared

This section compares the results generated by the five autocorrelation methods examined previously. The results are obtained from 12,251 line items in the RAF inventory with a non-constant demand size. Each data series contains between 50 and 99 observations.

Table C.2 presents summary statistics for individually significant lags, as well as line items that have significant autocorrelations as a whole. Through the use of the chi-square test, the high-low frequency method identifies substantially more individually significant lags than the other four methods that utilise correlation coefficients (58.8 percent against 6.9 to 13.4 percent). When considering the sign of the significant lags, the original data series and the high-low frequency method tend to have a low proportion of negative autocorrelations compared with the other three methods. The original data series has fewer line items that contain at least one significant lag (48.5 percent) while the high-low frequency method has the most (72.0 percent).

**Table C.2: Autocorrelation Summary Statistics - Demand Size.**

| Autocorrelation Method | Percentage of Individually Significant Lags | | | Percentage of Line Items ($n$ = 12,251) | |
|---|---|---|---|---|---|
| | Total | Proportion | | With Signif Lags | Signif as a Whole |
| | | Negative | Positive | | |
| Original Data Series | 6.9 | 7.9 | 92.1 | 48.5 | 18.9 |
| Log Transform | 9.2 | 16.0 | 84.0 | 55.1 | 25.5 |
| Spearman Rank-Order | 13.4 | 22.0 | 78.0 | 68.6 | 41.2 |
| High-Low Correlation | 12.8 | 21.4 | 78.6 | 66.5 | 38.9 |
| High-Low Frequency | 58.8 | 5.1 | 94.9 | 72.0 | $\approx$45.0 |

As determined by a regression analysis upon the results of the first four methods, there is a very strong linear relationship ($r^2$ = 0.999) between the percentage of line items with at least one significant lag and the percentage of line items with autocorrelations that are significant as a whole. Therefore, it is possible to determine an approximate percentage of line items that have significant autocorrelations as a whole for the high-low frequency method based on the observed percentage of line items with at least one significant lag.

As shown in the last column of Table C.2, the methods differ substantially in the number of line items they identify as significantly autocorrelated as a whole. Interestingly, each of the alternative methods identify a higher percentage (25.5 to about 45.0 percent) of line items with significant autocorrelations as a whole, compared with that obtained by the original data series (18.9 percent).

Attention is now given to the level of conformity as to which line items are actually identified as having autocorrelation. Table C.3 provides Venn diagram-type statistics for the level of conformity between four correlation coefficient methods, where a ✓ indicates autocorrelation is identified and a ✗ indicates autocorrelation has not been identified. It is not possible to include the high-low frequency method, as this method cannot consider autocorrelations as a whole in the same manner.

**Table C.3: Autocorrelation Method Conformity.**

| Original Series | Log Transform | Spearman Rank-Order | High-Low Correlation | Percentage of Observations |
|---|---|---|---|---|
| ✗ | ✗ | ✗ | ✗ | 48.0% |
| ✓ | ✓ | ✓ | ✓ | 11.8% |
| ✓ | ✓ | ✓ | ✗ | 2.1% |
| ✓ | ✓ | ✗ | ✓ | 0.1% |
| ✓ | ✗ | ✓ | ✓ | 1.2% |
| ✗ | ✓ | ✓ | ✓ | 7.6% |
| ✓ | ✓ | ✗ | ✗ | 0.6% |
| ✓ | ✗ | ✓ | ✗ | 0.6% |
| ✓ | ✗ | ✗ | ✓ | 0.5% |
| ✗ | ✓ | ✓ | ✗ | 2.7% |
| ✗ | ✓ | ✗ | ✓ | 0.1% |
| ✗ | ✗ | ✓ | ✓ | 10.6% |
| ✓ | ✗ | ✗ | ✗ | 2.0% |
| ✗ | ✓ | ✗ | ✗ | 0.5% |
| ✗ | ✗ | ✓ | ✗ | 4.6% |
| ✗ | ✗ | ✗ | ✓ | 7.0% |

The results indicate that all four methods agree on 48.0 percent of line items which do not have autocorrelations that are significant as a whole and, similarly they agree on a further 11.8 percent, which do. Beyond that there are a further 11.0 percent (2.1 + 0.1 +

1.2 + 7.6) where three methods agree there are autocorrelations, leaving a total of 29.2 percent of line items where less than three methods agree there are autocorrelations.

## Concluding Remarks

Determining the significance of autocorrelations when the data is skewed can lead to erroneous interpretations and for this reason alternative methods of identifying autocorrelations were investigated. While excluding the original data series from consideration, each alternative method identifies a varying level of autocorrelation in the demand size, ranging from 25.5 percent of line items to about 45 percent.

The rank-order method and the two high-low methods tend to indicate higher levels of autocorrelation than does the logarithm transformation method. However, the latter method examines the demand sizes, rather than ranks or comparisons against the median, and is therefore considered the more useful method as stock control methods themselves require knowledge about the demand sizes. The logarithm transformation method has been selected as the sole method for further autocorrelation analysis on this basis. In addition, this method benefits from the ease and simplicity of generating results, as well as the ability to produce the required statistics, including the identification of significant autocorrelations as a whole.

## Appendix D - The Friedman Test

The nonparametric Friedman test provides a method for testing whether $k$ related samples have been drawn from the same population of ordinal sample data, which is data consisting of rankings. Under this test the null and alternative hypotheses are:

$H_0$ : the $k$ populations have identical probability distributions, and

$H_1$ : at least two of the populations differ in location.

Kvanli *et al*. [45] describe the assumptions and procedure. It is assumed that the factor levels are applied in a random manner within each block, the number of blocks ($b$) or the number of factor levels ($k$) exceeds five, and the values within each block can be ranked.

The $k$ observations within each block are rank ordered, assigning the average of the tied positions in the event of ties within a block. Define $T_i$ as the total of the ranks for the $i^{th}$ population. The test statistic for the Friedman test is defined as:

$$FR = \frac{12}{bk(k+1)}\sum_{i=1}^{k} T_i^2 - 3b(k+1)$$

The distribution of the $FR$ statistic approximately follows a chi-square distribution with $k-1$ degrees of freedom. The Friedman test procedure is to reject the null hypothesis if $FR$ lies in the right-tail of the chi-square distribution, that is

reject $H_0$ if $FR > \chi^2_{\alpha,k-1}$

The SAS FREQ procedure produces Friedman's statistic as the *Row Mean Scores Differ* value by specifying the CMH2 option with SCORES=RANK and stratification by block.

## Appendix E - Measurement of Accuracy

This appendix presents a range of error measures comparing Croston's method with more traditional means of forecasting, including exponential smoothing, a one year moving average and a simple previous year average. In each case the statistics are averaged across a total of 18,750 sample line items. Optimal smoothing constants from a hold-out sample of 500 line items were used throughout.

Error measures are presented in Table E.1, firstly when comparing the forecast value with the one-period ahead demand, secondly when comparing the forecast value with the lead-time demand in all periods and finally when comparing the lead-time demand in periods of positive demand only. The minimum observation for each statistic is shown in bold-type.

In each case, the respective averages are provided for the mean absolute deviation (MAD), the MAD for forecast errors which exceed 5.0 percent above the actual value (denoted MAD+), the MAD for forecast errors more than 5.0 percent below the actual value (denoted MAD-), as well as the root mean square error (RMSE), the mean absolute percentage error (MAPE) and the median absolute percentage error (MdAPE). With values substantially greater than the tabulated chi-square value of 7.81 at the 5 percent significance level, Friedman's statistic indicates that there are differences in the forecasting performance in all cases, although the differences are not as prominent when considering lead-time demand in periods with positive demand only.

Table E.1: Measurement of Accuracy - Forecast Comparisons.

| Demand Aggreg'n | Error Measure | Expon'l Smooth. | Croston's Method | Moving Average | Prev Year Average | Friedman Statistic |
|---|---|---|---|---|---|---|
| **One-Period Ahead Demand (All Periods)** | | | | | | |
| Quarterly | MAD | 20.08 | 20.79 | **19.37** | 20.83 | 8,047.62 |
| | MAD + | 11.66 | 12.47 | **10.47** | 11.69 | 14,505.91 |
| | MAD - | 8.38 | **8.26** | 8.84 | 9.09 | 17,906.56 |
| | RMSE | **26.96** | 27.62 | 27.15 | 28.83 | 4,926.94 |
| | MAPE | **117.62** | 127.30 | 119.25 | 123.25 | 5,231.11 |
| | MdAPE | **73.87** | 74.92 | 75.76 | 77.08 | 7,538.83 |
| Monthly | MAD | 9.22 | 9.81 | **8.71** | 9.20 | 16,564.56 |
| | MAD + | 5.25 | 5.90 | **4.63** | 5.05 | 18,391.88 |
| | MAD - | 3.95 | **3.90** | 4.07 | 4.13 | 18,044.35 |
| | RMSE | 14.04 | 14.38 | **13.92** | 14.41 | 6,686.86 |
| | MAPE | 101.06 | **98.46** | 102.96 | 104.51 | 10,448.66 |
| | MdAPE | 76.58 | **70.09** | 79.95 | 80.03 | 14,157.96 |
| Weekly | MAD | 3.11 | 3.52 | **2.94** | 3.04 | 19,246.92 |
| | MAD + | 1.71 | 2.14 | **1.53** | 1.62 | 19,070.22 |
| | MAD - | 1.39 | **1.38** | 1.41 | 1.42 | 16,747.47 |
| | RMSE | 6.28 | 6.46 | **6.24** | 6.32 | 9,172.85 |
| | MAPE | 94.20 | **90.42** | 95.68 | 95.74 | 13,866.78 |
| | MdAPE | 87.33 | **82.20** | 89.71 | 89.47 | 18,694.12 |
| **Lead-Time Demand (All Periods)** | | | | | | |
| Quarterly | MAD | **14.88** | 18.60 | 15.29 | 16.66 | 8,359.42 |
| | MAD + | **8.55** | 11.31 | 8.82 | 10.00 | 13,977.09 |
| | MAD - | **6.27** | 7.24 | 6.40 | 6.60 | 5,319.75 |
| | RMSE | **19.56** | 25.16 | 19.86 | 21.45 | 7,961.72 |
| | MAPE | **173.29** | 303.95 | 181.07 | 204.63 | 4,863.52 |
| | MdAPE | **91.02** | 202.02 | 98.64 | 112.21 | 2,599.54 |
| Monthly | MAD | **5.00** | 5.77 | 5.18 | 5.69 | 3,936.33 |
| | MAD + | **2.89** | 3.93 | 3.00 | 3.44 | 10,806.45 |
| | MAD - | 2.09 | **1.81** | 2.16 | 2.23 | 21,698.94 |
| | RMSE | **6.63** | 7.16 | 6.79 | 7.36 | 1,436.55 |
| | MAPE | **174.76** | 342.52 | 182.48 | 212.49 | 3,337.34 |
| | MdAPE | **86.99** | 245.56 | 94.85 | 113.27 | 2,490.82 |

| Demand Aggreg'n | Error Measure | Expon'l Smooth. | Croston's Method | Moving Average | Prev Year Average | Friedman Statistic |
|---|---|---|---|---|---|---|
| **Lead-Time Demand (All Periods) - continued** | | | | | | |
| Weekly | MAD | **1.17** | 1.45 | 1.21 | 1.33 | 5,163.07 |
| | MAD + | **0.67** | 1.00 | 0.70 | 0.80 | 10,631.45 |
| | MAD - | 0.49 | **0.44** | 0.50 | 0.53 | 17,445.13 |
| | RMSE | **1.56** | 1.81 | 1.59 | 1.73 | 2,557.83 |
| | MAPE | **171.29** | 348.63 | 179.25 | 211.12 | 3,718.95 |
| | MdAPE | **84.08** | 242.09 | 92.25 | 109.78 | 2,920.55 |
| **Lead-Time Demand (Demand Only)** | | | | | | |
| Quarterly | MAD | 16.46 | 18.00 | **16.43** | 17.32 | 580.53 |
| | MAD + | 10.98 | 10.64 | **10.45** | 10.76 | 2,825.87 |
| | MAD - | **5.41** | 7.31 | 5.92 | 6.50 | 13,477.78 |
| | RMSE | **19.95** | 24.42 | 20.69 | 21.71 | 2,041.30 |
| | MAPE | 254.14 | **216.74** | 233.80 | 218.81 | 151.37 |
| | MdAPE | 174.06 | **128.27** | 145.42 | 137.81 | 184.16 |
| Monthly | MAD | 5.71 | 6.02 | **5.66** | 5.87 | 946.18 |
| | MAD + | 3.87 | 3.73 | 3.67 | **3.60** | 6,602.67 |
| | MAD - | **1.82** | 2.28 | 1.97 | 2.25 | 13,152.85 |
| | RMSE | **6.93** | 8.24 | 7.16 | 7.39 | 1,566.38 |
| | MAPE | 256.78 | 222.70 | 233.42 | **211.27** | 233.87 |
| | MdAPE | 169.18 | 132.54 | 137.33 | **130.44** | 281.95 |
| Weekly | MAD | 1.38 | 1.79 | **1.34** | 1.38 | 2,682.78 |
| | MAD + | 0.93 | 1.13 | 0.88 | **0.83** | 9,386.47 |
| | MAD - | **0.44** | 0.66 | 0.46 | 0.54 | 12,530.58 |
| | RMSE | **1.66** | 2.76 | 1.70 | 1.74 | 3,910.61 |
| | MAPE | 252.72 | 226.30 | 228.12 | **215.83** | 810.74 |
| | MdAPE | 164.97 | **126.53** | 128.44 | 133.06 | 71.11 |

## Appendix F - Median Absolute Percentage Error by Demand Pattern

In this appendix comparisons are made between the five identified demand patterns using the median absolute percentage error (MdAPE). The forecasting methods included in the analysis are exponential smoothing, Croston's method, a one year moving average and a simple previous year average. For each demand pattern the MdAPE is averaged across 3,750 sample line items, where optimal smoothing constants were obtained from hold-out samples of 100 line items for each pattern. Results are shown in Table F.1 for forecast comparisons with the one-period ahead demand, with the lead-time demand in all periods, and also with the lead-time demand in periods of positive demand only. The minimum observation in each instance is shown in bold-type.

**Table F.1: Median Absolute Percentage Error (MdAPE) by Demand Pattern.**

| Demand Aggreg'n | Demand Pattern | Expon'l Smooth. | Croston's Method | Moving Average | Prev Year Average | Friedman Statistic |
|---|---|---|---|---|---|---|
| **One-Period Ahead Demand (All Periods)** | | | | | | |
| Quarterly | Smooth | 50.53 | **47.76** | 51.29 | 53.50 | 492.86 |
| | Irregular | 83.41 | 77.92 | **73.45** | 79.37 | 325.86 |
| | Slow-moving | 67.39 | **49.26** | 77.69 | 74.43 | 4,517.36 |
| | Mildly Erratic | **86.05** | 103.52 | 88.41 | 90.70 | 2,126.64 |
| | Highly Erratic | **82.18** | 96.31 | 88.56 | 87.84 | 2,055.99 |
| Monthly | Smooth | 58.37 | **54.15** | 59.99 | 61.03 | 1,067.97 |
| | Irregular | 77.79 | **71.67** | 74.33 | 78.15 | 935.67 |
| | Slow-moving | 83.60 | **68.56** | 88.76 | 86.73 | 6,120.73 |
| | Mildly Erratic | 83.48 | **81.48** | 89.73 | 89.13 | 3,983.87 |
| | Highly Erratic | 79.76 | **74.66** | 87.04 | 85.22 | 4,068.97 |
| Weekly | Smooth | 76.09 | **73.59** | 77.43 | 77.77 | 1,379.48 |
| | Irregular | 81.23 | **77.00** | 83.85 | 83.83 | 2,241.26 |
| | Slow-moving | 95.22 | **89.44** | 96.64 | 96.31 | 5,892.41 |
| | Mildly Erratic | 92.63 | **86.57** | 95.69 | 95.23 | 5,192.18 |
| | Highly Erratic | 91.49 | **84.39** | 94.94 | 94.21 | 5,464.52 |

| Demand Aggreg'n | Demand Pattern | Expon'l Smooth. | Croston's Method | Moving Average | Prev Year Average | Friedman Statistic |
|---|---|---|---|---|---|---|
| **Lead-Time Demand (All Periods)** | | | | | | |
| Quarterly | Smooth | **52.15** | 64.20 | 53.34 | 60.34 | 592.82 |
| | Irregular | 82.67 | 165.64 | **81.55** | 95.86 | 264.00 |
| | Slow-moving | **82.73** | 168.28 | 94.57 | 103.02 | 728.60 |
| | Mildly Erratic | **116.89** | 320.16 | 130.33 | 151.79 | 791.34 |
| | Highly Erratic | **121.35** | 294.54 | 134.40 | 151.13 | 633.73 |
| Monthly | Smooth | **51.22** | 71.51 | 52.60 | 60.47 | 229.17 |
| | Irregular | 80.58 | 174.04 | **78.25** | 93.18 | 139.77 |
| | Slow-moving | **82.58** | 202.67 | 94.26 | 108.40 | 1,031.48 |
| | Mildly Erratic | **114.55** | 413.28 | 128.53 | 155.31 | 932.80 |
| | Highly Erratic | **106.11** | 366.72 | 120.71 | 149.14 | 973.31 |
| Weekly | Smooth | **50.01** | 71.68 | 51.32 | 58.57 | 330.99 |
| | Irregular | 76.15 | 141.77 | **75.63** | 88.66 | 168.93 |
| | Slow-moving | **80.08** | 201.40 | 91.31 | 101.71 | 1,033.40 |
| | Mildly Erratic | **108.87** | 424.43 | 125.00 | 152.65 | 881.01 |
| | Highly Erratic | **105.28** | 371.18 | 117.98 | 147.31 | 986.14 |
| **Lead-Time Demand (Demand Only)** | | | | | | |
| Quarterly | Smooth | 65.33 | **57.87** | 58.98 | 62.46 | 413.27 |
| | Irregular | 124.74 | **88.03** | 97.25 | 101.95 | 182.81 |
| | Slow-moving | 126.10 | **96.93** | 113.76 | 111.68 | 249.54 |
| | Mildly Erratic | 313.17 | **224.41** | 265.18 | 236.47 | 117.05 |
| | Highly Erratic | 277.05 | **199.58** | 223.62 | 202.65 | 118.52 |
| Monthly | Smooth | 63.72 | **57.10** | 57.14 | 60.47 | 271.08 |
| | Irregular | 119.48 | 92.60 | **92.16** | 94.01 | 90.75 |
| | Slow-moving | 129.66 | **105.10** | 114.58 | 114.58 | 269.40 |
| | Mildly Erratic | 307.32 | 240.05 | 251.69 | **223.26** | 129.51 |
| | Highly Erratic | 258.05 | 192.19 | 198.13 | **183.02** | 197.47 |
| Weekly | Smooth | 62.31 | 57.81 | **54.67** | 58.50 | 530.99 |
| | Irregular | 122.85 | **89.89** | 91.79 | 93.31 | 82.32 |
| | Slow-moving | 119.05 | **102.01** | 107.34 | 106.07 | 148.51 |
| | Mildly Erratic | 290.56 | 227.62 | 226.98 | **221.80** | 60.18 |
| | Highly Erratic | 256.24 | **175.34** | 182.58 | 206.81 | 102.10 |

# Appendix G - Syntetos and Boylan's Modifications to Croston's Method

## Part A: Expectation of the Inter-Demand Interval

Presented in Syntetos and Boylan [78].

(Assume that $\alpha = 1$ so that $p'_{t+1} = p_t$)

If we denote by $p_t$ the inter demand interval that follows the geometric distribution, including the first success (i.e. demand occurring period), and by $1/p_t$ the probability of demand occurrence at period $t$, we then have:

$$E\left(\frac{1}{p_t}\right) = \sum_{x=1}^{\infty} \frac{1}{x} \frac{1}{p} \left(1 - \frac{1}{p}\right)^{x-1}$$

$$= \frac{1}{p} \sum_{x=1}^{\infty} \frac{1}{x} \left(\frac{p-1}{p}\right)^{x-1}$$

for $p > 1$ (i.e. demand does not occur in every single time period)

$$= \frac{1}{p} \sum_{x=1}^{\infty} \frac{1}{x} \frac{((p-1)/p)^x}{((p-1)/p)^1}$$

$$= \frac{1}{p} \frac{1}{((p-1)/p)} \sum_{x=1}^{\infty} \frac{1}{x} \left(\frac{p-1}{p}\right)^x$$

$$= \frac{1}{p-1} \left[ \frac{p-1}{p} + \frac{1}{2}\left(\frac{p-1}{p}\right)^2 + \frac{1}{3}\left(\frac{p-1}{p}\right)^3 + \ldots \right]$$

$$= -\frac{1}{p-1} \log\left(\frac{1}{p}\right)$$

## Part B: An Expectation for Giving Unbiased Estimates

Presented in Syntetos and Boylan [78].

$$E\left(\frac{1}{p_t c^{p_t-1}}\right) = \sum_{x=1}^{\infty}\frac{1}{x}\frac{1}{p}\frac{1}{c^{x-1}}\left(1-\frac{1}{p}\right)^{x-1}$$

$$= \frac{1}{p}\sum_{x=1}^{\infty}\frac{1}{x}\left(\frac{1}{c}-\frac{1}{cp}\right)^{x-1}$$

$$= \frac{1}{p}\sum_{x=1}^{\infty}\frac{1}{x}\left(\frac{p-1}{cp}\right)^{x-1}$$

$$= \frac{1}{p}\left[1+\sum_{x=2}^{\infty}\frac{1}{x}\left(\frac{p-1}{cp}\right)^{x-1}\right]$$

As $c \to \infty,\ E\left(\frac{1}{p_t c^{p_t-1}}\right) \to \frac{1}{p}$

## Appendix H - Measurement of Accuracy - Croston's and Variants

This appendix compares error measure statistics for the forecasting methods classed as variations on Croston's method, including Croston's method itself, the revised Croston's method, the bias reduction method and the approximation method. The statistics shown in each case are the average MAD, the average MAD for forecast errors which exceed 5.0 percent above the actual value (denoted MAD+), the average MAD for forecast errors more than 5.0 percent below the actual value (denoted MAD-), as well as the average RMSE, the average MAPE and the average MdAPE, taken across 18,750 sample line items. Optimal smoothing constants for Croston's method were obtained from a hold-out sample of 500 line items and the other three methods utilise the same parameters.

Error measures are presented in Table H.1 comparing the forecast value, firstly with the one-period ahead demand, secondly with the lead-time demand in all periods and finally with the lead-time demand in periods of positive demand only. The minimum observation for each statistic is shown in bold-type. These results are comparable with those of the more traditional forecasting methods presented in Appendix E.

Once again, the Friedman's statistics are substantially greater than the tabulated chi-square value of 7.81 at the 5 percent significance level, indicating that there are differences in the forecasting performance in all cases.

Table H.1: Measurement of Accuracy - Croston's and Variants.

| Demand Aggreg'n | Error Measure | Croston's Method | Revised Croston's | Bias Reduction | Approxi-mation | Friedman Statistic |
|---|---|---|---|---|---|---|
| **One-Period Ahead Demand (All Periods)** | | | | | | |
| Quarterly | MAD | 20.79 | 20.95 | 20.52 | **19.43** | 28,657.69 |
| | MAD + | 12.47 | 12.62 | 12.11 | **8.95** | 38,441.71 |
| | MAD - | **8.26** | 8.27 | 8.36 | 10.45 | 35,997.03 |
| | RMSE | 27.62 | 27.73 | 27.46 | **26.68** | 20,843.25 |
| | MAPE | 127.30 | 129.24 | 124.38 | **110.68** | 3,311.13 |
| | MdAPE | 74.92 | 77.08 | 74.33 | **70.31** | 1,949.09 |
| Monthly | MAD | 9.81 | 10.17 | 9.70 | **9.50** | 35,639.46 |
| | MAD + | 5.90 | 6.30 | 5.75 | **5.35** | 37,719.84 |
| | MAD - | 3.90 | **3.86** | 3.94 | 4.14 | 38,443.72 |
| | RMSE | 14.38 | 14.61 | 14.31 | **14.22** | 22,889.16 |
| | MAPE | 98.46 | 101.60 | 97.88 | **95.94** | 1,660.28 |
| | MdAPE | **70.09** | 70.87 | 70.51 | 70.34 | 12,742.56 |
| Weekly | MAD | 3.52 | 3.64 | 3.49 | **3.47** | 42,165.62 |
| | MAD + | 2.14 | 2.27 | 2.10 | **2.07** | 42,418.92 |
| | MAD - | 1.38 | **1.37** | 1.39 | 1.40 | 44,254.66 |
| | RMSE | 6.46 | 6.52 | 6.45 | **6.44** | 23,204.50 |
| | MAPE | 90.42 | 90.76 | 90.48 | **90.32** | 21,041.16 |
| | MdAPE | 82.20 | **81.68** | 82.47 | 82.51 | 32,942.55 |
| **Lead-Time Demand (All Periods)** | | | | | | |
| Quarterly | MAD | 18.60 | 18.60 | 18.03 | **16.88** | 22,661.00 |
| | MAD + | 11.31 | 11.23 | 10.50 | **5.00** | 41,359.41 |
| | MAD - | **7.24** | 7.33 | 7.48 | 11.85 | 31,529.94 |
| | RMSE | 25.16 | 25.13 | 24.44 | **21.56** | 25,230.27 |
| | MAPE | 303.95 | 305.00 | 265.39 | **200.20** | 27,351.18 |
| | MdAPE | 202.02 | 204.63 | 178.15 | **139.13** | 1,797.17 |
| Monthly | MAD | 5.77 | 5.98 | 5.62 | **5.30** | 19,812.18 |
| | MAD + | 3.93 | 4.02 | 3.71 | **3.04** | 29,634.87 |
| | MAD - | **1.81** | 1.94 | 1.88 | 2.23 | 18,466.03 |
| | RMSE | 7.16 | 7.38 | 7.02 | **6.66** | 19,833.11 |
| | MAPE | 342.52 | 374.40 | 315.76 | **300.78** | 23,901.65 |
| | MdAPE | 245.56 | 272.44 | 225.57 | **215.70** | 7,746.19 |

| Demand Aggreg'n | Error Measure | Croston's Method | Revised Croston's | Bias Reduction | Approxi-mation | Friedman Statistic |
|---|---|---|---|---|---|---|
| **Lead-Time Demand (All Periods) - continued** | | | | | | |
| Weekly | MAD | 1.45 | 1.76 | 1.40 | **1.36** | 18,178.57 |
| | MAD + | 1.00 | 1.23 | 0.93 | **0.87** | 22,504.78 |
| | MAD - | **0.44** | 0.52 | 0.46 | 0.49 | 14,234.34 |
| | RMSE | 1.81 | 2.17 | 1.77 | **1.72** | 17,968.26 |
| | MAPE | 348.63 | 458.34 | 324.91 | **322.50** | 20,488.69 |
| | MdAPE | 242.09 | 323.18 | 225.05 | **223.82** | 9,061.46 |
| **Lead-Time Demand (Demand Only)** | | | | | | |
| Quarterly | MAD | 18.00 | 18.11 | 17.42 | **16.61** | 18,266.50 |
| | MAD + | 10.64 | 10.63 | 9.77 | **4.37** | 35,210.71 |
| | MAD - | **7.31** | 7.44 | 7.59 | 12.21 | 24,775.78 |
| | RMSE | 24.42 | 24.52 | 23.62 | **21.09** | 21,057.67 |
| | MAPE | 216.74 | 218.63 | 178.14 | **144.40** | 14,981.29 |
| | MdAPE | 128.27 | 135.32 | 105.94 | **94.13** | 544.67 |
| Monthly | MAD | 6.02 | 6.33 | 5.76 | **5.46** | 20,101.97 |
| | MAD + | 3.73 | 3.91 | 3.31 | **2.23** | 28,170.94 |
| | MAD - | **2.28** | 2.40 | 2.43 | 3.21 | 17,242.74 |
| | RMSE | 8.24 | 8.64 | 7.88 | **7.19** | 23,017.33 |
| | MAPE | 222.70 | 272.77 | 180.52 | **172.05** | 16,939.61 |
| | MdAPE | 132.54 | 173.61 | 107.10 | **105.60** | 3,681.15 |
| Weekly | MAD | 1.79 | 2.15 | 1.64 | **1.57** | 24,980.43 |
| | MAD + | 1.13 | 1.40 | 0.88 | **0.72** | 28,417.92 |
| | MAD - | **0.66** | 0.75 | 0.75 | 0.85 | 18,421.12 |
| | RMSE | 2.76 | 3.30 | 2.49 | **2.27** | 26,041.85 |
| | MAPE | 226.30 | 356.94 | **168.03** | 173.84 | 18,624.13 |
| | MdAPE | 126.53 | 216.48 | **97.20** | 101.13 | 8,318.53 |

# Appendix I - MdAPE by Demand Pattern - Croston's and Variants

This appendix presents median absolute percentage error (MdAPE) results for the forecasting methods presented as suitable for erratic demand patterns. The results are comparable with those of the traditional forecasting methods presented in Appendix F.

**Table I.1: MdAPE by Demand Pattern - Croston's and Variants.**

| Demand Aggregation | Demand Pattern | Croston's Method | Revised Croston's | Bias Reduction | Approxi-mation | Friedman Statistic |
|---|---|---|---|---|---|---|
| **One-Period Ahead Demand (All Periods)** | | | | | | |
| | **Smooth** | 47.76 | 48.14 | 48.15 | **47.11** | 163.40 |
| | **Irregular** | 77.92 | 78.93 | 76.63 | **71.39** | 49.15 |
| **Quarterly** | **Slow-moving** | 49.26 | **49.18** | 51.99 | 52.83 | 2,694.77 |
| | **Mildly Erratic** | 103.52 | 111.06 | 100.89 | **93.55** | 394.25 |
| | **Highly Erratic** | 96.31 | 98.35 | 94.21 | **86.93** | 196.15 |
| | **Smooth** | 54.15 | **53.45** | 54.83 | 54.83 | 1,241.72 |
| | **Irregular** | 71.67 | 73.40 | 71.57 | **70.90** | 634.79 |
| **Monthly** | **Slow-moving** | 68.56 | **65.71** | 69.88 | 70.35 | 7,229.87 |
| | **Mildly Erratic** | 81.48 | 85.43 | 81.39 | **81.01** | 3,118.60 |
| | **Highly Erratic** | 74.66 | 76.41 | 74.93 | **74.64** | 2,864.10 |
| | **Smooth** | 73.59 | **72.81** | 73.95 | 74.02 | 4,723.02 |
| | **Irregular** | 77.00 | **76.45** | 77.30 | 77.31 | 4,381.26 |
| **Weekly** | **Slow-moving** | 89.44 | **88.86** | 89.69 | 89.72 | 9,245.24 |
| | **Mildly Erratic** | 86.57 | **86.43** | 86.76 | 86.78 | 7,755.85 |
| | **Highly Erratic** | 84.39 | **83.88** | 84.67 | 84.69 | 7,666.26 |
| **Lead-Time Demand (All Periods)** | | | | | | |
| | **Smooth** | 64.20 | 65.02 | 61.58 | **56.90** | 157.35 |
| | **Irregular** | 165.64 | 160.63 | 157.25 | **127.87** | 119.67 |
| **Quarterly** | **Slow-moving** | 168.28 | 172.39 | 141.95 | **103.59** | 2,153.94 |
| | **Mildly Erratic** | 320.16 | 329.12 | 273.13 | **212.16** | 1,014.98 |
| | **Highly Erratic** | 294.54 | 298.91 | 258.86 | **196.43** | 998.74 |
| | **Smooth** | 71.51 | 80.43 | 67.45 | **63.77** | 955.58 |
| | **Irregular** | 174.04 | 187.62 | 164.36 | **154.68** | 927.95 |
| **Monthly** | **Slow-moving** | 202.67 | 229.87 | 182.86 | **174.60** | 2,920.40 |
| | **Mildly Erratic** | 413.28 | 458.61 | 377.42 | **363.69** | 1,750.02 |
| | **Highly Erratic** | 366.72 | 406.16 | 336.11 | **322.07** | 1,910.38 |

| Demand Aggregation | Demand Pattern | Croston's Method | Revised Croston's | Bias Reduction | Approxi-mation | Friedman Statistic |
|---|---|---|---|---|---|---|
| **Lead-Time Demand (All Periods) - continued** | | | | | | |
| | **Smooth** | 71.68 | 107.57 | 67.50 | **66.53** | 2,863.49 |
| | **Irregular** | 141.77 | 184.61 | 132.91 | **131.79** | 1,607.88 |
| **Weekly** | **Slow-moving** | 201.40 | 283.92 | 184.90 | **183.95** | 2,681.71 |
| | **Mildly Erratic** | 424.43 | 567.86 | 394.94 | **393.33** | 1,419.93 |
| | **Highly Erratic** | 371.18 | 471.92 | 345.02 | **343.49** | 1,476.46 |
| **Lead-Time Demand (Demand Only)** | | | | | | |
| | **Smooth** | 57.87 | 59.39 | 55.95 | **53.76** | 247.11 |
| | **Irregular** | 88.03 | 88.35 | 81.24 | **76.25** | 167.26 |
| **Quarterly** | **Slow-moving** | 96.93 | 107.88 | 76.40 | **65.71** | 647.78 |
| | **Mildly Erratic** | 224.41 | 240.92 | 172.03 | **151.48** | 364.43 |
| | **Highly Erratic** | 199.58 | 209.02 | 160.46 | **135.86** | 394.75 |
| | **Smooth** | 57.10 | 65.05 | 52.70 | **51.32** | 525.72 |
| | **Irregular** | 92.60 | 104.79 | 81.95 | **79.88** | 629.77 |
| **Monthly** | **Slow-moving** | 105.10 | 152.69 | 79.77 | **78.63** | 1,197.01 |
| | **Mildly Erratic** | 240.05 | 327.15 | 186.78 | **185.77** | 1,026.68 |
| | **Highly Erratic** | 192.19 | 257.39 | 150.49 | **148.72** | 1,029.57 |
| | **Smooth** | 57.81 | 84.72 | 52.24 | **52.23** | 3,045.74 |
| | **Irregular** | 89.89 | 123.57 | **78.23** | 79.13 | 2,634.72 |
| **Weekly** | **Slow-moving** | 102.01 | 209.66 | **71.67** | 75.67 | 1,488.57 |
| | **Mildly Erratic** | 227.62 | 403.59 | **166.37** | 175.77 | 1,283.46 |
| | **Highly Erratic** | 175.34 | 306.61 | **128.88** | 135.67 | 1,228.89 |

## Appendix J - Effect of Autocorrelation on Forecasting Performance

This appendix considers the effect of autocorrelation and crosscorrelation on forecasting performance. The analysis compares the performance of exponential smoothing and Croston's method at various levels of autocorrelation and crosscorrelation. In each case the line items have been classified according to the significance of their autocorrelations of the logarithmic transformation on the individual transaction data:

(i) If the correlations are not significant on the whole a *not signif* classification is given, otherwise

(ii) If the sum of the individually significant correlations is less than zero a *negative* classification is given, or

(iii) If the sum of the individually significant correlations is greater than zero, a *positive* classification is given, alternatively

(iv) If the correlations are significant on the whole, but there are no individually significant correlations, then these line items are not considered.

Based on a sample size of 18,750 line items, the tables in this appendix present the average MAPE and the average MdAPE. Results are presented for autocorrelations in the demand size, autocorrelations in the interval between transactions and crosscorrelations between the demand size and interval respectively. Bold type indicates the best forecasting method under each negative, nil and positive autocorrelation classification.

**Table J.1: Demand Size Autocorrelation.**

| Demand Aggreg'n | Error Measure | Negative | | Nil | | Positive | |
|---|---|---|---|---|---|---|---|
| | | Expon'l Smooth. | Croston's Method | Expon'l Smooth. | Croston's Method | Expon'l Smooth. | Croston's Method |
| **One-Period Ahead Demand (All Periods)** | | | | | | | |
| Quarterly | MAPE | **110.90** | 117.13 | **116.94** | 126.75 | **125.99** | 135.20 |
| | MdAPE | **72.33** | 73.34 | **73.67** | 74.86 | **78.24** | 79.59 |
| Monthly | MAPE | 105.90 | **102.91** | 100.33 | **97.67** | 105.91 | **104.33** |
| | MdAPE | 81.88 | **75.50** | 76.15 | **69.59** | 77.93 | **72.86** |
| Weekly | MAPE | 95.31 | **91.65** | 94.08 | **90.25** | 96.05 | **92.71** |
| | MdAPE | 87.50 | **82.31** | 87.31 | **82.15** | 87.55 | **82.70** |
| **Lead-Time Demand (All Periods)** | | | | | | | |
| Quarterly | MAPE | **170.25** | 275.24 | **170.07** | 305.27 | **215.92** | 327.98 |
| | MdAPE | **91.56** | 186.38 | **87.79** | 202.39 | **130.10** | 222.08 |
| Monthly | MAPE | **172.52** | 361.75 | **174.47** | 341.79 | **183.90** | 352.66 |
| | MdAPE | **88.75** | 263.25 | **87.08** | 246.04 | **86.72** | 235.48 |
| Weekly | MAPE | **168.47** | 359.25 | **171.38** | 348.22 | **176.64** | 361.86 |
| | MdAPE | **82.96** | 244.69 | **84.40** | 243.02 | **83.12** | 237.80 |
| **Lead-Time Demand (Demand Only)** | | | | | | | |
| Quarterly | MAPE | 283.24 | **240.87** | 248.70 | **211.55** | 294.29 | **262.86** |
| | MdAPE | 219.75 | **174.53** | 168.02 | **121.64** | 209.77 | **175.70** |
| Monthly | MAPE | 293.73 | **251.71** | 252.62 | **220.06** | 272.89 | **230.70** |
| | MdAPE | 218.54 | **181.83** | 164.49 | **128.37** | 186.21 | **145.93** |
| Weekly | MAPE | 289.99 | **257.87** | 248.43 | **224.60** | 270.27 | **224.37** |
| | MdAPE | 205.86 | **176.74** | 161.50 | **123.94** | 173.59 | **121.41** |

**Table J.2: Transaction Interval Autocorrelation.**

| Demand Aggreg'n | Error Measure | Negative | | Nil | | Positive | |
|---|---|---|---|---|---|---|---|
| | | Expon'l Smooth. | Croston's Method | Expon'l Smooth. | Croston's Method | Expon'l Smooth. | Croston's Method |
| **One-Period Ahead Demand (All Periods)** | | | | | | | |
| Quarterly | MAPE | **109.25** | 114.66 | **116.91** | 126.40 | **126.65** | 139.81 |
| | MdAPE | 71.68 | **71.55** | **73.73** | 74.72 | **77.30** | 80.11 |
| Monthly | MAPE | 102.05 | **97.14** | 100.18 | **97.88** | 107.27 | **104.82** |
| | MdAPE | 76.05 | **68.89** | 76.66 | **70.20** | 75.93 | **70.15** |
| Weekly | MAPE | 93.04 | **89.02** | 93.86 | **90.01** | 96.80 | **94.22** |
| | MdAPE | 87.34 | **82.06** | 87.32 | **82.11** | 87.12 | **82.24** |
| **Lead-Time Demand (All Periods)** | | | | | | | |
| Quarterly | MAPE | **166.98** | 276.58 | **170.15** | 302.72 | **214.54** | 348.65 |
| | MdAPE | **88.77** | 180.02 | **87.96** | 202.46 | **125.16** | 219.85 |
| Monthly | MAPE | **173.20** | 311.53 | **173.15** | 343.99 | **191.15** | 360.34 |
| | MdAPE | **92.24** | 238.15 | **86.34** | 246.79 | **85.50** | 235.45 |
| Weekly | MAPE | **168.82** | 315.64 | **170.29** | 349.82 | **184.88** | 371.07 |
| | MdAPE | **89.75** | 238.71 | **83.50** | 241.72 | **81.02** | 243.40 |
| **Lead-Time Demand (Demand Only)** | | | | | | | |
| Quarterly | MAPE | 245.13 | **231.89** | 252.87 | **210.93** | 284.62 | **271.91** |
| | MdAPE | 176.43 | **149.79** | 171.81 | **121.51** | 204.84 | **185.72** |
| Monthly | MAPE | 243.37 | **217.28** | 256.65 | **221.95** | 274.87 | **239.02** |
| | MdAPE | 158.40 | **129.10** | 169.83 | **132.15** | 185.01 | **149.80** |
| Weekly | MAPE | 230.21 | **217.45** | 253.90 | **227.92** | 275.07 | **230.67** |
| | MdAPE | 148.59 | **120.12** | 166.52 | **127.78** | 169.56 | **122.76** |

**Table J.3: Size and Interval Crosscorrelation.**

| Demand Aggreg'n | Error Measure | Negative | | Nil | | Positive | |
|---|---|---|---|---|---|---|---|
| | | Expon'l Smooth. | Croston's Method | Expon'l Smooth. | Croston's Method | Expon'l Smooth. | Croston's Method |
| **One-Period Ahead Demand (All Periods)** | | | | | | | |
| Quarterly | MAPE | **121.46** | 128.14 | **117.27** | 128.69 | **115.67** | 117.26 |
| | MdAPE | **73.01** | 73.50 | **73.77** | 75.46 | 75.37 | **73.10** |
| Monthly | MAPE | 105.92 | **100.98** | 99.93 | **98.16** | 102.90 | **97.56** |
| | MdAPE | 77.11 | **69.43** | 76.15 | **70.18** | 78.81 | **70.35** |
| Weekly | MAPE | 95.17 | **90.81** | 93.95 | **90.32** | 94.43 | **90.24** |
| | MdAPE | 87.83 | **82.24** | 87.08 | **82.11** | 88.54 | **82.81** |
| **Lead-Time Demand (All Periods)** | | | | | | | |
| Quarterly | MAPE | **169.59** | 281.18 | **170.88** | 309.05 | **193.39** | 295.64 |
| | MdAPE | **93.07** | 195.93 | **87.61** | 203.24 | **111.27** | 199.47 |
| Monthly | MAPE | **177.76** | 322.39 | **172.49** | 347.74 | **185.69** | 329.02 |
| | MdAPE | **94.64** | 234.67 | **86.38** | 250.38 | **82.43** | 225.81 |
| Weekly | MAPE | **172.85** | 324.92 | **169.77** | 353.54 | **179.21** | 341.52 |
| | MdAPE | **90.62** | 232.33 | **83.59** | 244.65 | **80.21** | 237.42 |
| **Lead-Time Demand (Demand Only)** | | | | | | | |
| Quarterly | MAPE | 262.17 | **224.22** | 249.25 | **210.09** | 276.26 | **252.98** |
| | MdAPE | 181.87 | **136.45** | 169.92 | **122.22** | 191.67 | **159.61** |
| Monthly | MAPE | 277.76 | **232.87** | 248.35 | **216.00** | 285.49 | **254.89** |
| | MdAPE | 188.22 | **145.09** | 164.78 | **129.39** | 175.46 | **138.43** |
| Weekly | MAPE | 277.05 | **249.87** | 246.74 | **221.37** | 262.86 | **231.99** |
| | MdAPE | 187.05 | **153.71** | 162.43 | **123.67** | 155.47 | **113.82** |

## Appendix K - Effect of Smoothing Parameters by Demand Pattern

Optimal smoothing constant values by demand pattern are presented in this appendix, firstly for exponential smoothing and then for Croston's method. Using MAPE as the performance measure, the optimal smoothing constant values by demand pattern are compared with the optimal values as a whole. The optimal values were produced from a hold-out sample of 500 line items composed of equally represented demand patterns.

Optimal smoothing constant values for exponential smoothing by demand pattern are presented in Table K.1. MAPE was calculated for increments of 0.01 in each case.

**Table K.1: Optimal Smoothing Constant by Demand Pattern - Expon. Smoothing.**

| Demand Aggregation | Demand Pattern | Overall | | Demand Pattern | | MAPE Improvement Percentage (a to b) |
|---|---|---|---|---|---|---|
| | | Smooth Const. | MAPE (a) | Smooth Const. | MAPE (b) | |
| **One-Period Ahead Demand (All Periods)** | | | | | | |
| Quarterly | Smooth | 0.18 | 82.39 | 0.22 | 82.24 | 0.18% |
| | Irregular | 0.18 | 229.69 | 0.23 | 228.77 | 0.40% |
| | Slow-moving | 0.18 | 68.46 | 0.01 | 60.07 | 12.26% |
| | Mildly Erratic | 0.18 | 93.08 | 0.07 | 89.67 | 3.66% |
| | Highly Erratic | 0.18 | 115.83 | 0.74 | 106.97 | 7.65% |
| | **Overall** | **-** | **118.71** | **-** | **114.45** | **3.59%** |
| Monthly | Smooth | 0.05 | 115.78 | 0.14 | 111.50 | 3.70% |
| | Irregular | 0.05 | 134.12 | 0.08 | 132.38 | 1.30% |
| | Slow-moving | 0.05 | 83.33 | 0.01 | 75.27 | 9.67% |
| | Mildly Erratic | 0.05 | 84.53 | 0.01 | 80.79 | 4.42% |
| | Highly Erratic | 0.05 | 97.83 | 0.02 | 96.26 | 1.60% |
| | **Overall** | **-** | **103.20** | **-** | **99.71** | **3.38%** |

| Demand Aggregation | Demand Pattern | Overall | | Demand Pattern | | MAPE Improvement Percentage (a to b) |
|---|---|---|---|---|---|---|
| | | Smooth Const. | MAPE (a) | Smooth Const. | MAPE (b) | |
| **One-Period Ahead Demand (All Periods) - continued** | | | | | | |
| Weekly | **Smooth** | 0.01 | 103.52 | 0.03 | 102.54 | 0.95% |
| | **Irregular** | 0.01 | 89.51 | 0.02 | 89.15 | 0.40% |
| | **Slow-moving** | 0.01 | 95.34 | 0.01 | 95.34 | 0.00% |
| | **Mildly Erratic** | 0.01 | 93.61 | 0.01 | 93.61 | 0.00% |
| | **Highly Erratic** | 0.01 | 92.80 | 0.01 | 92.80 | 0.00% |
| | **Overall** | **-** | **94.96** | **-** | **94.69** | **0.28%** |
| **Lead-Time Demand (All periods)** | | | | | | |
| Quarterly | **Smooth** | 0.43 | 70.37 | 0.29 | 68.86 | 5.15% |
| | **Irregular** | 0.43 | 190.69 | 0.56 | 188.74 | 1.02% |
| | **Slow-moving** | 0.43 | 101.23 | 0.26 | 94.29 | 6.86% |
| | **Mildly Erratic** | 0.43 | 191.66 | 0.66 | 190.13 | 0.80% |
| | **Highly Erratic** | 0.43 | 224.53 | 0.50 | 223.46 | 0.48% |
| | **Overall** | **-** | **155.71** | **-** | **153.14** | **1.65%** |
| Monthly | **Smooth** | 0.16 | 68.86 | 0.11 | 67.74 | 1.63% |
| | **Irregular** | 0.16 | 198.55 | 0.20 | 197.62 | 0.47% |
| | **Slow-moving** | 0.16 | 99.87 | 0.09 | 94.58 | 5.30% |
| | **Mildly Erratic** | 0.16 | 197.39 | 0.18 | 197.27 | 0.06% |
| | **Highly Erratic** | 0.16 | 227.98 | 0.20 | 226.80 | 0.52% |
| | **Overall** | **-** | **158.65** | **-** | **156.93** | **1.08%** |
| Weekly | **Smooth** | 0.04 | 71.11 | 0.03 | 69.66 | 2.04% |
| | **Irregular** | 0.04 | 195.64 | 0.05 | 194.72 | 0.47% |
| | **Slow-moving** | 0.04 | 102.37 | 0.02 | 95.43 | 6.78% |
| | **Mildly Erratic** | 0.04 | 187.59 | 0.05 | 187.57 | 0.01% |
| | **Highly Erratic** | 0.04 | 225.28 | 0.05 | 223.90 | 0.61% |
| | **Overall** | **-** | **156.40** | **-** | **154.26** | **1.37%** |
| **Lead-Time Demand (Demand Only)** | | | | | | |
| Quarterly | **Smooth** | 0.19 | 83.27 | 0.22 | 83.05 | 0.26% |
| | **Irregular** | 0.19 | 232.33 | 0.27 | 229.53 | 1.21% |
| | **Slow-moving** | 0.19 | 127.18 | 0.15 | 125.26 | 1.51% |
| | **Mildly Erratic** | 0.19 | 272.11 | 0.13 | 267.08 | 1.85% |
| | **Highly Erratic** | 0.19 | 361.84 | 0.22 | 360.75 | 0.30% |
| | **Overall** | **-** | **208.94** | **-** | **206.81** | **1.02%** |

| Demand Aggregation | Demand Pattern | Overall | | Demand Pattern | | MAPE Improvement Percentage (a to b) |
|---|---|---|---|---|---|---|
| | | Smooth Const. | MAPE (a) | Smooth Const. | MAPE (b) | |
| **Lead-Time Demand (Demand Only) - continued** | | | | | | |
| Monthly | **Smooth** | 0.06 | 86.10 | 0.06 | 86.10 | 0.00% |
| | **Irregular** | 0.06 | 244.50 | 0.06 | 244.50 | 0.00% |
| | **Slow-moving** | 0.06 | 131.73 | 0.05 | 131.61 | 0.09% |
| | **Mildly Erratic** | 0.06 | 331.63 | 0.05 | 330.19 | 0.43% |
| | **Highly Erratic** | 0.06 | 369.67 | 0.07 | 367.27 | 0.65% |
| | **Overall** | **-** | **225.48** | **-** | **224.77** | **0.31%** |
| Weekly | **Smooth** | 0.01 | 83.87 | 0.01 | 83.87 | 0.00% |
| | **Irregular** | 0.01 | 226.87 | 0.01 | 226.87 | 0.00% |
| | **Slow-moving** | 0.01 | 133.11 | 0.01 | 133.11 | 0.00% |
| | **Mildly Erratic** | 0.01 | 289.64 | 0.01 | 289.64 | 0.00% |
| | **Highly Erratic** | 0.01 | 377.90 | 0.02 | 366.16 | 3.11% |
| | **Overall** | **-** | **216.34** | **-** | **214.18** | **1.00%** |

Optimal smoothing constant values for Croston's method by demand pattern are presented in Table K.2. MAPE was calculated for increments of 0.1 on this occasion.

**Table K.2: Optimal Smoothing Constants by Demand Pattern - Croston's Method.**

| Demand Pattern | Overall | | | Demand Pattern | | | MAPE Improvement Percentage (a to b) |
|---|---|---|---|---|---|---|---|
| | Smooth Const's | | MAPE (a) | Smooth Const's | | MAPE (b) | |
| | Demand Size | Demand Interval | | Demand Size | Demand Interval | | |
| **One-Period Ahead Demand (All Periods)** | | | | | | | |
| **Quarterly** | | | | | | | |
| **Smooth** | 0.4 | 0.3 | 81.90 | 0.4 | 0.2 | 81.11 | 0.96% |
| **Irregular** | 0.4 | 0.3 | 236.48 | 0.3 | 0.2 | 235.33 | 0.49% |
| **Slow-moving** | 0.4 | 0.3 | 56.02 | 0.9 | 0.0 | 50.15 | 10.48% |
| **Mildly Erratic** | 0.4 | 0.3 | 100.14 | 0.0 | 0.5 | 94.76 | 5.37% |
| **Highly Erratic** | 0.4 | 0.3 | 131.50 | 0.9 | 0.3 | 126.91 | 3.49% |
| **Overall** | **-** | **-** | **122.11** | **-** | **-** | **118.66** | **2.87%** |

| Demand Pattern | Overall | | | Demand Pattern | | | MAPE Improvement Percentage (a to b) |
|---|---|---|---|---|---|---|---|
| | Smooth Const's | | MAPE (a) | Smooth Const's | | MAPE (b) | |
| | Demand Size | Demand Interval | | Demand Size | Demand Interval | | |
| **One-Period Ahead Demand (All Periods) - continued** | | | | | | | |
| **Monthly** | | | | | | | |
| **Smooth** | 0.2 | 0.1 | 105.77 | 0.2 | 0.0 | 105.18 | 0.56% |
| **Irregular** | 0.2 | 0.1 | 128.42 | 0.2 | 0.1 | 128.42 | 0.00% |
| **Slow-moving** | 0.2 | 0.1 | 71.24 | 0.5 | 0.0 | 64.20 | 9.88% |
| **Mildly Erratic** | 0.2 | 0.1 | 75.85 | 0.1 | 0.1 | 75.45 | 0.53% |
| **Highly Erratic** | 0.2 | 0.1 | 95.77 | 0.0 | 0.2 | 93.40 | 2.47% |
| **Overall** | **-** | **-** | **95.45** | **-** | **-** | **93.45** | **2.16%** |
| **Weekly** | | | | | | | |
| **Smooth** | 0.1 | 0.0 | 102.98 | 0.1 | 0.1 | 101.44 | 1.50% |
| **Irregular** | 0.1 | 0.0 | 87.16 | 0.1 | 0.1 | 86.75 | 0.47% |
| **Slow-moving** | 0.1 | 0.0 | 88.49 | 0.1 | 1.0 | 87.82 | 0.76% |
| **Mildly Erratic** | 0.1 | 0.0 | 86.33 | 0.1 | 0.0 | 86.33 | 0.00% |
| **Highly Erratic** | 0.1 | 0.0 | 86.03 | 0.0 | 0.0 | 85.98 | 0.06% |
| **Overall** | **-** | **-** | **89.86** | **-** | **-** | **89.67** | **0.59%** |
| **Lead-Time Demand (All Periods)** | | | | | | | |
| **Quarterly** | | | | | | | |
| **Smooth** | 0.9 | 0.4 | 87.45 | 0.3 | 0.4 | 80.05 | 8.46% |
| **Irregular** | 0.9 | 0.4 | 216.10 | 0.9 | 0.4 | 216.10 | 0.00% |
| **Slow-moving** | 0.9 | 0.4 | 199.70 | 0.8 | 0.4 | 199.62 | 0.04% |
| **Mildly Erratic** | 0.9 | 0.4 | 388.19 | 0.0 | 0.5 | 356.94 | 8.05% |
| **Highly Erratic** | 0.9 | 0.4 | 374.50 | 1.0 | 0.4 | 373.38 | 0.30% |
| **Overall** | **-** | **-** | **252.56** | **-** | **-** | **244.61** | **3.16%** |
| **Monthly** | | | | | | | |
| **Smooth** | 0.1 | 0.3 | 83.23 | 0.2 | 0.3 | 83.11 | 0.14% |
| **Irregular** | 0.1 | 0.3 | 241.59 | 0.7 | 0.3 | 240.83 | 0.31% |
| **Slow-moving** | 0.1 | 0.3 | 212.32 | 0.5 | 0.4 | 205.32 | 3.30% |
| **Mildly Erratic** | 0.1 | 0.3 | 375.90 | 0.0 | 0.4 | 352.08 | 6.34% |
| **Highly Erratic** | 0.1 | 0.3 | 449.98 | 0.3 | 0.3 | 436.74 | 2.94% |
| **Overall** | **-** | **-** | **272.31** | **-** | **-** | **263.73** | **3.30%** |

| Demand Pattern | Overall | | | Demand Pattern | | | MAPE Improvement Percentage (a to b) |
|---|---|---|---|---|---|---|---|
| | Smooth Const's | | MAPE (a) | Smooth Const's | | MAPE (b) | |
| | Demand Size | Demand Interval | | Demand Size | Demand Interval | | |
| **Lead-Time Demand (All Periods) - continued** | | | | | | | |
| **Weekly** | | | | | | | |
| **Smooth** | 0.0 | 0.3 | 99.23 | 0.1 | 0.2 | 88.40 | 10.91% |
| **Irregular** | 0.0 | 0.3 | 246.51 | 0.1 | 0.2 | 234.62 | 4.82% |
| **Slow-moving** | 0.0 | 0.3 | 219.38 | 0.6 | 0.3 | 207.28 | 5.52% |
| **Mildly Erratic** | 0.0 | 0.3 | 340.73 | 0.0 | 0.4 | 340.32 | 0.12% |
| **Highly Erratic** | 0.0 | 0.3 | 422.40 | 0.2 | 0.3 | 417.85 | 1.08% |
| **Overall** | **-** | **-** | **263.53** | **-** | **-** | **257.69** | **3.00%** |
| **Lead-Time Demand (Demand Only)** | | | | | | | |
| **Quarterly** | | | | | | | |
| **Smooth** | 0.9 | 0.5 | 84.97 | 0.4 | 0.4 | 78.48 | 7.64% |
| **Irregular** | 0.9 | 0.5 | 205.52 | 0.9 | 0.3 | 202.62 | 1.41% |
| **Slow-moving** | 0.9 | 0.5 | 102.78 | 1.0 | 0.4 | 102.29 | 0.48% |
| **Mildly Erratic** | 0.9 | 0.5 | 232.72 | 0.8 | 0.5 | 232.48 | 0.10% |
| **Highly Erratic** | 0.9 | 0.5 | 266.59 | 1.0 | 0.5 | 261.65 | 1.85% |
| **Overall** | **-** | **-** | **174.56** | **-** | **-** | **171.60** | **1.84%** |
| **Monthly** | | | | | | | |
| **Smooth** | 0.5 | 0.4 | 88.66 | 0.2 | 0.3 | 82.13 | 7.37% |
| **Irregular** | 0.5 | 0.4 | 228.48 | 0.1 | 0.2 | 221.40 | 3.10% |
| **Slow-moving** | 0.5 | 0.4 | 108.44 | 1.0 | 0.4 | 102.43 | 5.54% |
| **Mildly Erratic** | 0.5 | 0.4 | 281.86 | 0.0 | 0.5 | 262.94 | 6.71% |
| **Highly Erratic** | 0.5 | 0.4 | 318.42 | 0.5 | 0.4 | 318.42 | 0.00% |
| **Overall** | **-** | **-** | **199.67** | **-** | **-** | **192.60** | **3.90%** |
| **Weekly** | | | | | | | |
| **Smooth** | 0.6 | 0.3 | 96.70 | 0.1 | 0.2 | 82.51 | 14.67% |
| **Irregular** | 0.6 | 0.3 | 219.00 | 0.1 | 0.2 | 199.31 | 8.99% |
| **Slow-moving** | 0.6 | 0.3 | 112.57 | 1.0 | 0.3 | 111.19 | 1.23% |
| **Mildly Erratic** | 0.6 | 0.3 | 239.70 | 0.8 | 0.4 | 232.41 | 3.04% |
| **Highly Erratic** | 0.6 | 0.3 | 298.72 | 1.0 | 0.3 | 290.32 | 2.81% |
| **Overall** | **-** | **-** | **190.16** | **-** | **-** | **179.38** | **5.67%** |

# Appendix L - Best Forecasting Methods According to MdAPE

Table L.1 provides a summary of the best forecasting methods by demand pattern and type of forecast according to the median absolute percentage error (MdAPE). These results were generated from a sample of 18,750 line items with equal representation by demand pattern.

**Table L.1: Best Forecasting Methods by Demand Pattern (Using MdAPE).**

| Demand Aggreg'n | Demand Pattern | Type of Forecast | | | | | |
|---|---|---|---|---|---|---|---|
| | | One-Period Ahead Demand - All Periods | | Lead-Time Demand - All Periods | | Lead-Time Demand - Demand Only | |
| | | Method | MdAPE | Method | MdAPE | Method | MdAPE |
| Quarterly | **Smooth** | Approx | 47.11 | ES | 52.15 | Approx | 53.76 |
| | **Irregular** | Approx | 71.39 | MA | 81.55 | Approx | 76.25 |
| | **Slow-moving** | Revised | 49.18 | ES | 82.73 | Approx | 65.71 |
| | **Mildly Erratic** | ES | 86.05 | ES | 116.89 | Approx | 151.48 |
| | **Highly Erratic** | ES | 82.18 | ES | 121.35 | Approx | 135.86 |
| | **Overall** | **Approx** | **70.31** | **ES** | **91.02** | **Approx** | **94.13** |
| Monthly | **Smooth** | Revised | 53.45 | ES | 51.22 | Approx | 51.32 |
| | **Irregular** | Approx | 70.79 | MA | 78.25 | Approx | 79.88 |
| | **Slow-moving** | Revised | 65.71 | ES | 82.58 | Approx | 78.63 |
| | **Mildly Erratic** | Approx | 81.01 | ES | 114.55 | Approx | 185.71 |
| | **Highly Erratic** | Approx | 74.64 | ES | 106.11 | Approx | 148.72 |
| | **Overall** | **Croston** | **70.09** | **ES** | **86.99** | **Approx** | **105.60** |
| Weekly | **Smooth** | Revised | 72.81 | ES | 50.01 | Approx | 52.23 |
| | **Irregular** | Revised | 76.45 | ES | 75.63 | Bias Red | 78.23 |
| | **Slow-moving** | Revised | 88.86 | ES | 80.08 | Bias Red | 71.67 |
| | **Mildly Erratic** | Revised | 86.43 | ES | 108.87 | Bias Red | 166.37 |
| | **Highly Erratic** | Revised | 83.88 | ES | 105.28 | Bias Red | 128.88 |
| | **Overall** | **Revised** | **81.68** | **ES** | **84.08** | **Bias Red** | **97.20** |

## Appendix M - Average Implied Stock-Holdings by Demand Pattern

The tables in this appendix present the calculated implied stock-holdings from 11,203 line items in total. Results are compared between exponential smoothing, Croston's method, the previous year average method and the approximation method, with updating every period as well as reordering every quarter. Stock-holding measurements are made every period.

In the tables which follow, a nomenclature is used to signify the various demand aggregation, update/reorder interval and measurement interval combinations. These take the form (*D*,*U*,*M*) where *D* signifies the demand aggregation, *U* signifies the update interval and *M* signifies the measurement interval. Thus, (M,M,Q) refers to a monthly demand aggregation with a monthly update interval and a quarterly measurement interval.

**Table M.1: Average Implied Stock-Holdings - Smooth Demand.**

| Demand Aggregation | Nomen-clature | Expon'l Smooth. | Croston's Method | Prev Year Average | Approxi-mation | Friedman Statistic by Method |
|---|---|---|---|---|---|---|
| **Quarterly** | (Q,Q,Q) | 68.81 | 68.20 | 78.68 | 68.13 | 1,183.22 |
| **Monthly** | (M,M,M) | 68.66 | 66.42 | 74.47 | 66.20 | 933.48 |
| **Weekly** | (W,W,W) | 66.61 | 65.48 | 72.73 | 65.33 | 713.99 |
| **Friedman Statistic by Aggregation** | | 233.50 | 31.38 | 173.16 | 27.11 | - |
| **Quarterly** | (Q,Q,Q) | 68.81 | 68.20 | 78.68 | 68.13 | 1,183.22 |
| **Monthly** | (M,Q,M) | 74.10 | 71.89 | 78.43 | 71.66 | 847.43 |
| **Weekly** | (W,Q,W) | 74.86 | 73.46 | 78.61 | 73.31 | 663.68 |
| **Friedman Statistic by Aggregation** | | 1536.19 | 484.85 | 790.22 | 492.43 | - |

**Table M.2: Average Implied Stock-Holdings - Irregular Demand.**

| Demand Aggregation | Nomen-clature | Expon'l Smooth. | Croston's Method | Prev Year Average | Approxi-mation | Friedman Statistic by Method |
|---|---|---|---|---|---|---|
| **Quarterly** | (Q,Q,Q) | 128.72 | 127.59 | 154.42 | 127.38 | 978.66 |
| **Monthly** | (M,M,M) | 135.73 | 130.49 | 150.79 | 130.19 | 845.90 |
| **Weekly** | (W,W,W) | 135.59 | 127.46 | 146.28 | 127.34 | 989.31 |
| **Friedman Statistic by Aggregation** | | 746.69 | 151.56 | 32.18 | 149.20 | - |
| **Quarterly** | (Q,Q,Q) | 128.72 | 127.59 | 154.42 | 127.38 | 978.66 |
| **Monthly** | (M,Q,M) | 137.55 | 132.30 | 153.64 | 131.95 | 823.59 |
| **Weekly** | (W,Q,W) | 138.86 | 130.36 | 153.96 | 130.23 | 960.22 |
| **Friedman Statistic by Aggregation** | | 1,487.68 | 217.29 | 425.91 | 229.94 | - |

**Table M.3: Average Implied Stock-Holdings - Slow-Moving Demand.**

| Demand Aggregation | Nomen-clature | Expon'l Smooth. | Croston's Method | Prev Year Average | Approxi-mation | Friedman Statistic by Method |
|---|---|---|---|---|---|---|
| **Quarterly** | (Q,Q,Q) | 5.76 | 5.69 | 6.65 | 5.66 | 92.78 |
| **Monthly** | (M,M,M) | 6.06 | 5.63 | 6.49 | 5.62 | 131.71 |
| **Weekly** | (W,W,W) | 6.04 | 5.53 | 6.44 | 5.53 | 132.38 |
| **Friedman Statistic by Aggregation** | | 527.11 | 152.86 | 770.89 | 149.62 | - |
| **Quarterly** | (Q,Q,Q) | 5.76 | 5.69 | 6.65 | 5.66 | 92.78 |
| **Monthly** | (M,Q,M) | 6.23 | 5.78 | 6.64 | 5.76 | 98.64 |
| **Weekly** | (W,Q,W) | 6.28 | 5.71 | 6.64 | 5.71 | 163.44 |
| **Friedman Statistic by Aggregation** | | 325.06 | 43.85 | 260.14 | 43.88 | - |

**Table M.4: Average Implied Stock-Holdings - Mildly Erratic Demand.**

| Demand Aggregation | Nomen-clature | Expon'l Smooth. | Croston's Method | Prev Year Average | Approxi-mation | Friedman Statistic by Method |
|---|---|---|---|---|---|---|
| **Quarterly** | (Q,Q,Q) | 23.75 | 24.01 | 28.88 | 23.93 | 158.96 |
| **Monthly** | (M,M,M) | 26.14 | 22.94 | 28.83 | 22.89 | 238.42 |
| **Weekly** | (W,W,W) | 26.01 | 22.18 | 28.56 | 22.18 | 271.39 |
| **Friedman Statistic by Aggregation** | | 811.92 | 141.31 | 573.92 | 151.20 | - |
| **Quarterly** | (Q,Q,Q) | 23.75 | 24.01 | 28.88 | 23.93 | 158.96 |
| **Monthly** | (M,Q,M) | 26.24 | 22.90 | 28.83 | 22.85 | 247.11 |
| **Weekly** | (W,Q,W) | 26.56 | 22.26 | 28.91 | 22.26 | 308.68 |
| **Friedman Statistic by Aggregation** | | 444.84 | 2.06 | 162.03 | 1.99 | - |

**Table M.5: Average Implied Stock-Holdings - Highly Erratic Demand.**

| Demand Aggregation | Nomen-clature | Expon'l Smooth. | Croston's Method | Prev Year Average | Approxi-mation | Friedman Statistic by Method |
|---|---|---|---|---|---|---|
| **Quarterly** | (Q,Q,Q) | 24.51 | 23.48 | 28.64 | 23.46 | 224.72 |
| **Monthly** | (M,M,M) | 25.55 | 22.64 | 27.81 | 22.60 | 326.33 |
| **Weekly** | (W,W,W) | 25.61 | 22.28 | 27.65 | 22.27 | 350.83 |
| **Friedman Statistic by Aggregation** | | 573.62 | 111.05 | 401.78 | 109.95 | - |
| **Quarterly** | (Q,Q,Q) | 24.51 | 23.48 | 28.64 | 23.46 | 224.72 |
| **Monthly** | (M,Q,M) | 26.37 | 23.15 | 28.65 | 23.14 | 325.93 |
| **Weekly** | (W,Q,W) | 26.47 | 22.69 | 28.58 | 22.69 | 350.07 |
| **Friedman Statistic by Aggregation** | | 436.57 | 0.00 | 241.80 | 0.41 | - |

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| Page for Printing | Number on Page | Item | Page for Printing | Number on Page | Item | Page for Printing | Number on Page | Item |
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| 36 | 19 | Figure 2.1 | 138 | 121 | Figure 5.11 | 208 | 191 | Figure 7.11 |
| 69 | 52 | Figure 4.1 | 139 | 122 | Figure 5.12 | 242 | 225 | Figure 8.1 |
| 70 | 53 | Figure 4.2 | 139 | 122 | Figure 5.13 | 243 | 226 | Figure 8.2 |
| 71 | 54 | Figure 4.3 | 140 | 123 | Figure 5.14 | 244 | 227 | Figure 8.3 |
| 71 | 54 | Figure 4.4 | 147 | 130 | Figure 6.1 | 245 | 228 | Figure 8.4 |
| 72 | 55 | Figure 4.5 | 148 | 131 | Figure 6.2 | 245 | 228 | Figure 8.5 |
| 73 | 56 | Figure 4.6 | 148 | 131 | End of Figure 6.2 | 246 | 229 | Figure 8.6 |
| 74 | 57 | Figure 4.7 | 150 | 133 | Figure 6.3 | 250 | 233 | Figure 9.1 |
| 75 | 58 | Figure 4.8 | 150 | 133 | Figure 6.4 | 256 | 239 | Figure 9.2 |
| 77 | 60 | Figure 4.9 | 153 | 136 | Figure 6.5 | 256 | 239 | Figure 9.3 |
| 80 | 63 | Figure 4.10 | 155 | 138 | Figure 6.6 | 259 | 242 | Figure 9.4 |
| 82 | 65 | Figure 4.11 | 155 | 138 | Figure 6.7 | 261 | 244 | Figure 9.5 |
| 83 | 66 | Figure 4.12 | 156 | 139 | Figure 6.8 | 262 | 245 | Figure 9.6 |
| 85 | 68 | Figure 4.13 | 157 | 140 | Figure 6.9 | 264 | 247 | Figure 9.7 |
| 86 | 69 | Figure 4.14 | 159 | 142 | Figure 6.10 | 265 | 248 | Figure 9.8 |
| 87 | 70 | Figure 4.15 | 160 | 143 | Figure 6.11 | 266 | 249 | Figure 9.9 |
| 88 | 71 | Figure 4.16 | 161 | 144 | Figure 6.12 | 267 | 250 | Figure 9.10 |
| 90 | 73 | Figure 4.17 | 164 | 147 | Figure 6.13 | 269 | 252 | Figure 9.11 |
| 91 | 74 | Figure 4.18 | 171 | 154 | Figure 6.14 | 270 | 253 | Figure 9.12 |
| 92 | 75 | Figure 4.19 | 172 | 155 | Figure 6.15 | 274 | 257 | Figure 9.13 |
| 93 | 76 | Figure 4.20 | 172 | 155 | End of Figure 6.15 | 280 | 263 | Figure 9.14 |
| 94 | 77 | Figure 4.21 | 180 | 163 | Table 7.2 | 333 | 316 | Graph |
| 99 | 82 | Figure 5.1 | 193 | 176 | Figure 7.1 | 336 | 319 | Figure C.1 |
| 105 | 88 | Figure 5.2 | 193 | 176 | Figure 7.2 | 337 | 320 | Figure C.2 |
| 121 | 104 | Figure 5.3 | 194 | 177 | Figure 7.3 | 338 | 321 | Figure C.3 |
| 128 | 111 | Figure 5.4 | 195 | 178 | Figure 7.4 | 340 | 323 | High-Low |
| 132 | 115 | Figure 5.5 | 196 | 179 | Figure 7.5 | 342 | 325 | Figure C.4 |
| 133 | 116 | Figure 5.6 | 201 | 184 | Figure 7.6 | | | |
| 134 | 117 | Figure 5.7 | 201 | 184 | Figure 7.7 | | | |
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